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Numerical Methods for Partial Differential Equations in Financial Derivatives Pricing: A Comparative Review from Classical Methods to Deep Learning

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Abstract. Pricing of financial derivatives can generally be reduced to solving partial differential equations (PDEs). Starting from the classical Black-Scholes framework, pricing models that better reflect market characteristics, such as stochastic volatility and jump-diffusion models, have been developed, and classical numerical methods, such as the finite difference method, have long dominated their solution. However, as the dimensionality of pricing problems rises, grid-based methods face a sharp growth in computational cost, the so-called curse of dimensionality, while deep learning methods such as physics-informed neural networks (PINNs) have recently offered new approaches to high-dimensional PDEs. This paper systematically reviews and compares classical numerical methods (finite difference, finite element and spectral methods) with deep learning methods in terms of convergence, computational efficiency and applicability, and discusses their suitability for free-boundary problems, such as the early exercise of American options, and for high-dimensional multi-asset pricing. A literature review combined with comparative analysis was adopted, and the methods were compared across numerical stability, computational complexity and dimensional adaptability. The analysis shows that classical methods rest on relatively mature convergence theory and produce stable results for low-dimensional problems but carry a heavy computational burden in high dimensions, whereas deep learning methods are more efficient in high-dimensional settings, although their training stability and theoretical convergence guarantees remain to be improved. On this basis, the paper offers a reference for selecting numerical methods in different pricing scenarios and identifies hybrid frameworks combining the strengths of both categories as a promising direction for future research.

Keywords: Partial differential equations, Financial derivatives pricing, Numerical methods, Physics-informed neural networks, Deep learning

1. Introduction

The sound pricing of financial derivatives is one of the core problems in financial engineering. Since the establishment of the Black-Scholes model, complex pricing problems have generally been reduced to solving partial differential equations (PDEs) or their variational inequalities [1]. As

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market features have become more complex, pricing models have developed from single-factor modifications, such as stochastic volatility and jump processes [2, 3], to composite models that combine long memory with multiple factors [4-6]. The finite difference method has long dominated low-dimensional pricing problems [7, 8] and has gradually been extended to finite element methods and high-order collocation techniques for more complex geometries and boundary conditions [9-12]. For the free-boundary problem posed by the early exercise of American options, research has advanced from basic boundary-tracking algorithms [1] to solutions under complex contract terms and jump-diffusion dynamics [13-15]. In recent years, deep learning methods have evolved from direct fitting of option prices [16, 17] to physics-informed neural networks (PINNs) and backward stochastic differential equation (BSDE) architectures that incorporate PDE mechanisms, offering a new way to address the "curse of dimensionality" [18-20]. However, existing studies mostly focus on improving a single method in specific scenarios, and few systematically compare classical numerical methods with deep learning methods in terms of convergence, computational efficiency and free-boundary handling.

To fill this gap, this study takes PDE-based pricing models as its subject and, combining a literature review with comparative analysis, examines the applicability and performance of both categories of methods in different pricing scenarios. The study will reveal their respective strengths and limitations, provide a theoretical basis for algorithm selection in complex option pricing, and offer a foundation for future hybrid solution frameworks integrating the advantages of both [18-20].

2. Theoretical foundations of PDE-based pricing

2.1. Black-Scholes-Merton framework and its assumptions

The Black-Scholes-Merton (BSM) model is the foundation of derivatives pricing theory. It assumes that the price S_t of the underlying asset follows a geometric Brownian motion (GBM):

$$dS_t=\mu S_t dt+\sigma S_t dW_t \tag{1}$$

where μ is the expected return, σ is the constant volatility and W_t is a standard Brownian motion. Within this framework, the no-arbitrage principle and a dynamic Delta hedging strategy imply that the derivative price $V(S,t)$ satisfies the classical second-order parabolic partial differential equation (PDE):

$$\frac{\partial V}{\partial t}+rS\frac{\partial V}{\partial S}+\frac{1}{2}\sigma^2S^2\frac{\partial^2V}{\partial S^2}-rV=0 \tag{2}$$

where r is the risk-free rate. With the corresponding terminal and boundary conditions, closed-form solutions for European call and put options can be derived. However, the assumptions of constant volatility and continuous price movements deviate markedly from the volatility smile/skew and the fat-tailed returns observed in real financial markets [5]. This limitation has pushed subsequent models to expand in two directions: introducing price jumps and long-memory processes to capture discontinuous shocks, and randomizing volatility and interest rates to fit complex dynamics.

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2.2. Extensions with jump-diffusion models and fractional processes

To address sudden market shocks that cause discontinuous jumps in asset prices, jump processes have been introduced into the traditional PDE framework. Based on the Kou jump-diffusion model, Gan derived the partial integro-differential equation (PIDE) for European options under jump diffusion [2]:

$$\frac{\partial v}{\partial \tau}-\frac{1}{2}\sigma^2x^2\frac{\partial^2 v}{\partial x^2}-(r-\lambda\zeta)x\frac{\partial v}{\partial x}+(r+\lambda)v-\lambda\int_0^{\infty}v(xy)f(y)dy=0 \tag{3}$$

where λ is the jump intensity and $f(y)$ is the probability density function of the jump size, and the integral term expresses the non-local effect of jumps [2]. Meanwhile, Geng and Zhou introduced fractional Brownian motion (with Hurst exponent H) into the jump-diffusion framework, proposed a mixed fractional jump-diffusion model, and derived a pricing solution for path-dependent Asian options [4]. This evolution shows that pricing dynamics have moved from describing a single non-classical feature to the coupled modeling of several complex stochastic mechanisms, namely long memory and integral jumps.

2.3. Stochastic volatility models: the Heston model and its extensions

Another mainstream extension treats volatility itself as a stochastic process [3]. Within the classical Heston stochastic volatility framework, Deng studied the pricing and Delta hedging of European chooser options, where the variance v_t follows a CIR mean-reverting process [3]:

$$dv_t=\kappa(\theta-v_t)dt+\xi\sqrt{v_t}dW_t^v \tag{4}$$

A correlation coefficient ρ exists between the Brownian motions driving the asset price and the volatility. Because single-factor stochastic volatility models cannot fit steep and smooth implied volatility surfaces at the same time, Wang et al. combined the square-root term of the Heston model with the 3/2 model and proposed the 4/2 stochastic volatility model, whose volatility term takes the form $(c_1\sqrt{v_t}+c_2/\sqrt{v_t})$, markedly improving parameter fitting accuracy and surface characterization [5]. Furthermore, Wu et al. constructed the Heston-HW-MEJ composite model, which integrates the Hull-White stochastic interest rate process, Heston stochastic volatility and mixed-exponential jumps (MEJ) [6], with the following system of stochastic differential equations:

$$\begin{cases} dx_t=\left(r_t-\lambda h-\frac{v_t}{2}\right)dt+\sqrt{v_t}dW_t+Y_tdN_t \\ dv_t=\alpha(\beta-v_t)dt+\sigma_v\sqrt{v_t}dW_t^v \\ dr_t=\kappa(\theta-r_t)dt+\sigma_r dW_t^r \\ Cov\left(dW_t dW_t^v\right)=\rho dt \end{cases} \tag{5}$$

These results fully reflect the theoretical evolution of stochastic volatility models from single-factor CIR modification to the coordinated modeling of multiple high-dimensional factors (interest rate, volatility and jumps).

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3. Numerical solution methods for pricing PDEs

3.1. Finite difference method

Owing to the early exercise feature, the pricing of American options is essentially a free-boundary problem, and the finite difference method (FDM), thanks to its simple implementation and mature theory, has become the earliest and most widely used numerical tool for such problems. Xu systematically compared the stability and convergence of explicit, implicit and Crank-Nicolson schemes for the variational inequality of American put options [7]. For the early exercise boundary, Li and Jin proposed different schemes combining the implicit and explicit methods and determined the optimal exercise boundary $S_f(t)$ by numerical iteration [1]. Zhang examined the theoretical properties of discretization schemes for American option PDEs in terms of truncation error and algorithmic convergence [8].

To improve the accuracy and efficiency of locating the exercise boundary, Zhao et al. proposed a predictor-corrector technique that approaches the optimal exercise boundary at each time step through explicit tracking [13], while Xue et al. established a free-boundary fitting algorithm for American stock options with specific dividend terms [21]. As pricing problems extend to multi-asset settings, the alternating direction implicit (ADI) scheme has been introduced into high-dimensional PDE solving. Zhang et al. constructed a high-dimensional FDM scheme for multi-asset American options, which decomposes the high-dimensional operator into a series of one-dimensional tridiagonal systems and effectively reduces the computational burden caused by increasing grid dimensions [22].

3.2. Finite element and finite volume methods

The finite difference method relies on regular grids and is limited in handling irregular computational domains or path-dependent options, so the finite element method (FEM) and the finite volume method (FVM) have been introduced to broaden the applicability of numerical methods. For the non-local partial integro-differential equation (PIDE) derived from the Merton jump-diffusion model, Gan applied the finite volume method for spatial discretization and built the discrete scheme on flux conservation over control volumes, offering an effective way to handle non-local pricing equations with jump terms [2].

Yu transformed the option pricing PDE into a weak form in a Sobolev space and used the Ritz-Galerkin variational principle and finite element basis functions to approximate option prices over complex domains [9]. On this basis, Song et al. combined a state-variable dimension reduction technique with the finite element method to price American lookback options with path-dependent features [10]. For European down-and-out barrier call options, Wu proved that under the condition $\omega = \frac{\sigma^2 \Delta t}{(\ln u)^2} = 1$ the finite element, finite difference and CRR binomial tree methods are equivalent, and gave an asymptotic expansion of the finite element solution showing that, as the number of time steps $n \to \infty$, it converges to the Black-Scholes analytical solution at order $O(1/\sqrt{n})$, which provides solid support for the theoretical completeness of such methods for options with complex boundaries [11].

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3.3. Spectral and collocation methods

To further improve the convergence speed and smoothness of numerical discretization, some studies have turned to high-order collocation methods. Wu et al. introduced cubic trigonometric B-spline basis functions to build a collocation scheme for the Black-Scholes model [12]. At node x_i, the numerical solution of the option price is expressed as:

$$u(x,t)=\sum_{l=-3}^{n-1}\omega_1(t)\varphi_{3,l}(x) \tag{6}$$

where $\omega_l(t)$ is a time-dependent unknown and $\varphi_{3,l}(x)$ is the cubic trigonometric B-spline basis function. Compared with traditional low-order difference and finite element methods, this method markedly improves the smoothness of spatial discretization and the convergence speed [12].

4. Free-boundary problems of american options

4.1. Formulation of the free-boundary problem and perpetual american options

Unlike European options, American options can be exercised at any time before maturity. The option price $V(S,t)$ must therefore satisfy both the partial differential equation and the constraint that the early exercise payoff is not lower than the holding value, so the problem is, in essence, a linear complementarity problem (LCP) or variational inequality:

$$\begin{cases} LV-rV\le 0 \\ V(S,t)-h(S)\ge 0 \\ (LV-rV)(V(S,t)-h(S))=0 \end{cases} \tag{7}$$

where L is the differential operator with respect to the asset price. The optimal exercise boundary (the free boundary $S_f(t)$) divides the solution domain into a holding region ($V>h(S)$, where the PDE holds with equality) and an exercise region ($V=h(S)$). The free boundary $S_f(t)$ is an unknown function that must be solved together with the option price and satisfies the smooth pasting condition (for American put options, $\frac{\partial V}{\partial S}\Big|_{S=S_f}=-1$). This is the core mathematical difficulty that distinguishes American option pricing from European option pricing.

For the special case of the perpetual American option, whose maturity tends to infinity, the option price no longer depends on time t ($\frac{\partial V}{\partial t}=0$), and the PDE reduces to an ordinary differential equation (ODE) or an integro-differential equation. Under a general jump-diffusion model, Jiang and Luo substituted the variable $x=\ln(S/S_\infty)$ for the logarithmic price, introduced the Green function $G(x,y)$ and multi-order convolution kernels $G^{(k)}(x,y)$, and proved that the optimal exercise boundary S_∞ (i.e., S_f^*) satisfies the following exact explicit expression in the form of an infinite series [15]:

$$S_\infty=\frac{1-\left[F_1'(0)+\sum_{k=1}^{\infty}\lambda^k\int_0^\infty G_x^{(k)}(0,y)F_1(y)dy\right]}{2+\left[F_2'(0)+\sum_{k=1}^{\infty}\lambda^k\int_0^\infty G_x^{(k)}(0,y)F_2(y)dy\right]} \tag{8}$$

where λ is the jump intensity and $F_1(y)$ is the known non-homogeneous term under the corresponding linearized operator. The study further proved that, under the smooth junction

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condition (continuity of the first derivative at the free boundary), the generality of the jump probability density function $p(y)$ does not affect the existence and uniqueness of this explicit solution, which provides an important theoretical benchmark for boundary behavior and numerical pricing of American options under jump risk [15].

4.2. Numerical treatment of the early exercise boundary

For American options with additional contract terms, the geometry and structure of the free boundary become more complicated. Gao and Liang studied continuously paid American installment options and pointed out that, because such options involve continuous premium payments, the holder must make two decisions at every time node: whether to exercise early and whether to pay the next premium [14].

This dual decision mechanism splits the optimal exercise region into several subregions, and the free boundary takes a dual or even composite structure: an upper exercise boundary $S_f(t)$ (triggering early exercise) and a lower abandonment boundary $S_d(t)$ (the option is abandoned when the stock price is so low that continuing to pay premiums is not worthwhile) exist at the same time. The study used the penalty method and an implicit difference scheme to decouple and numerically compute this composite free boundary [14]. Compared with the single exercise boundary of standard American options, such composite boundary problems demand higher grid-capturing accuracy and stability from numerical algorithms, which also reflects the expanding research trend of free-boundary problems as contract terms become more complex.

5. Frontier methods: deep learning for high-dimensional PDEs

5.1. Deep learning methods based on the Feynman-Kac representation and backward stochastic differential equations

When traditional grid-based numerical methods solve high-dimensional derivatives pricing PDEs, the number of grid nodes and the computational complexity grow exponentially with the dimension d of the underlying assets ($O(N^d)$). This "curse of dimensionality" has driven the academic community to explore new solution approaches based on deep learning.

Early studies relied mainly on purely data-driven nonlinear fitting. Sun et al. used deep neural networks to approximate the gradient of the option price with respect to the state variables and solved option pricing under a behavioral asset model that incorporates investor irrationality and herding effects, providing a feasible pricing method for behavioral options, which have no analytical solutions and are constrained by the curse of dimensionality in high dimensions [16].

In contrast to purely data-driven black-box fitting, Teng, building on the Pardoux-Peng theory of backward stochastic differential equations and the nonlinear Feynman-Kac theorem, rigorously transformed the high-dimensional PDE system:

$$\frac{\partial V}{\partial t} + LV - rV = 0 \tag{9}$$

(where L is the diffusion differential operator, $V(t,X_t)$ denotes the option price and r is the risk-free rate) into an equivalent decoupled forward-backward stochastic differential equation (FBSDE) system [19]:

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$$Y_t=\phi(X_T)+\int_t^T g(s,X_s,Y_s,Z_s)ds-\int_t^T Z_s^\top dW_s \tag{10}$$

where $\phi(X_T)$ is the terminal payoff of the option and $g(\cdot)$ is the BSDE generator; the option price is $Y_t=V(t,X_t)$, and the hedging term $Z_t=\nabla V(t,X_t)^\top\sigma(t,X_t)$ is approximated by deep neural networks that parameterize (Y_t,Z_t) at each discrete time step [19]. By minimizing, through stochastic gradient descent (SGD), the joint loss composed of the discrete BSDE path residual and the terminal payoff, the solution of the high-dimensional PDE becomes an end-to-end network training process, which in theory offers a feasible path past the curse of dimensionality faced by traditional numerical methods [19]. Wang further extended this line of research to the deep learning-based numerical computation of backward stochastic systems and its application to derivatives pricing [20].

5.2. Physics-informed neural networks (PINNs)

Beyond the BSDE architecture, physics-informed neural networks (PINNs) offer another mesh-free solution path that embeds the physical and mathematical constraints of financial PDEs directly into the training of the neural network.

Zhang substituted the operator residual of the PDE itself into the loss function of the neural network [18]. The PDE residual operator is defined as:

$$R_{PDE}(\hat{u}_\theta)=D\hat{u}_\theta-f \tag{11}$$

where $\hat{u}_\theta$ is the approximate PDE solution output by the neural network, D is the PDE differential operator and f is the source term on the right-hand side of the equation. PINN sets the optimization objective for the network parameters θ as the minimization of a composite loss function [18]:

$$\begin{aligned}L(\theta;x,t) &= \frac{\lambda_d}{N_d}\sum_{(x,t)\in\Omega_t}\|D(\partial t,\partial x,\partial xx,\ldots)\hat{u}(x,t;\theta)-f(x,t)\|^2 \\ &+\frac{\lambda_b}{N_b}\sum_{(x,t)\in\partial\Omega\times(0,T)}\|B(\partial x,\partial xx,\ldots)\hat{u}(x,t;\theta)-g(x,t)\|^2 \\ &+\frac{\lambda_i}{N_i}\sum_{x\in\Omega}\|\hat{u}(x,0;\theta)-u_0(x)\|^2,\end{aligned} \tag{12}$$

where, the hyperparameters $\lambda_d,\lambda_b,\lambda_i$ are the weights of the PDE residual, boundary condition and initial condition terms, respectively, and N_d,N_b,N_i are the numbers of randomly sampled points used to enforce the equation and the boundary and initial conditions [18]. The advantage of this method is that the model completely abandons the spatial grid partitioning of traditional finite difference or finite element methods and can theoretically adapt naturally to input variables of arbitrary dimension, providing a mesh-free alternative to the Deep BSDE architecture for solving high-dimensional financial pricing equations.

6. Comparative analysis of numerical methods

6.1. Accuracy and convergence

In terms of the completeness of convergence theory, classical numerical methods already show a fairly clear hierarchy. Among finite difference schemes, the explicit scheme is limited by the mesh-

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ratio stability condition (e.g., $\Delta t/\Delta x^2 \leq 1/2$), whereas the implicit and Crank-Nicolson schemes are unconditionally stable and second-order convergent ($O(\Delta t^2+\Delta x^2)$); even so, their truncation errors remain bound to the grid step size, and the convergence rate is strictly limited by the order of the difference scheme itself.

By comparison, the cubic trigonometric B-spline collocation method adopted by Wu et al., as a high-order method, usually achieves a better convergence rate and smoothness at the same grid density thanks to the continuous high-order smoothness of its basis functions between nodes [12], which shows a clear accuracy advantage of high-order collocation methods over traditional low-order difference schemes. In addition, the free-boundary fitting algorithm of Xue et al. for options with specific terms shows that the refinement of boundary treatment also directly affects the accuracy of the overall numerical solution [21]. On the whole, classical methods all possess rigorous error bounds and convergence-order proofs.

6.2. Computational efficiency and the curse of dimensionality

For high-dimensional derivatives pricing, the ADI difference scheme designed by Zhang et al. for multi-asset American options effectively eases the growth of grid size through operator splitting at moderate dimensions (e.g., two to three), but it still relies fundamentally on spatial grid partitioning, so storage and computation continue to grow exponentially with the number of assets ($O(N^d)$) [22], and the curse of dimensionality cannot be fundamentally overcome. Deep learning methods bypass the limitation of grid discretization: the frameworks of Teng and Wang that convert high-dimensional PDEs into BSDE/FBSDE problems through the Feynman-Kac representation [19, 20], and the PINN method of Zhang [18], all replace full grid coverage with training on randomly sampled points in high-dimensional space, so that, in theory, the computational complexity of a single iteration grows polynomially rather than exponentially with the dimension d. The deep neural network pricing algorithm designed by Li for high-dimensional basket options further verifies this high-dimensional efficiency advantage on multi-asset portfolio options [17].

However, the training of deep learning methods relies on non-convex optimization, and their convergence speed and result stability depend heavily on the network architecture and hyperparameter selection, so they do not yet have convergence guarantees as mature as those of classical numerical methods.

6.3. Applicability and difficulty of boundary treatment

In handling the payoff structures and boundary conditions of different option types, the applicability of the methods differs markedly. For European options, whose payoff at maturity is a known piecewise continuous function, all methods can cope well. For American options, which have an unknown optimal exercise boundary, the comparative study of Xu shows that different difference schemes differ significantly in stability when approximating the free boundary [7], and the study of Gao and Liang on installment options further shows that the more complex the boundary structure (e.g., a dual exercise boundary), the more difficult numerical decoupling and tracking become [14].

For barrier options, whose payoff is discontinuous at the barrier, a non-smooth payoff markedly lowers the theoretical convergence order of finite element methods, and PINNs are also prone to vanishing or oscillating gradients when handling the residual loss at non-smooth interfaces. The closed-form solution for perpetual American options obtained by Jiang and Luo under the jump-diffusion framework [15] shows that problems with a theoretical analytical benchmark make it

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easier to verify the approximation accuracy of numerical methods near the boundary, whether classical or deep learning-based.

7. Conclusion

This paper systematically reviewed and compared classical numerical methods and deep learning methods for solving the partial differential equations (PDEs) of financial derivatives pricing. The results show that the two categories are complementary. In low-dimensional scenarios, classical methods (finite difference, finite element and spline collocation) offer mature convergence orders and error control and yield highly stable results. In high-dimensional scenarios, the Feynman-Kac-based backward stochastic differential equation (BSDE) architecture and physics-informed neural networks (PINNs) show a clear efficiency advantage in overcoming the "curse of dimensionality" through mesh-free sampling. For free-boundary problems and non-smooth payoffs, the two categories face different challenges: the accuracy of grid decoupling for classical methods and the stability of non-convex optimization for deep learning methods. These findings answer the core question raised in the introduction: the choice of method should weigh the asset dimension, the boundary complexity and the need for theoretical convergence guarantees.

The limitations of this paper are twofold. First, the comparison synthesizes theoretical results and numerical schemes from the literature and includes no direct numerical experiments under a unified hardware and data setting, so its quantitative depth remains limited. Second, the discussion of the training uncertainty of deep learning methods does not examine network topology, hyperparameter sensitivity or the influence of loss-term weights, which calls for empirical verification. Future research can focus on two directions: a hybrid solution framework combining the theoretical rigor of classical methods with the high-dimensional adaptability of deep learning; and the adaptation of deep learning to the free boundaries of American options and complex barrier options, aiming at new numerical paradigms with both theoretical convergence bounds and high-dimensional generalization, which would support robust and efficient pricing of complex financial derivatives.

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