Proceedings of CONF-APMM 2025 Symposium: Applied Mathematics and Quantitative Modeling
DOI: 10.54254/2753-8818/2026.PU35024

Mathematical Modeling and Vibration Reduction Performance Analysis of the Cable-NES System

Zhenghang Chen

School of Mathematics and Statistics, Xidian University, Xi'an, China
3253488729@qq.com

Abstract. This paper mainly discusses the application potential of NES in vibration reduction of stay cables. By establishing a mathematical model of the combined stay cable and NES system, it derives the motion equations based on Hamilton's principle, so as to analyze the dynamic characteristics of the coupled system. On this basis, numerical simulation methods are adopted to explore the vibration reduction effect of NES under different excitation conditions. The research results show that compared with traditional vibration reduction devices, NES exhibits superior vibration reduction performance in a wide frequency domain, especially in suppressing the main resonance peak, and has stronger adaptability and response capability. Through amplitude-frequency response analysis and random response analysis, this paper reveals the important role of NES in reducing the amplitude of stay cables, improving structural stability and prolonging service life. In addition, this paper evaluates the feasibility and design optimization potential of NES in practical engineering applications, and emphasizes its wide adaptability in complex dynamic environments.

Keywords: cable-stayed bridge, nonlinear energy sink, vibration reduction technology, dynamic characteristics, mathematical model

1. Introduction

With the continuous breakthrough in the span of long-span cable-stayed bridges, stay cables, as the core load-bearing components, have increasingly prominent characteristics of large slenderness ratio, low self-stiffness and low structural damping, making them extremely sensitive to the dynamic effect of wind loads. Natural wind is not an ideal uniform steady flow. The random pulsation components contained in it will exert continuous random excitation on stay cables, which in turn causes random vibration of the structure. This type of vibration is one of the most common and most frequent forms of wind-induced vibration of stay cables [1].

Aiming at the wind-induced vibration problem caused by the high flexibility and low damping characteristics of stay cables, as well as the limitations of traditional linear vibration reduction technologies such as narrow frequency band and insufficient robustness, this paper takes the cable-NES coupled system as the core research object, and carries out systematic research on the vibration reduction mechanism, numerical simulation and parameter optimization of nonlinear energy sinks for stay cables. Through the progressive steps of theoretical modeling, equation derivation,

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numerical solution and characteristic analysis, this paper reveals the coupled dynamic law between NES and stay cables, and verifies the effectiveness of NES in wide-frequency vibration reduction [2].

2. Mathematical modeling and vibration reduction performance analysis of the cable-NES system

2.1. Research on vibration reduction of NES as nonlinear stiffness

The core feature of NES lies in its strong nonlinear stiffness characteristic. For the most typical cubic nonlinear NES, the restoring force and displacement satisfy the constitutive relation F=kx3. Different from the linear force-displacement response of traditional linear vibration reduction devices, this strong nonlinear characteristic without linear terms endows NES with unique nonlinear dynamic behavior [3]. Its core vibration reduction mechanism is nonlinear Targeted Energy Transfer (TET): during vibration, the vibration energy of the main structure (such as stay cables) will be irreversibly and unidirectionally transferred to the attached NES subsystem through nonlinear coupling, and then the transferred energy is dissipated by the damping elements configured in NES itself. The whole process does not require external energy input and belongs to a completely passive vibration control technology [4].

The nonlinear stiffness of NES breaks the limitation of frequency tuning. Its vibration reduction frequency band covers an extremely wide range, and it can realize vibration control on multiple natural modes of stay cables at the same time, which perfectly adapts to the wide-band random buffeting response under pulsating wind excitation. Meanwhile, NES has strong robustness, and has good adaptability to fluctuations of excitation frequency and changes of structural parameters (such as attenuation of cable force and structural aging during service), and will not lose vibration reduction capability due to small parameter deviations [5].

2.2. Dynamic modeling of the stay cable system

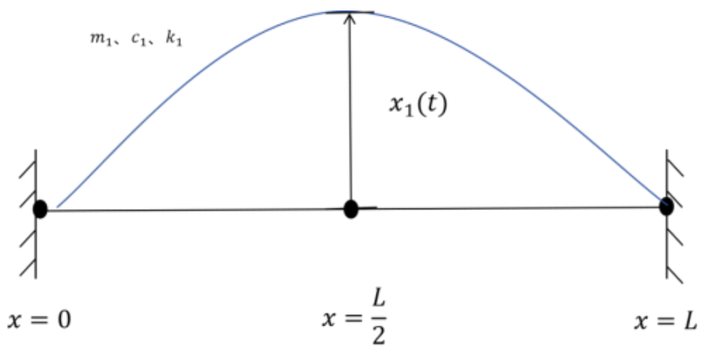

Figure 1. Mechanical model of the cable system

As shown in Figure 1, this paper equates the engineering stay cable to a single-degree-of-freedom linear oscillator fixed at both ends, and selects the mid-span lateral displacement of the cable as the core physical quantity to characterize the overall vibration response, so as to describe the vibration law of the dominant mode of the cable. The modeling process only focuses on the lateral vibration characteristics in the vertical plane, ignores the axial telescopic deformation, torsional vibration effect and high-order mode coupling of the cable, and focuses on the attenuation law of the main vibration energy.

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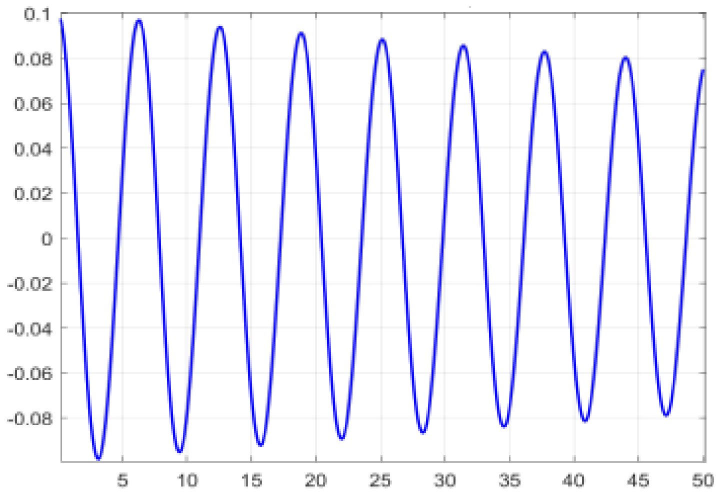

Figure 2. Time-history response curve of the cable system without NES

As shown in Figure 2, on the MATLAB platform, the reduced-order first-order differential equations are compiled into a custom right-hand side function. After inputting structural parameters such as system mass, stiffness and damping, as well as initial vibration conditions, the differential equations are solved by the fourth-order Runge-Kutta method, and the time-history response curve of the cable system without NES is obtained.

3. Dynamic modeling of the cable-NES system

3.1. Mechanical model

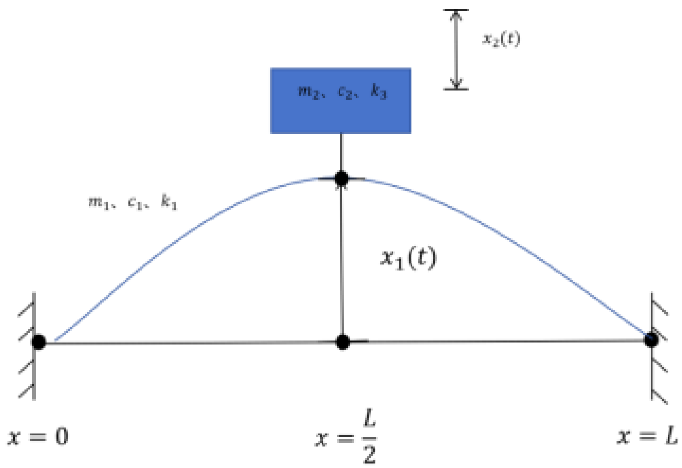

Figure 3. Mechanical model of the Suo-NES system

As shown in Figure 3, the stay cable is simplified as a single-degree-of-freedom linear oscillator simply supported at both ends, and its overall lateral vibration characteristics are characterized by the mid-span displacement of the cable. The Nonlinear Energy Sink (NES) adopts a classic single-degree-of-freedom essentially nonlinear configuration, consisting of a small mass block, linear damping elements and pure cubic nonlinear springs, without linear stiffness terms, to ensure its wide-band targeted energy transfer characteristics.

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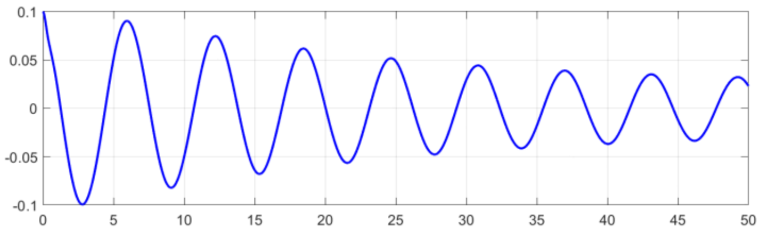

Figure 4. Time-history response curve of the cable in the cable-NES coupling system

As shown in Figure 4, Figure 5 and Figure 6, on the MATLAB platform, the reduced-order first-order differential equations are compiled into a custom right-hand side function. After inputting structural parameters such as system mass, stiffness and damping, as well as initial vibration conditions, the differential equations are solved by the fourth-order Runge-Kutta method, and the time-history response curves of the cable and the NES device in the cable-NES coupled system are obtained respectively.

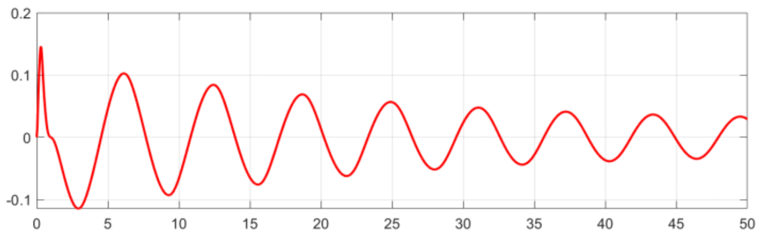

Figure 5. NES time-history response curve in the SDOF-NES coupled system

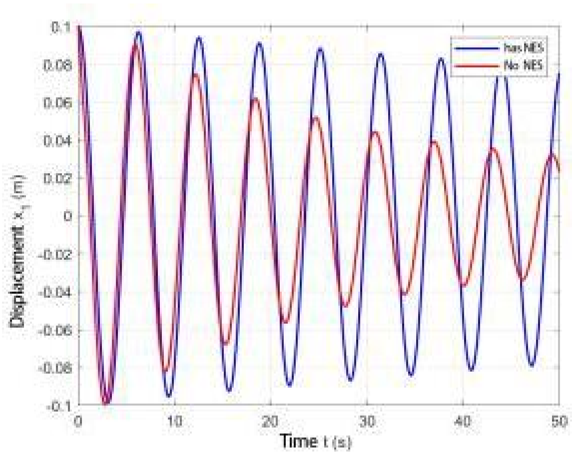

Figure 6. Comparison of time-history response curves with and without NES cable

3.2. Amplitude-frequency response analysis

Based on the free vibration equation of the cable system mentioned above, a harmonic excitation $F_0 \sin(\omega t)$ is applied to the main system (stay cable), where F_0 is the excitation amplitude (equivalently matched with the initial displacement disturbance), and $\omega = 2\pi f$ is the excitation angular frequency (f is the excitation frequency, which can vary continuously). The forced vibration equation is constructed as follows:

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$$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 = F_0\sin(\omega t) \tag{1}$$

Based on the free vibration equation of the cable-NES system mentioned above, a harmonic excitation $F_0\sin(\omega t)$ is applied to the main system (stay cable), where F_0 is the excitation amplitude (equivalently matched with the initial displacement disturbance), and $\omega = 2\pi f$ is the excitation angular frequency (f is the excitation frequency, which can vary continuously). The forced vibration equation is constructed as follows:

$$\begin{cases} m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 = c_2(\dot{x}_2 - \dot{x}_1) + k_3(x_2 - x_1)^3 + F_0\sin(\omega t) \\ m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_3(x_2 - x_1)^3 = 0 \end{cases} \tag{2}$$

The core parameters of the two types of equations remain consistent with those above (cable: $m_1 = 1.0\ kg$, $c_1 = 0.01\ N \cdot s/m$, $k_1 = 1.0\ N/m$; NES: : $m_2 = 0.05\ kg$, $c_2 = 0.20\ N \cdot s/m$, $k_3 = 1000\ N/m^3$) to ensure the coherence and consistency of the analysis.

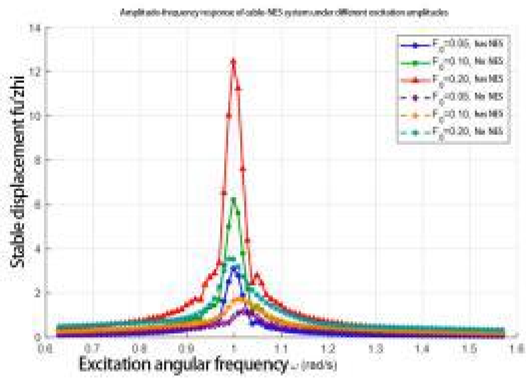

Figure 7. Frequency response comparison of the Suo-NES system

As shown in Figure 7, the steady-state displacement amplitude-frequency response curves of the cable structure with and without the NES (Nonlinear Energy Sink) device under three different excitation amplitudes ($F_0 = 0.05N$,$F_0 = 0.10N$,$F_0 = 0.20N$) are presented. Time-history analysis shows the dynamic time-sequence process of energy transfer, while amplitude-frequency analysis quantifies the energy suppression effect at different frequencies. Both together confirm the scientificity and high efficiency of the vibration reduction mechanism of the cable-NES system under high nonlinear stiffness.

3.3. Random response analysis

First, the dynamic equation of the pure cable system without NES under random excitation is established:

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 = F(t) \tag{3}$$

On this basis, the dynamic equation of the cable-NES coupled system under random excitation is established:

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$$\begin{cases} m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 = c_2(\dot{x}_2 - \dot{x}_1) + k_3(x_2 - x_1)^3 + F(t) \\ m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_3(x_2 - x_1)^3 = 0 \end{cases} \tag{4}$$

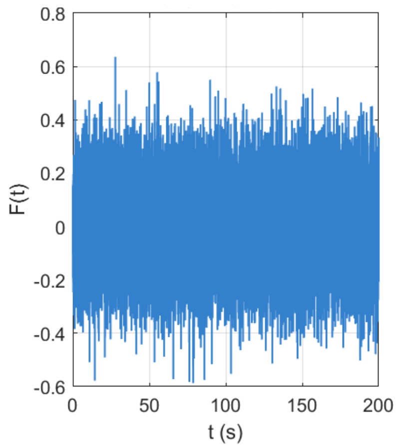

Figure 8. Time history diagram of Gaussian white noise random excitation

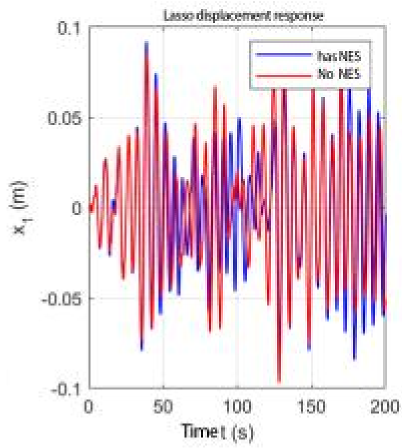

Figure 9. Comparison of random displacement responses of cables

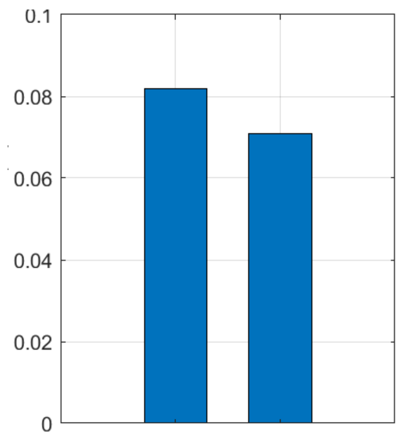

Figure 10. Comparison of RMS average values of cable displacement

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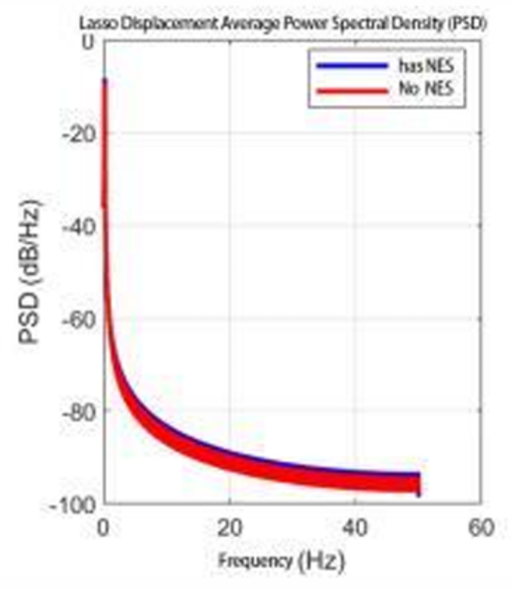

Figure 11. Comparison of average power spectral density of cable displacement

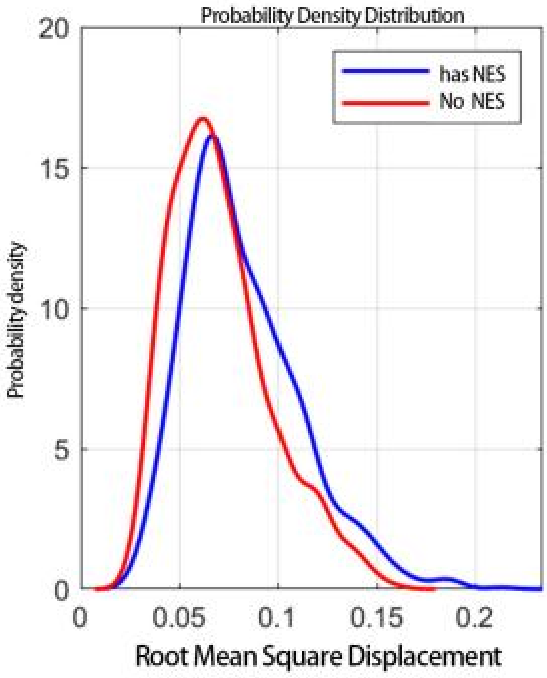

Figure 12. Comparison of probability density of cable displacement response

As shown in Figure 8, Figure 9, Figure 10, Figure 11 and Figure 12, the calculation results show that after adding NES, the root mean square value of the cable displacement response is significantly reduced, and the power spectrum is effectively suppressed in the full frequency band, especially in the low-frequency main resonance region. At the same time, the probability density function of cable displacement changes from a wide-tailed distribution to a narrow-peak distribution, and the probability of large-amplitude vibration decreases significantly. The above phenomena indicate that under random excitation, NES can still quickly and irreversibly transfer the vibration energy of the

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cable to itself through the nonlinear targeted energy transfer (TET) mechanism and dissipate it through damping.

3.4. Parameter optimization of NES

As shown in Figure 13, Figure 14, Figure 15 and Figure 16, in the random response analysis, by comparing the probability density function diagrams of the cable displacement response in detail, it can be clearly observed that the vibration suppression effect of the Nonlinear Energy Sink (NES) on the stay cable structure does not reach the expected ideal state, and its vibration reduction performance needs to be further improved.

Table 1. Table of random response analysis

No.	Parameter Symbol	Physical Meaning	Value	Unit
1	m_1	Equivalent modal mass of stay cable	1.0	*kg*
2	c_1	Linear damping coefficient of stay cable	0.005	*N ·s/m*
3	k_1	Linear stiffness coefficient of stay cable	1.0	*N/m*
4	m_2	Mass of NES subsystem	0.15	*kg*
5	c_2	Linear damping coefficient of NES	0.4	*N ·s/m*
6	k_3	Cubic nonlinear stiffness coefficient of NES	800	N/m^3

Energy dissipation and transfer capacity, and finally achieve a better and more stable vibration control effect on stay cables.

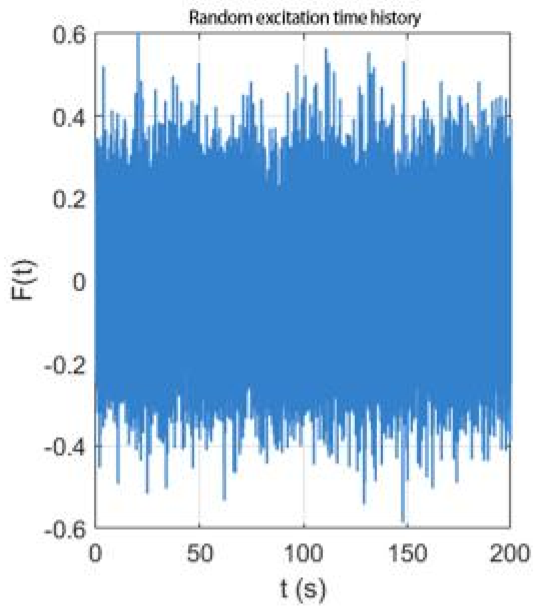

Figure 13. Time history diagram of Gaussian white noise random excitation(2)

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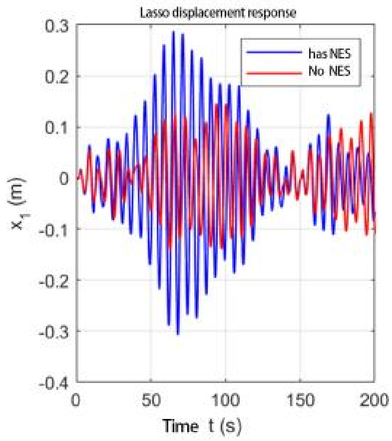

Figure 14. Comparison of random displacement responses of cables(2)

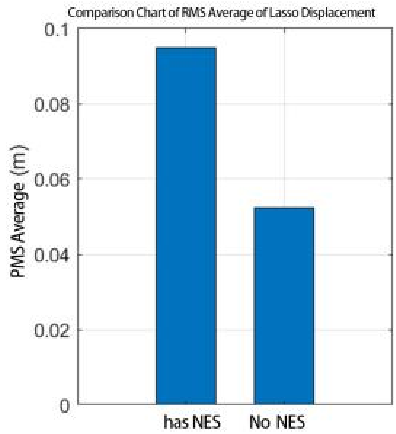

Figure 15. Comparison of RMS average values of cable displacement(2)

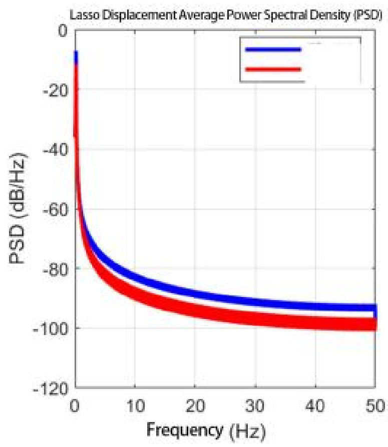

Figure 16. Comparison of average power spectral density of cable displacement(2)

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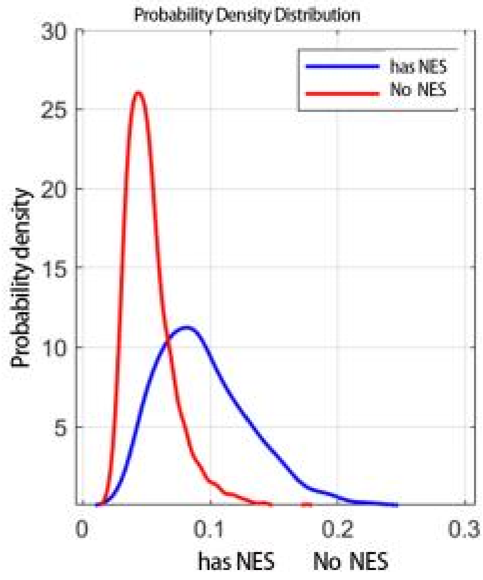

Figure 17. Comparison of probability density of cable displacement response(2)

As shown in Table 1, under the excitation of Gaussian white noise, it can be seen from the time history of cable displacement response that after adding the optimally configured NES, the cable displacement amplitude is significantly suppressed, the vibration peak is greatly reduced, and the response process tends to be stable faster.

As shown in Figure 17, it can be seen from the average power spectral density (PSD) of cable displacement that under the condition without NES, the energy is mainly concentrated in the low frequency band, forming an obvious resonance peak; after adding NES, the PSD curve of the whole frequency band moves down significantly, especially the resonance peak near the first-order natural frequency of the cable is greatly flattened. The probability density distribution curve further shows that the displacement RMS distribution under the condition with NES is more concentrated in the low value range, the peak value increases significantly and the position shifts to the low displacement direction, indicating that NES effectively reduces the probability of large-amplitude vibration of the cable and improves the stability of the system response.

4. Conclusion

Taking the wind-induced vibration control of stay cables of long-span cable-stayed bridges as the engineering background, aiming at the limitations of traditional linear vibration reduction devices such as narrow frequency band and easy detuning, this paper carries out systematic research on the dynamic modeling, vibration reduction mechanism and control effect of the cable-NES (Nonlinear Energy Sink) coupled system. Through theoretical modeling, numerical simulation and comparative analysis of multiple working conditions, the vibration reduction performance of NES under different excitation environments is comprehensively verified from three dimensions: time domain, frequency domain and statistical significance.

Relying on the nonlinear targeted energy transfer (TET) mechanism, NES can realize efficient suppression of free vibration, main resonance and random vibration of stay cables without frequency tuning, and shows excellent vibration reduction effects in time domain, frequency domain and statistical significance. The research results clearly clarify the dynamic behavior and energy transfer law of the cable-NES system, define the influence law of nonlinear stiffness, damping and mass

DOI: 10.54254/2753-8818/2026.PU35024

parameters on the vibration reduction effect, and provide reliable theoretical basis and numerical support for wind-induced vibration control of stay cables.

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