

Reynolds Number and Bio-Inspired Bacterial Swimming

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Abstract. Bacteria can swim by rotating helical flagella. Their swimming is different from the movement of macroscopic objects because bacteria live at a low Reynolds number. At a low Reynolds number, viscous force is more dominant than inertia. This project investigated how Reynolds number, viscosity, and helical geometry affect bacterial swimming. First, literature related to low-Reynolds-number swimming, flagellum hydrodynamics, bacterial swimming, and helical swimmer geometry was reviewed. Then, a MATLAB model calculated a relative swimming speed as the flagellum's pitch angle was changed. Finally, a larger swimmer was tested with its helix-driven tail rotating in different solutions. The swimmer was fastest in water and slowest in the fluid with the highest resistance. The results support the main hypothesis: increasing resistance decreases the speed of the motor-driven swimmer. The experiment also shows that a helical tail can produce forward motion if drag is unequal along and across the tail.

Keywords: Reynolds number, bacterial swimming, microswimmer, helical flagellum, viscosity, MATLAB

1. Introduction

Bacterial swimming is a good example of small-scale physics. Many bacteria swim with one or more helical flagella. Direct experiments confirmed that bacteria use rotating flagella to swim [1, 2]. Using a flagellum is similar to using a small propeller, but fluid dynamics around small swimmers are different from fluid dynamics around macroscopic swimmers, such as boats.

Reynolds number is a dimensionless factor relating inertia and viscosity. Bacteria have a low Reynolds number, so viscosity has a much larger effect than inertia. Bacteria cannot coast, and their propulsion efficiency is low [3]. In this project, the relationship among low-Reynolds-number swimming, viscosity, and helical geometry was investigated with a numerical model. The main research question was how different fluid forces and helical geometry affect bio-inspired swimmer performance.

The project had three objectives. The first objective was to understand the topic of low-Reynolds-number swimming. The second objective was to build a MATLAB model to calculate the effect of pitch angle. The third objective was to build a larger swimmer and test it in different solutions.

2. Background theory

2.1. Reynolds number

Reynolds number, Re , can be calculated with density, speed, body length, and viscosity:

$$Re = (\rho U L) / \mu \quad (1)$$

where density is ρ , speed is U , characteristic body length is L , and dynamic viscosity is μ . A lower Reynolds number means that viscous forces are greater than inertial forces. A target Reynolds number of less than or equal to 0.1 was selected for the scaled up swimmer in this project.

2.2. Low-Reynolds-number swimming

At a low Reynolds number, a microswimmer cannot coast. It must keep changing the fluid in ways that are not simply the reverse of its last movement. Purcell illustrated this idea with the "scallop theorem." A rotating helix can avoid this problem because its rotation is coupled to forward motion. Flagellar hydrodynamic theory expands this concept to predict swimming motion with slender flagella [4, 5].

2.3. Helical flagellum and unequal drag forces

Thin flagella experience different types of resistance parallel to or perpendicular to their slender bodies. Resistive force theory describes these two types of resistance with different drag coefficients. The perpendicular drag coefficient is often larger than the parallel drag coefficient [4]. A simple relationship used when constructing the MATLAB model is:

$$\xi_{\text{perpendicular}} = 2\xi_{\text{parallel}} \quad (2)$$

Rotating a helical tail can generate forward force because drag is asymmetric along and across the tail. The pitch angle, ψ , can be modified to better the forward propulsion. Both numerical and experimental studies also suggest that helix geometry can greatly impact on propulsion [5]. Extremely low or high pitch angles will probably not be optimal for propulsion.

3. Methods

3.1. Literature review

The literature review discussed three related areas. First, Taylor [6], Purcell [3], and Lauga and Powers [7] provided the background for microorganism swimming and hydrodynamics. Second, Gray and Hancock [4] and Lighthill [5] extended the fluid drag theory to slender flagella. Finally, Berg and Anderson directly demonstrated that bacteria swim by rotating their flagella. Berg and Turner [2] and Magariyama and Kudo [8] studied important variables in swimming in polymer solutions. Chattopadhyay [9] measured the low mechanical efficiency of bacterial propulsion. Rodenborn [10] examined the comparison between experiments and the mathematical prediction for a rotating helical flagellum. These articles served as the foundation for the MATLAB model and prototype design.

3.2. MATLAB model

The MATLAB model varied the pitch angle from 0 to $\pi/2$, and found a relative forward swimming speed at each angle. They used the drag coefficients, based on the resistive force theory [4]. They were constant, with parallel and perpendicular values different due to the slender-body approximation. All other swimmer design parameters were held constant. The maximum speed from the model was set equal to 1 for comparison.

The MATLAB model served as a first design-estimation tool rather than as a tool for predicting all details of real bacterial swimming. The model helped to estimate an effective range of helix-shape parameters.

3.3. Prototype design and fluid tests

A bio inspired swimmer was built with a body length of about 10 cm and a helix-driven tail. The design was based on the idea that a rotating helical tail can convert rotary motion into forward motion. The main relevant design parameters were body length, body diameter, tail length, helix radius, pitch angle, motor speed, and motor torque.

The swimmer was tested in three different solutions: water, soap solution, and laundry detergent solution. These solutions were chosen to provide a simple increase in resistance. Swimming speed was calculated from swimming distance and travel time. Propulsion and stability were also observed. The same swimmer body and motor setup were used whenever possible.

4. Results

4.1. MATLAB pitch angle result

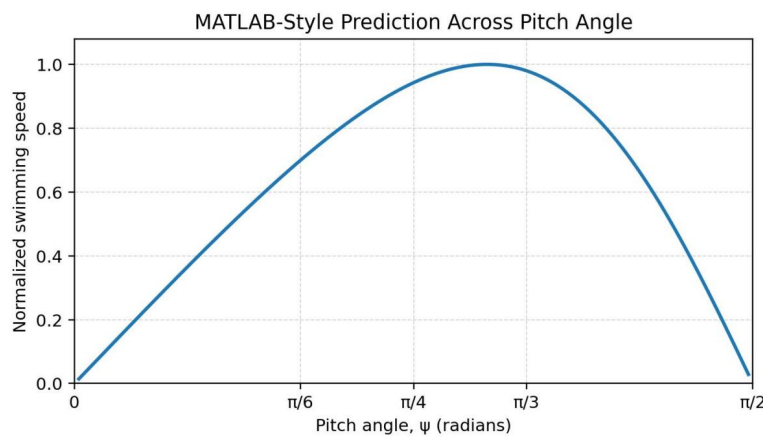


Figure 1. Normalized swimming speed calculated when pitch angle was changed from 0 to $\pi/2$

The numerical model predicts the highest propulsion uses a middle angle, as shown in Figure 1. This seems logical, since the angle range was consistent with earlier work [5] which showed helix geometry influences propulsion. A very small or very large pitch angle would not be expected to generate maximum propulsion.

4.2. Prototype fluid test results

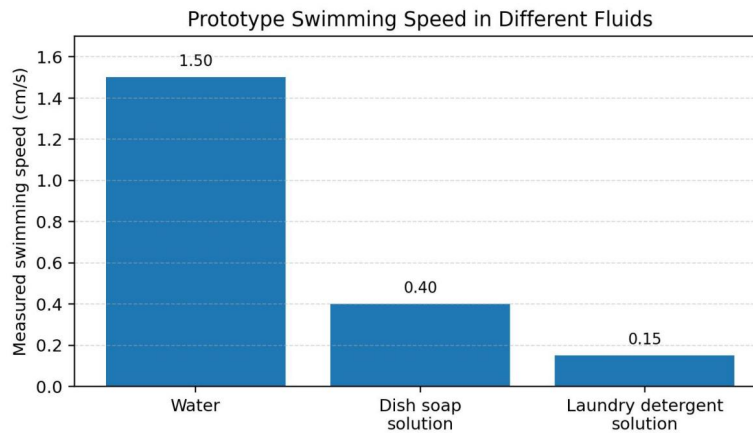


Figure 2. Estimated swimmer speed in three different solutions

Table 1. Estimated swimmer speed and observed behavior in different solutions

Fluid	Estimated Speed (cm/s)	Observed Behavior
Water	1.50	Fastest and most stable swimming
Soap solution	0.40	Slower swimming with more resistance
Laundry detergent solution	0.15	Slowest and most difficult to control

The swimmer had its highest estimated speed in water at 1.5 cm/s, as shown in Figure 2 and Table 1. Speed decreased to 0.4 cm/s in the soap solution and 0.15 in the laundry detergent solution. Control over the swimmer was harder in more viscous solutions. Fluid resistance seems to be the main factor controlling speed. Water allowed the fastest propulsion.

5. Discussion

Both the numerical model and the physical experiment do help answer the research question and help support the main hypothesis. The MATLAB model shows the helix geometry is important for propulsion. The physical model showed that fluid resistance influences the swimmer's performance. These results suggest that swimmer function can be influenced by the balance of helix geometry, motor rotation, and fluid resistance.

The swimmer's velocity was less in the more viscous solutions because the effects of the higher drag on the swimmer were not compensated. In other words, the same motor that drove the swimmer close to its limit in regular water was not powerful enough to drive it at the same speed in the more viscous solutions.

In some cases, real bacteria in viscous solutions exhibit different behavior than the simplified larger swimmer. Berg and Turner [2] discovered that *E. coli* swims faster in some viscous polymer solutions. Magariyama and Kudo [8] made a model with two different apparent viscosities around the cell body and the flagellum, to explain this phenomenon. This clearly shows that one viscosity value should not be used to characterize an entire microscopic fluid environment.

The model prototype was much larger than a bacterium, and this increases the importance of other physical phenomena. In addition, the experiment shows that the performance of a swimmer is

also dependent upon body- and tail-induced drag, in addition to fluid viscosity. A small robot swimming in water does not necessarily have low Reynolds number like a bacterium.

The prototype design was based on known motor settings, but the amount of motor power was not measured. Real *E. coli* also has low propulsion efficiency. Chattopadhyay et al [9]. calculated this efficiency and discovered that only a small portion of the motor power was used towards useful forward motion. This again shows that speed is not the only factor in swimmer performance.

6. Limitations and future improvement

Several factors influenced the accuracy of this work. First, since the viscous fluid was not characterized using a viscometer, the tests were primarily qualitative. Second, the prototype speed was obtained from the known distance traveled and time, and thus there may have been human error in measuring the time. Third, as the fluid viscosity increased, the motor's rotation speed and torque presumably decreased. Fourth, the body also rotated, or shifted to one side during the tests, which indicates that not all of the energy from the motor was converted into forward propulsive motion.

Subsequent work should include measurements of the fluid density and viscosity, measurements of motor rotation speed and torque, and determination of speed using video tracking. One should 3D-print tail shapes with known controlled radii, controlled pitch angles, controlled lengths, and known wire thicknesses. The Reynolds number should be calculated for each experiment. The MATLAB model should be made more realistic by including body drag, motor torque, tail drag, and reverse body rotation. The accuracy of this several experimental results could then be compared to the refined model.

7. Conclusion

The bacterial flagella-generated swimming was studied by combining numerical methods (MATLAB program) and physical modeling. Fluid dynamics shows that the motion at low Reynolds number is dominated by viscous forces. The rotating helical tail can achieve forward motion since there is more drag on the tail when it chases certain directions. The MATLAB model presents that a pitch angle in a middle interval is more suitable for propulsion than 0 or $\pi/2$. The experimental result shows that the prototype swims fastest in water, but slows down greatly in soap and detergent-containing solution because of the higher resistance. Taking into account the simplifications made in both the model and the experiment, a solid relationship exists between fluid dynamics, bacterial swimming, and bio-inspired engineering.

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