

A Review of Fluid Dynamics in Blood: From Macroscopic Laminar Flow to Microscopic Boundary Layer

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Abstract. In modern society, cardiovascular disease has become the leading cause of death among humans. This paper focuses on the fluid dynamics of blood in blood vessels, the changes in blood flow under atherosclerosis and their impact on the vessel wall, and the microfluidic effect of blood flow in capillaries. The aim of this research is to quantify the forces exerted by blood on blood vessels at different radii and the changes in blood flow under atherosclerosis, thereby deriving a more universal formula to predict the formation of atherosclerosis and the damage to blood vessels caused by vascular diseases, thus enabling clinical application. The research methodology employed in this study is literature analysis. This paper finds that under different vessel diameters, different flow states result in different flow formulas representing the shear stress on the vessel wall. Furthermore, the complex blood flow situation when plaques appear in blood vessels requires simulation analysis to better analyze the time-averaged wall shear stress and oscillatory shear index of blood flow on the vessel wall, and ultimately to calculate the degree of vascular damage caused by plaque formation.

Keywords: Poiseuille's Law, Atherosclerosis, Fluid Dynamics, Capillaries

1. Introduction

Poiseuille's law can be used to derive the relationship between flow rate, pressure, and vessel radius in blood vessels. The derivation of shear stress also utilizes the Carson model to correct for blood flow under microscopic conditions. For blood flowing in microvessels, the viscosity coefficient needs to be determined based on the actual flow conditions within the microvessels for further correction. Current research on the fluid dynamics of blood in blood vessels has become more refined, reaching the level of non-Newtonian fluid unification and microcirculatory two-phase flow. Conventional blood analysis primarily uses Newtonian fluid dynamics, with viscosity coefficients as the main analytical method. However, at the microscopic level, with vessel diameters of 5-10 μm , there are special cases requiring non-Newtonian fluid dynamics studies. This paper employs a research method that extends and derives from the literature and the basic principles of Poiseuille's law. This research is at the intersection of fluid mechanics and biomedicine, providing a fundamental formula reference for vascular medicine and the development of microfluidic chips.

2. Fundamental theory of viscous fluid flow in pipes and Poiseuille's law

2.1. Basic laws of laminar flow and viscous flow in circular pipes

Larger blood vessels, such as the aorta and major blood vessels in the limbs, have diameters $>500 \mu\text{m}$. Blood flows very rapidly in these vessels, exhibiting a very high shear rate and shear-thinning properties. Furthermore, two-phase flow characteristics do not need to be considered in large blood vessels. In other words, for blood vessels with diameters $>500 \mu\text{m}$, the overall blood flow is uniform. Therefore, blood in this case can be viewed using a Newtonian fluid model.

However, the flow pattern must be considered, using the Reynolds number to determine whether the flow is laminar or turbulent. Newton's law of viscosity only applies in laminar flow. The basic formula for the Reynolds number is as follows.

$$\text{Re} = \frac{\rho v D}{\mu} \quad (1)$$

In the formula, blood density, dynamic viscosity, coronary trunk diameter, mean resting blood flow velocity, diameter, and mean flow velocity are given. The final solution yields: for coronary arteries: $\text{Re} \approx 180\text{--}360$; for ordinary large arteries in the limbs: $\text{Re} \approx 300\text{--}525$. The calculated result of this formula < 2000 is considered laminar flow, and Newton's law of viscosity applies. It can be used to derive Poiseuille's law.

In the formula, the blood density is $\rho = 1050 \text{ kg/m}^3$, the dynamic viscosity is $\mu = 0.0035 \text{ Pa} \cdot \text{s}$, the diameter of the main coronary artery is $D = 2.5 \sim 4.0 \text{ mm}$, the mean resting blood flow velocity is $v = 15 \sim 30 \text{ cm/s}$, and the diameter is $D = 3.0 \sim 5.0 \text{ mm}$,

Average flow velocity: $v = 20 \sim 35 \text{ cm/s}$. The final solutions are: coronary arteries: $\text{Re} \approx 180\text{--}360$; common large arteries in the limbs: $\text{Re} \approx 300\text{--}525$. When $\text{Re} < 1000$, it is laminar flow; when $\text{Re} > 2000$, it is turbulent flow. If the calculated result is Less than 2000, it is considered laminar flow, and Newton's law of viscosity applies. This can be used to derive Poiseuille's law.

2.2. Derivation of Poiseuille's law

The derivation of Poiseuille's law requires Newton's law of viscosity, the basic formula of which is:

$$f = \eta \frac{du}{dy} dS \quad (2)$$

This formula can calculate the magnitude of internal friction in a fluid. From Figure 1, we take the layer of fluid between the fluid and the blood vessel wall as our research object. Figure 1 shows the force analysis of blood flow in blood vessels:

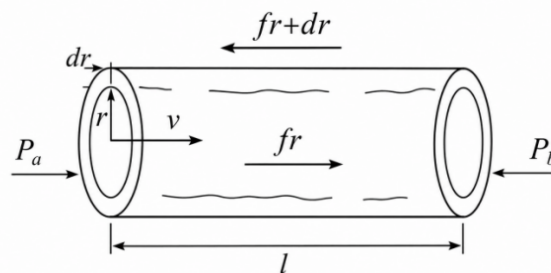


Figure 1. Force analysis diagram when the dr boundary layer is the object of study

By analyzing the frictional forces in the dr layer and applying the principles of steady flow, the fundamental equation of Poiseuille's law is ultimately obtained.

$$Q = \frac{\pi(P_a - P_b)R^4}{8\eta l} \quad (3)$$

3. Shear stress on the blood vessel wall

3.1. WSS

The WSS (Wave Surface Stress) was first proposed by Lutz et al. in 1977. It refers to the force exerted by blood flow on the vessel wall per unit area of the vessel's inner surface. For any specific shape of channel and any flow pattern, there is a basic formula for calculating WSS:

$$\tau = \mu \frac{du}{dy} \quad (4)$$

By combining Poiseuille's law formula with formula (4), we can finally obtain the formula for the shear stress of the blood vessel wall under ideal conditions:

$$\tau_w = \frac{4\eta Q}{\pi R^3} \quad (5)$$

Normally, blood vessels are not ideal. For vessels with large diameters, the influence of the heart's pulse on blood flow needs to be considered. In this case, blood flow is no longer ideal, and the ideal shear stress formula is no longer applicable.

3.2. WSS with pulsating circulation

For the influence of cardiac pulsation on blood flow in large-diameter blood vessels such as the aorta and large arteries, a dimensionless number, the Womersley number, can be introduced for reference [1]. Its formula is:

$$\alpha = R\sqrt{\frac{\omega\rho}{\mu}} \quad (6)$$

This formula essentially measures the ratio of inertial force to viscous force in pulsating blood flow, where R is the vessel radius, ω is the angular frequency, ρ is the blood density, and μ is the blood dynamic viscosity.

Based on classic blood parameters $\rho = 1050 \text{ kg/m}^3$, heart rate 70 beats/min, $\omega \approx 7.35 \text{ rad/s}$, $\mu = 0.0035 \text{ Pa}\cdot\text{s}$. Therefore, when the aortic radius $R = 1.5\text{-}5 \text{ mm}$, $\alpha = 3\text{-}12$, and α must be corrected. For small vessels in the microcirculation system, $R < 0.5 \text{ mm}$, $\alpha < 1$, no correction is needed. Therefore, Poiseuille's law formula for larger diameter vessels needs to be corrected.

In this case, the blood flow velocity is no longer constant but fluctuates with time, so the original Poiseuille law formula becomes invalid.

The Navier-Stokes equations for laminar flow in a circular tube for Newtonian fluids [2]:

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v_r \frac{\partial u}{\partial r} \right) = -\frac{\partial p}{\partial x} + \mu \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right] \quad (7)$$

For steady radial velocity flow according to the classical Poiseuille formula, we obtain:

$$0 = -\frac{\partial p}{\partial x} + \mu \left(\frac{d^2 u}{dr^2} + \frac{1}{r} \frac{du}{dr} \right) \quad (8)$$

Integrating, we can obtain Poiseuille's classic formula (7).

In the case of non-stationarity $\frac{\partial u}{\partial t} \neq 0$, (9) The equation becomes:

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) \quad (9)$$

The axial pressure is periodic and can be decomposed into a simple harmonic form:

$$-\frac{\partial p}{\partial x} = P' e^{i\omega t} \quad (10)$$

Velocity can also be written in the form of a period:

$$\rho \frac{\partial u}{\partial t} = i\omega \rho U \quad (11)$$

Combining this with (10), we can obtain the final formula:

$$i\omega \rho U = P' + \mu \left(\frac{d^2 U}{dr^2} + \frac{1}{r} \frac{dU}{dr} \right) \quad (12)$$

Solving equations (11) and (12) simultaneously, we get:

$$\frac{d^2 U}{dr^2} + \frac{1}{r} \frac{dU}{dr} - i\alpha^2 U = -\frac{P'R^2}{\mu} \quad (13)$$

Integrating it yields:

$$u(r) = \frac{R^2}{i\alpha^2 \mu} \frac{dP}{dz} \left[1 - \frac{J_0(i^{3/2} \alpha \frac{r}{R})}{J_0(i^{3/2} \alpha)} \right] \quad (14)$$

Solving the equations in (4) and (4) simultaneously, we obtain the formula for the shear stress of the aortic vessel wall:

$$\tau_w = \frac{8\mu Q}{\pi \alpha R^3} \cdot \frac{i^{3/2} J_1(\alpha i^{3/2})}{J_0(\alpha i^{3/2})} \quad (15)$$

J_0 and J_1 are zero-order and first-order Bessel functions of the first kind, respectively, and α are Womersley numbers.

Large vessels highlight the influence of cardiac pulsation on the blood fluidity. This formula can effectively correct the shear stress situation of Poiseuille's law under pulsating conditions.

3.3. WSS with pulsating circulation

For capillaries, i.e., those with a diameter of $d=5-10\mu m$, the diameter of the vessel is approximately equal to the diameter of a red blood cell, and the Newtonian fluid dynamics completely fail.

Therefore, the Casson equation is used for correction, i.e., a new viscosity coefficient μ_a is generated, which is calculated by the Casson model [3].

The formula for the Carson model is:

$$\sqrt{\tau} = \sqrt{\tau_y} + \sqrt{\mu_\infty \dot{\gamma}} \quad (16)$$

τ : shear stress, τ_y : yield stress, μ_∞ : high shear viscosity, $\dot{\gamma}$: shear rate

The basic formula for calculating apparent viscosity μ_a is:

$$\mu_a = \frac{\tau}{\dot{\gamma}} \quad (17)$$

The shear rate can be obtained by removing the viscosity coefficient from formula (5):

$$\dot{\gamma}_w = \frac{4Q}{\pi R^3} \quad (18)$$

The viscosity coefficient in Poiseuille's law is corrected, and the following simultaneous solutions are obtained:

$$Q = \frac{\pi R^4 \Delta P}{8L \cdot \left(\sqrt{\mu_\infty} + \sqrt{\frac{\tau_y \pi R^3}{4Q}} \right)^2} \quad (19)$$

This formula essentially transforms the constant viscosity coefficient of a Newtonian fluid into a varying apparent viscosity coefficient, corresponding to the shear-thinning characteristic of blood—that is, the viscosity of blood changes with different shear rates. In microvessels, the shear rate of blood is extremely high at the vessel wall, while it is essentially zero at the vessel's axis. This is because, due to the axial migration of red blood cells within the vessel, the blood becomes a plug flow, meaning the viscosity is extremely high at the axis and very low near the vessel wall. This amplifies the non-Newtonian fluid behavior of blood to an extreme degree in extremely small vessel diameters.

Meanwhile, because blood migrates axially in capillaries, there is a medium—plasma—between red blood cells and blood vessels, which makes blood exhibit two-phase flow properties in blood vessels. Therefore, a correction coefficient should be added to μ_a in formula (18).

The commonly used empirical microscale correction factor $\alpha(R)$ represents the correction coefficient, and $\alpha(R)$ is a function obtained from the experimental fit.

$$\alpha(R) = 1 - \exp\left(-\frac{R}{R_0}\right) \quad (20)$$

R_0 is the characteristic tube diameter $\approx 50\mu\text{m}$. When $R > 100\mu\text{m}$, $\alpha(R) \approx 1$, and the microscale effect disappears. When $R < 50\mu\text{m}$, $\alpha(R) < 1$, which reflects viscosity decline. This is the Faringquist effect [4], which indicates that the smaller the tube diameter, the higher the proportion of cell-free plasma layer, the more obvious the drag reduction effect, and the lower the apparent viscosity [4].

The final formula for Poiseuille's law modified for use in microvessels is:

$$Q = \frac{\pi R^4 \Delta P}{8L \cdot \alpha \left(R \right) \cdot \left(\sqrt{\mu_{\infty}} + \sqrt{\frac{\tau_y \pi R^3}{4Q}} \right)^2} \quad (21)$$

The shear stress of the blood vessel wall can be obtained by combining formula (8):

$$\tau_w = \frac{4Q \cdot \alpha \left(R \right) \cdot \left(\sqrt{\mu_{\infty}} + \sqrt{\frac{\tau_y \pi R^3}{4Q}} \right)^2}{\pi R^3} \quad (22)$$

The modified shear stress applies to microvessels. Furthermore, compared to vessels with a diameter greater than 500 μm , the radius R of a microvessel is much smaller than that of a large artery, resulting in a Womersley number $a \ll 1$. Viscous forces are much greater than pulsating inertial forces, and the pulsating effect of the heart is negligible for the blood in capillaries. Therefore, we can directly modify the Poiseuille law formula under ideal conditions.

4. Analysis of blood flow in actual medical practice

4.1. Effects of WSS OSI on atherosclerosis and plaque formation

WSS (Wave Spectrum Score) can predict the location of potential vascular lesions. In other words, WSS values can predict which areas of the blood vessels are likely to form plaques and which areas with plaques pose a serious threat to the blood vessels. This has guiding significance for the diagnosis and treatment of vascular diseases.

Medical research has shown that atherosclerotic plaques first form in blood vessels in low WSS regions. Furthermore, the derivation of formula (14) reveals a positive correlation between Blood Flow Velocity and Wall Shear Stress, meaning that Blood Flow Velocity is positively correlated with the ability to inhibit plaque formation [5].

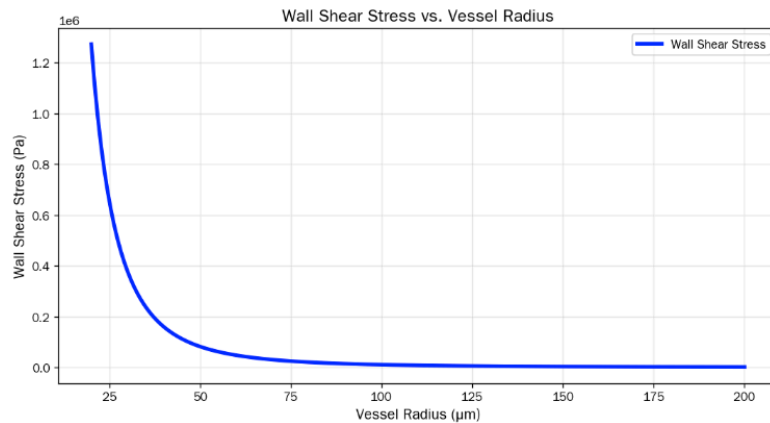


Figure 2. Curves showing the influence of vessel radius on wall shear stress under the conditions of formula (15)

When plaques form, they mostly only form in large blood vessels, so in most cases, the bias caused by non-Newtonian fluids does not need to be considered. Under the conditions of Equation 15, when plaques form, the vessel radius decreases, and the shear stress on the vessel wall increases, as shown in Figure 2.

It is evident that changes in blood vessel diameter significantly impact shear stress. When plaque deposition is severe, leading to shear stress exceeding 4.84 Pa, this surpasses the plaque's fibrous cap tolerance threshold, causing the cap to thin or even rupture. This exposes the plaque contents, activating the coagulation system and resulting in thrombosis and acute stroke. This conclusion aligns with medical findings, but practical considerations within blood vessels are crucial. During plaque formation, the flow is no longer simple laminar; turbulence and oscillations become more prevalent, necessitating the incorporation of the OSI oscillatory shear index and the TAWSS time-averaged wall shear stress. These parameters require real-time CFD simulation for a more comprehensive analysis of complex flow conditions and the corresponding blood dynamics during plaque formation.

4.2. Blood flow and simulation analysis within the vessel wall during plaque formation

Figure 3 shows the characteristics of blood flow in blood vessels with plaques.

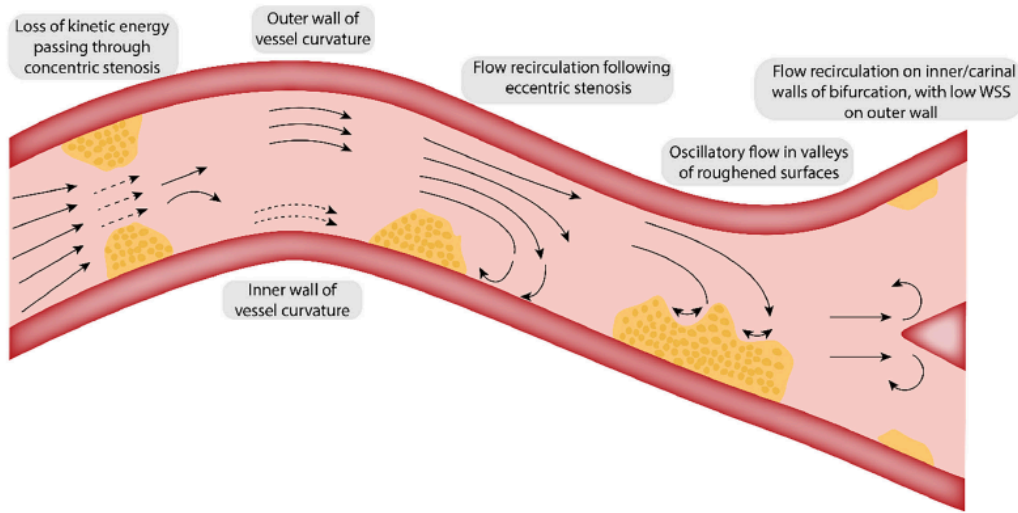


Figure 3. Different characteristics of blood flow under different plaque formation conditions and different flow states formed simultaneously [6]

As shown in Figure 4, blood flow accelerates when entering areas with plaques, resulting in kinetic energy loss. The blood flow is subject to centrifugal force, and there is significant shear force on the outer wall of the vessel at bends, while the opposite is true for the inner wall, where blood flow is slower and shear force is lower. This makes this location a prone area for plaque formation. In Figure 4, asymmetric plaques exacerbate flow separation, forming significant vortices and backflow downstream of narrowing. The uneven surface of the plaque traps blood flow, creating reciprocating flow and leading to a high oscillatory shear index. Backflow zones easily form on the outer wall of bifurcations, where wall shear stress is low, making it a high-risk location for atherosclerotic plaques.

The blood flow in blood vessels was simulated using ANSYS CDX [7]. The values of TAWSS and OSI were obtained, and the formulas are as follows:

$$TAWSS = \frac{1}{T} \int_0^T |WSS| dt \quad (23)$$

$$OSI = 0.5 \left(1 - \frac{\left| \int_0^T WSS dt \right|}{\int_0^T |WSS| dt} \right) \quad (24)$$

In CFD simulations, the Navier-Stokes equations are used to represent fluid flow.
Continuity equation:

$$\nabla \cdot \mathbf{u} = 0 \quad (25)$$

Momentum equation:

$$\rho(\mathbf{u} \cdot \nabla \mathbf{u}) = -\nabla p + \nabla \cdot \boldsymbol{\tau} \quad (26)$$

The shear stress formula is shown in formula (4) above.

The pulsating heart rate at the inlet is simulated using the experimentally measured velocity waveform (see Figure 4):

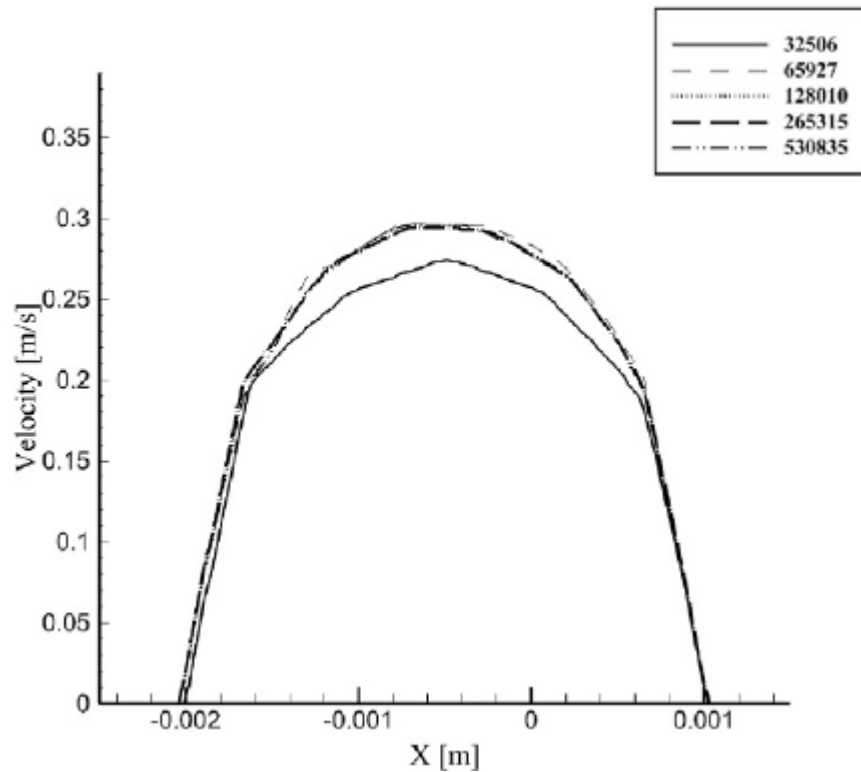


Figure 4. Image of blood flow velocity under the influence of blood pulsation as measured experimentally [7]

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$$P = (R_1 + R_2)Q - R_2 C \frac{dP}{dt} + R_1 R_2 C \frac{dQ}{dt} \quad (27)$$

The following CFD simulation analysis of blood vessels is shown in Figure 5.

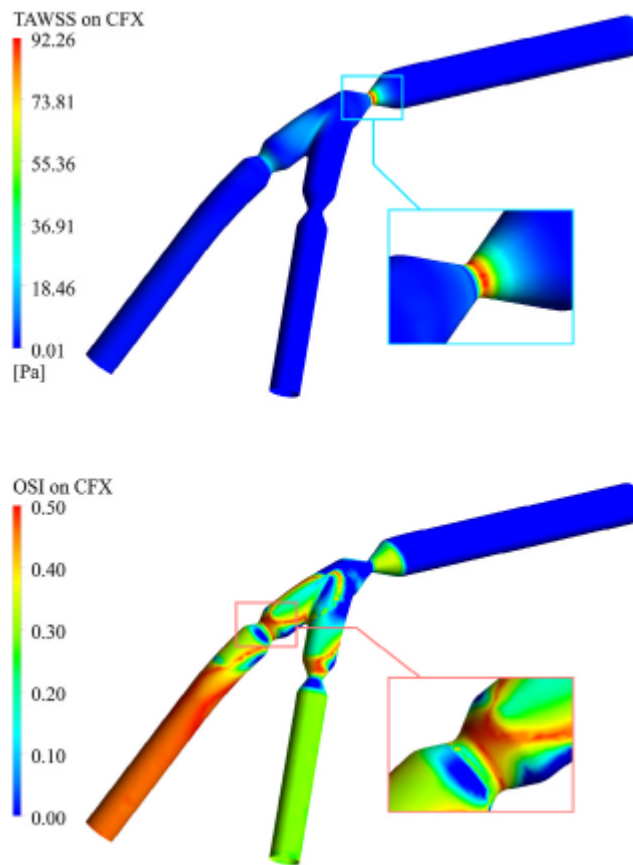


Figure 5. Simulation analysis of TAWSS and OSI was performed respectively, and the effect of blood on the blood vessel wall was determined by its color distribution [7]

The simulation results reveal the typical hemodynamic distribution characteristics of coronary artery lesions and confirm that low TAWSS and high OSI areas are high-risk biomechanical environments for atherosclerosis progression.

5. Conclusion

This paper primarily discusses the macroscopic and microscopic flow of blood, analyzing the differences between plaque formation conditions and those in ordinary large-diameter and microvascular systems. It describes the blood flow patterns and shear stress characteristics at different locations, enabling the prediction of plaque formation and disappearance. The paper concludes that in larger-diameter vessels, a Womersley correction to Poiseuille's law is needed to achieve a high degree of fit with blood flow in rigid, direct-flow vessels. For microvessels, the Carson model and correction coefficients must be introduced to make the blood flow more realistic. For vessels with plaques and atherosclerosis, the flow becomes highly complex, requiring simulation analysis for a more complete fit. This paper does not provide a detailed explanation of the follow-up investigation into whether the simulation results match reality. Future research using AI could reduce the number of CFD simulations, allowing robots to predict plaque formation based on vessel shape.

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