Proceedings of CONF-APMM 2026 Symposium: Applied Mathematics and Quantitative Modeling
DOI: 10.54254/2753-8818/2026.PU38921

Analysis and Optimization of Flow Characteristics in Hollow Microneedle Lumen Based on Poiseuille's Law

Lu Qian

School of Physics and Electronic Engineering, Yancheng Teachers University, Yancheng, China
q2441567413@163.com

Abstract. Microneedle patches represent an innovative transdermal drug delivery technology that offers painless administration and improved patient compliance. An in-depth understanding of the hydrodynamic characteristics in microneedle channels is crucial for optimizing drug delivery efficiency. This paper explores the flow characteristics of drug solutions within microneedle channels, focusing on the applicability of Poiseuille's law and the influence of geometric parameters on flow rate. Based on Poiseuille's law, an analytical flow model of microneedle array is established in this study. The results show that the inner lumen radius is the most critical parameter (S_R =4), significantly higher than those of pressure and needle length. When R = 35μm, L = 250μm, ΔP = 30kPa, the optimal design can achieve a maximum flow rate of 4741 μL/min. This study provides a theoretical basis and rapid design methodology for geometric parameter optimization of microneedle patches, offering valuable reference for advancing transdermal drug delivery technology.

Keywords: Microneedle, Poiseuille's law, Sensitivity analysis, Response surface optimization

1. Introduction

Transdermal drug delivery systems (TDDS) have evolved significantly from passive patches to microneedles (MNs) to overcome the barrier of the stratum corneum, providing controlled drug release, improved bioavailability, and enhanced patient compliance [1]. Since the first scopolamine patch was approved by the U.S. Food and Drug Administration in 1979, transdermal drug delivery technology has been continuously improved, and macromolecular drugs including mRNA, siRNA, insulin and vaccines have been able to be delivered [1]. Hollow microneedles (HMNs) have attracted wide attention as a potential alternative to traditional hypodermic needles because of their cylindrical lumen for direct administration and less tissue damage than solid microneedles [2].

However, the existing research mainly relies on computational fluid dynamics tools such as COMSOL Multiphysics for lumen flow analysis [3]. Most studies only analyze the geometric parameters such as lumen diameter, needle length, and inlet pressure in isolation, whereas research on the influence of multi-parameter coupling effect on the flow characteristics of the lumen is still limited [3]. Furthermore, the systematic design criteria for optimizing the size of the microneedle lumen under physiological constraints remain insufficient [4].

© 2026 The Authors. This is an open access article distributed under the terms of the Creative Commons Attribution License 4.0 (https://creativecommons.org/licenses/by/4.0/).

DOI: 10.54254/2753-8818/2026.PU38921

This study aims to establish an analytical flow model of the hollow microneedle lumen based on Poiseuille's law, systematically carry out parameter sensitivity analysis, and obtain engineering design criteria with practical guiding significance through the response surface optimization method.

2. Theoretical model

2.1. Poiseuille's law and flow regime

Poiseuille's law describes the steady laminar flow of an incompressible Newtonian fluid through a straight cylindrical channel. For a circular lumen of radius R and length L, the volumetric flow rate Q is given by:

$$Q = \frac{\pi R^4 \Delta P}{8 \mu L} \tag{1}$$

where ΔP is the pressure differential across the lumen, and μ is the dynamic viscosity of the fluid. This equation hods under three conditions: the flow is laminar, the fluid is incompressible and Newtonian, and the channel is cylindrical with fully developed flow [5].

The flow regime is characterized by the Reynolds number:

$$Re = \frac{\rho v D}{\mu} \tag{2}$$

where ρ is the fluid density, v is the average velocity, and D is the lumen diameter. Flow is laminar when Re < 2300 [5].

In the application of hollow microneedles, the inner lumen diameter is usually 40-70 μm, the needle length is 250-700 μm, and the inlet pressure is 10-30 kPa [2]. The working fluid is water at 37 ° C, with ρ = 997 kg / m^3 and $\mu = 0.8949 \times 10^3 Pa \cdot s$ [3].

2.2. Baseline calculation and laminar flow verification

A baseline case is calculated using representative parameters R = 25 μm, L = 500 μm, and ΔP = 20 kPa:

$$Q = \frac{\pi R^4 \Delta P}{8 \mu L} = 6.85 \times 10^{-9} m^3/s \approx 0.411 \mu L/min \tag{3}$$

$$v = \frac{Q}{\pi R^2} = 3.94 m/s \tag{4}$$

$$Re = \frac{\rho v D}{\mu} = 194 \tag{5}$$

To verify that the laminar flow condition (Re < 2300) holds across the entire parameter range, an extreme value analysis is conducted. The Reynolds number can be expressed as $Re = \frac{\rho R^3 \Delta P}{4 \mu^2 L}$, which increases with R and ΔP, and decreases with L. At the baseline (R=25μm, ΔP=20kPa, L=500μm), the Reynolds number is 194. The maximum Re of 1601 occurs at the upper bounds of R and ΔP and the lower bound of L (R = 35μm, ΔP =30kPa, L=250μm), while the minimum Re of 45 is obtained at R=20μm, ΔP=10kPa, and L =700μm. Since the maximum Reynolds number is well below the

DOI: 10.54254/2753-8818/2026.PU38921

critical value of 2300, the laminar flow condition is satisfied for all parameter combinations within the study range, confirming the applicability of Poiseuille's equation.

3. Sensitivity analysis

3.1. Parameter definition

Based on the literature survey, the parameter ranges for hollow microneedle lumen flow are defined as follows. The lumen diameter ranges from 40 to 70 μm, the needle length from 250 to 700 μm, and the inlet pressure from 10 to 30 kPa [2]. The working fluid is water at 37°C, with a density of 997 kg/m³ and a dynamic viscosity of 0.8949×10^{-3} Pa·s [3].

3.2. Sensitivity index

According to Saltelli et al., the sensitivity index is defined as the sigma-normalized derivative:

$$S_{Z_i}^{\sigma} = \frac{\sigma_{Z_i}\partial Y}{\sigma_Y \partial Z_i} \tag{6}$$

where $\frac{\partial Y}{\partial Z_i}$ is the partial derivative of the output Y with respect to the input parameter Z_i, σ_{Z_i} is the standard deviation of Z_i, and σ_Y is the standard deviation of Y. This measure represents the percentage change in output caused by a one percent change in the input parameter [6].

Applying this definition to Poiseuille's equation, the analytical sensitivity indices are derived as S_R=4, $S_{\Delta P}$=1, and S_L=-1. Since Poiseuille's equation is a separable power-law model without parameter interactions, the local analysis is equivalent to global sensitivity analysis, and the total-effect index S_{Ti} equals the first-order index S_i [6].

These results indicate that the lumen radius has the strongest influence on flow rate, which is proportional to the fourth power of radius, linearly proportional to pressure, and inversely proportional to needle length. This finding is consistent with numerical modelling results reported for hollow microneedle drug delivery systems [7].

3.3. Multi-parameter interaction

To investigate the coupling effects between parameters, a factorial analysis is performed by simultaneously varying two parameters while fixing the third at its baseline value.

Figure 1(a) presents the contour plot of flow rate as a function of lumen radius and inlet pressure. The contours are denser along the R-axis than the ΔP-axis, confirming that radius has a stronger influence on flow rate than pressure, consistent with the sensitivity analysis. Figure 1(b) shows the interaction between the inner lumen radius and the needle length. The flow velocity increases with the increase of radius and decreases with the increase of needle length, while the gradient along the radius axis is steeper. These results further confirm the dominant position of radius as a key design parameter.

DOI: 10.54254/2753-8818/2026.PU38921

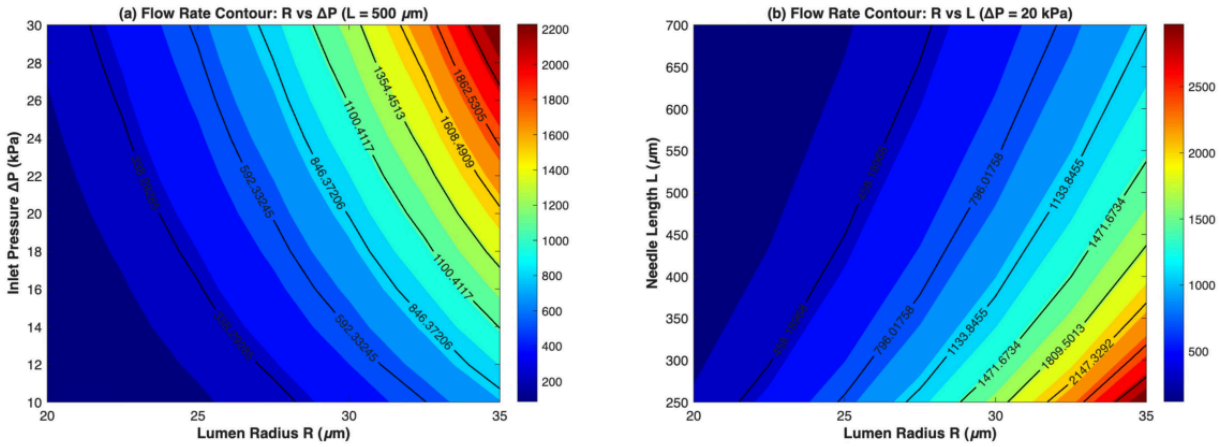


Figure 1. Combined contour plots

Multi-parameter analysis confirms that the lumen radius R is the most critical design parameter. The next section will determine the optimal parameter combination under physiological constraints through the response surface optimization method.

4. Response surface optimization

4.1. Optimization problem formulation

Based on the sensitivity analysis results in Section 3, the lumen radius R is identified as the dominant design parameter. As noted by Wu and Hamada, once a small number of important variables have been identified, response surface methodology can be used to explore their effects and identify optimal operating conditions [8]. Therefore, a response surface optimization analysis was carried out to maximize the flow rate under actual constraints.

The objective is to maximize the volumetric flow rate Q, which is expressed by Poiseuille's equation $Q = \frac{\pi R^4 \Delta P}{8\mu L}$, within the parameter ranges defined in Section 3.1. The optimization is subject to three constraints adopted from the literature [2]. The lumen radius R is constrained between 20 and 35 μm, reflecting typical manufacturing limitations for hollow microneedles. The needle length L ranges from 250 to 700 μm, which ensures penetration through the stratum corneum, a layer with a thickness of approximately 10-20 μm [9]. The inlet pressure ΔP is limited to the range of 10 to 30 kPa, representing typical operating conditions reported in hollow microneedle studies.

4.2. Response surface analysis

A response surface is constructed by evaluating the flow rate across the feasible parameter space. Since Poiseuille's equation indicates that flow rate increases with R^4 and ΔP, and decreases with L, the optimization favors the upper bounds of R and ΔP, and the lower bound of L.

The optimal needle length is determined as the lower limit of 250 μm. This choice was supported by the clinical research results of Makvandi et al., which reported that shorter microneedles (180-280 μm) had significantly lower pain than longer microneedles [9]. In addition, the study also showed that pain perception increased from 5% to 37% (an increase of seven times) when the length of the microneedle increased from 480 μm to 1450 μm (an increase of three times) [9]. Since 250

DOI: 10.54254/2753-8818/2026.PU38921

μm has exceeded the thickness of the stratum corneum by 10-20 μm, the length of the needle ensures sufficient skin penetration while minimizing patient discomfort [9].

The optimal inlet pressure is determined to be the upper limit of 30kPa. Since the Poiseuille equation shows that the flow rate is proportional to the pressure ($Q \propto \Delta P$), the higher the pressure, the greater the flow rate. Therefore, the upper limit of the reported parameter range is selected to maximize the flow rate [2]. This value is within the working pressure range of 10-100 kPa reported by Gera and Burra, and is expected to be well tolerated by patients with minimal discomfort [10].

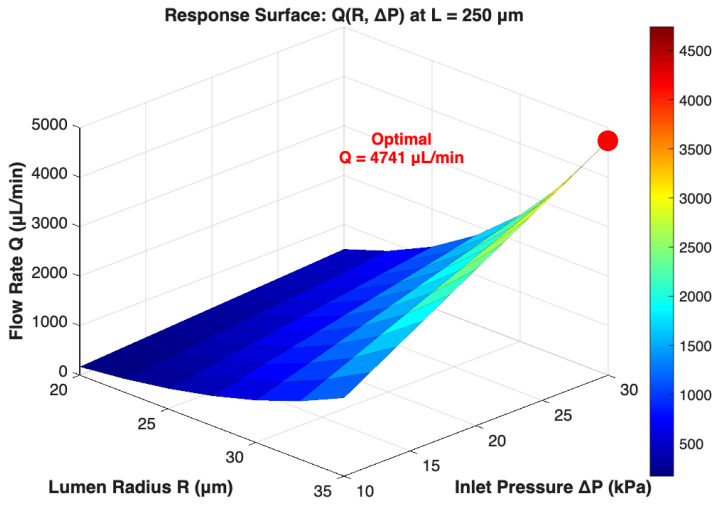


Figure 2. 3D response surface of flow rate as a function of R and dP (fixed L = 250 μm)

Figure 2 illustrates the response surface of flow rate with respect to lumen radius and inlet pressure, with needle length fixed at L = 250 μm. The surface exhibits a steep gradient along the R-axis and a moderate gradient along the ΔP-axis, consistent with the sensitivity analysis results. The optimal point is located at the upper bounds of both R and ΔP, yielding a maximum flow rate of 4741 μL/min.

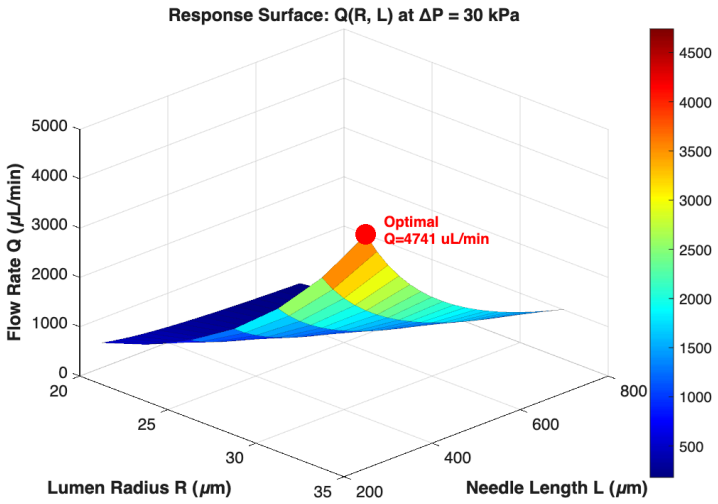


Figure 3. 3D response surface of flow rate as a function of R and L (fixed dP = 30 kPa)

DOI: 10.54254/2753-8818/2026.PU38921

Figure 3 shows the response surface for lumen radius and needle length, with inlet pressure fixed at $\Delta P = 30$ kPa. The flow rate increases sharply with radius and decreases with length. The optimal point is located at maximum R and minimum L, confirming the parameter selection rationale discussed above.

4.3. Optimal design

Based on the response surface analysis, the optimal parameter combination for maximizing the flow rate within the parameter range reported in the literature is: R = 35μm, L = 250μm, ΔP = 30kPa, and the maximum flow rate is 4741μL/min. The optimal inner lumen radius takes its upper limit because the flow velocity is the most sensitive to this parameter (S_R=4). The optimal needle length reaches its lower limit, which is consistent with the clinical findings that shorter microneedles can minimize patient discomfort. Moreover, the length still exceeds the thickness of the stratum corneum by 10-20 μm [7]. The optimal pressure is set to the upper limit to maximize the flow rate, because Q is proportional to ΔP.

5. Conclusion

In this study, an analytical flow model of the hollow microneedle lumen was established based on the Poiseuille equation, and sensitivity analysis and response surface optimization were systematically carried out. The results show that the inner lumen radius is the most significant parameter affecting the flow velocity, and its sensitivity index is 4. Followed by inlet pressure ($S_{\Delta p}$ =1) and needle length (S_L=−1). The optimal design achieves a maximum flow rate of 4741 μL/min under the conditions of R = 35 μm, L = 250 μm and ΔP = 30 kPa, which provides practical guidance for the design of hollow microneedles.

However, there are some limitations in this study. The analytical model assumes that the lumen is a straight cylinder and the fluid is a Newtonian fluid. In reality, however, the actual pharmaceutical preparations may exhibit non-Newtonian characteristics such as shear thinning. In addition, the model does not consider the effect of temperature on fluid viscosity, and the viscosity may change significantly under clinical conditions. Future research should be combined with the experimental verification of the prepared microneedles, and the model should be extended to non-Newtonian fluid effects and temperature-dependent viscosity changes to improve the accuracy and practical applicability of the optimization results.

References

[1] Shelke, O. S., Kalambhe, D. R., & Bankar, M. M. (2026). New transdermal technologies and analysis of perspectives shift of transdermal drug delivery from patch to microneedles. *Regenerative Engineering and Translational Medicine*.

[2] Nadda, R., Prakash, R., Das, D. B., & Donnelly, R. F. (2025). Incorporating fluid-structure interactions for modelling of pyramidal hollow microneedles for transdermal drug delivery. *Journal of Drug Delivery Science and Technology, 114*, 107561.

[3] Anbazhagan, G., Suseela, S. B., & Sankararajan, R. (2023). Effect of hollow microneedle geometry structure on mechanical stability and microfluidic flow for transdermal drug delivery applications. *Microfluidics and Nanofluidics, 27*, 25.

[4] Zhang, X., Gu, Q., Sui, X., et al. (2025). Design and optimization of hollow microneedle spacing for three materials using finite element methods. *Scientific Reports, 15*, 652.

[5] White, F. M. (2016). *Fluid mechanics* (8th ed.). McGraw-Hill, New York.

DOI: 10.54254/2753-8818/2026.PU38921

[6] Saltelli, A., Ratto, M., Andres, T., Campolongo, F., Cariboni, J., Gatelli, D., Saisana, M., & Tarantola, S. (2007). *Global sensitivity analysis: The primer*. John Wiley & Sons.
[7] Prakash, R., Nadda, R., Das, D. B., & Donnelly, R. F. (2025). Numerical modelling for coupled flow, transport, pharmacokinetic and pharmacodynamic of tablet-loaded hollow microneedle patch for transdermal drug delivery. *International Journal of Pharmaceutics, 686*, 126309.
[8] Wu, C. F. J., & Hamada, M. (2021). *Experiments: Planning, analysis, and optimization* (3rd ed.). John Wiley & Sons.
[9] Makvandi, P., Kirkby, M., Hutton, A. R., et al. (2021). Engineering microneedle patches for improved penetration: Analysis, skin models and factors affecting needle insertion. *Nano-Micro Letters, 13*, 93.
[10] Gera, A. K., & Burra, R. K. (2025). Design of hollow tapered PMMA polymeric microneedles for enhanced structural stability and drug delivery efficiency. *AIP Advances, 15*, 015034.