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Applications of Residue Theorem in Modern Mathematical Physics

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Abstract. The residue theorem is a widely recognized concept in mathematics. Years of thorough research have already validated its theoretical standing. It still offers massive convenience and holds significant weight when applied to modern mathematical physics. This paper first introduces the basic definition and core theoretical foundation of the residue theorem, then walks through its actual uses in integral calculations covering magnetic force, topological phase transitions, and fidelity susceptibility. It uses one-dimensional models for clear, straightforward derivation work, turning complex real integrals into contour integrals. One can get analytical solutions simply by calculating the residues of these contour integrals. A key part of using the residue theorem is finding all the singularities involved. The results make it clear that the residue theorem works as a highly efficient and rigorous method for solving tricky physical integrals in both classical electromagnetism and quantum condensed matter physics, especially when stacked up against traditional definite integral calculation methods. It works as a bridge connecting mathematical theory and real physical applications. This unified framework makes relevant concepts easier to grasp. It will help push for more extensive use of the residue theorem in related research going forward.

Keywords: residue theorem, contour integration, singularities

1. Introduction

Complex analysis has been a core mathematical framework for modern mathematical physics for decades. It gives researchers rigorous analytical tools to tackle problems that would otherwise be impossible to solve in areas like classical electromagnetism and quantum condensed matter physics. The residue theorem sits right at the center of complex integral theory. It is a landmark finding built on Cauchy's integral theorems and Laurent series expansion, and it bridges pure mathematical formalism to hands-on physical problem-solving. Conventional real-variable integration techniques often run into computational roadblocks when dealing with trigonometric integrals, singular integrals, and integrals with periodic boundary conditions. The residue theorem works differently. It turns the calculation of global closed-contour integration into a simple algebraic sum of residues at isolated singularities that lie inside the contour. This key simplification fixes long-standing computational headaches when evaluating integrals. It also builds a rigorous mathematical connection between the local singular behavior of complex functions and their global integral properties [1, 2].

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For decades, people have widely used the residue theorem in classical engineering fields, and electromagnetism is the most prominent area of these fields. As early as 1998, Sachuerfu showed this theorem really works for solving electromagnetic integral equations, which describe the force exerted on current-carrying conductors [3]. In recent years, topological condensed matter physics has developed at a fast pace. The residue theorem has become a key analytical tool for frontier research under this trend. It helps people calculate topological invariants in closed form, characterize quantum phase transitions precisely, and verify bulk-boundary correspondence in low-dimensional topological systems directly [4]. Researchers have done a lot of work on this theorem already. But most existing studies either only focus on its rigorous mathematical derivation for teaching, or only apply it to a single, separate physical scenario [5]. Hardly any studies have built a unified framework that fits teaching logic, and connects the theorem's basic mathematical principles to its two sets of applications in classical engineering and frontier quantum physics. This gap has made it harder for students to understand the method, and also holds back its cross-domain application.

Section 2 lays out the complete theoretical foundation of the residue theorem. One first introduces three core preliminary results from complex analysis that underpin the theorem. These are Cauchy's Integral Theorem, Cauchy's Integral Formula, and Laurent Series Expansion. Next, one formalizes the definition of the residue at an isolated singularity, then present the full mathematical statement of the residue theorem. Another thing worth mentioning is the author also summarizes three standard, widely applicable methods for residue calculation. These methods include rules for simple first-order poles, the general formula for higher-order poles, and the unit circle substitution method for trigonometric integrals over a 0 to 2π interval. It is all pretty straightforward once people map each part step by step. Section 3 Applications is the core of this work. It introduces how the residue theorem can be applied across different fields to solve physical problems. The section is split into two subsections. Finally, the author wraps up the core findings of this work. The residue theorem works for both classical and quantum physical domains, this universal applicability is a key point to note. The author also covers its potential real-world uses in rapidly developing fields, these include topological photonics and quantum information processing.

2. Theoretical foundation of residue theorem

2.1. Related works

Current studies on the residue theorem usually fall into two connected research areas. The first area focuses on theory and teaching, digging into the core mathematical logic supporting the theorem. The second is applied research, where people use the theorem to solve physics problems that are unique to different professional fields. It clicks pretty easily once people get these two categories sorted out. The author usually talks about mathematical physics education. Most mainstream studies in this area put a lot of focus on the residue theorem, and they mostly center on its strict mathematical derivations.

They also cover the basic supporting results behind it: Cauchy's integral theorem, Cauchy's integral formula, and Laurent series expansion, plus standard rules for calculating residues at different types of singularities. Another thing worth noting is the point Meng and Guan [1] raised. They point out a key flaw in traditional teaching methods. Standard mathematical physics curricula put formal derivations of the theorem first, but rarely tie these mathematical operations to actual real-world physical problems. This overfocus on abstract formal rules often makes students unable to link the theorem's mathematical framework to what it can actually do in practice. It drags down

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both learning effects and students' interest in the content as a whole. It is a pretty noticeable gap in current teaching designs.

When it comes to applied research, the residue theorem is widely used in two core physics fields. These two fields are classical electromagnetism and topological condensed matter physics. Looking at classical electromagnetism specifically, Sachuerfu [3] was the first to create a systematic set of steps. These steps are used to calculate the trigonometric integral that defines the Lorentz force on a circular current-carrying coil placed close to an infinite straight wire. Traditional methods for solving this integral depend on unwieldy trigonometric identities and multi-step variable substitutions. This makes them really easy to lead to calculation mistakes. Sachuerfu's method works differently. It converts the real-valued trigonometric integral into a closed contour integral on the complex unit circle. This lets researchers get a closed-form solution just by summing residues in a simple way. It directly shows how much more efficient the theorem is than old techniques. It is a pretty clear improvement, honestly.

Topological condensed matter physics is a fast-moving research area these days. The residue theorem has become one of the most essential tools for analyzing and characterizing topological phases in this field. The one-dimensional (1D) Su-Schrieffer-Heeger (SSH) model is the classic example of a 1D topological insulator. Meng and Guan [1] leveraged the residue theorem to work out an exact analytical expression for the second-order displacement. This quantity acts as a topological marker that can tell trivial and topological phases apart. They first mapped the Brillouin zone integral to a contour integral around the unit circle. Using this method, they pinned down the topological phase transition boundary very accurately. The whole study serves as a straightforward, easy-to-follow example of how useful the residue theorem can be in quantum physics research. Another line of work builds on this foundation. Luo applied the residue theorem to the 1D XY model, which is a core model for studying low-dimensional quantum magnetism [2]. He first set up the integral expression for fidelity susceptibility, a widely used metric to detect quantum phase transitions. He then worked out its closed analytical form through residue calculation. This study offers a rigorous method to locate quantum critical points in spin chains, and also gives more proof that the residue theorem works well across a range of quantum many-body systems.

Researchers have made plenty of advances in this area, but current studies still operate in separate silos. Pedagogical works almost never include the latest cutting-edge physical applications, and applied studies treat the residue theorem as nothing more than a simple computational tool. They do not try to connect its use back to the core mathematical principles it is built on. No existing research has put together a unified, step-by-step framework that moves smoothly from the theorem's basic mathematical fundamentals to cross-domain applications in both classical electromagnetism and topological quantum physics. This paper fills this exact gap. It builds out this kind of framework, with two clear goals in mind. First, it aims to provide a comprehensive pedagogical resource, and second, it wants to show the cross-cutting utility of the theorem. It is pretty straightforward in its scope, no extra fluff added here.

2.2. Preliminary theorems and core formulations

The residue theorem wasn't built out of nowhere. It rests on three key results from complex function theory. These three findings together set up all the theoretical basis needed for its formal definition.

One of the theorems is Cauchy's Integral Theorem, which states that if a function $f(z)$ is analytic everywhere in a simply connected domain$D$, and C is any piecewise smooth simple closed contour lying entirely in the domain D, then $\oint_c f(z)dz = 0$. This theorem raises a core question at

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once that when the closed contour Cencloses points where$f(z)$cannot be analytic, such as singularities, then the contour integral will no longer vanish. Furthermore, a direct and critical corollary of Cauchy's Integral Theorem, Cauchy's Integral Formula, quantifies the integral contribution of a single isolated singularity, laying the core intuition for the residue concept. It says that under the same condition mentioned above and $\overline{D} = D \cup C$, with z_0, an any point inside C , then $f(z_0) = \frac{1}{2\pi i} \oint_c \frac{f(z)}{z-z_0} dz.$ For higher-order derivatives, $f^{(n)}(z_0) = \frac{n!}{2\pi i} \oint_c \frac{f(z)}{(z-z_0)^{n+1}} dz, \ \ n = 1,2,3,\ldots$ This formula demonstrates that the integral around a contour enclosing an isolated singularity is fully determined by the local behaviour of the function at that singularity, which is the core logic of Residue Theorem. Laurent Series Expansion is the last one of the groundworks. For $f(z)$ analytic at point z_0 , it can be expanded into a Taylor series in the neighbourhood of z_0 . However, when $f(z)$is only analytic in the deleted neighbourhood $0 < |z - z_0| < R$, where z_0 is an isolated singularity, then the Taylor series is no longer applicable. Instead, the author uses the Laurent series, which consists of both non-negative power terms, which are the analytic parts, and negative power terms (principal parts). It is the coefficient of the term $(z - z_0)^{-1}$in the expansion that defines the residue of $f(z)$ at z_0 , providing the direct mathematical basis for the formal definition of residue.

2.3. Core definitions of residue and residue theorem

The residue is the coefficient of the term $(z - z_0)^{-1}$ in the Laurent series expansion of an analytic function around an isolated singularity z_0 , which quantifies the local singularity of the function. For a function $f(z)$ analytic in a deleted neighbourhood of a singularity z_0 , its Laurent expansion is shown as:

$$f(z) = \sum_{n=-\infty}^{\infty} a_n (z - z_0)^n \tag{1}$$

The coefficient a_{-1} is the residue of $f(z)$ at z_o denoted as $Res[f(z), z_0]$.

Residue theorem transforms closed-contour integration into summation of residues at singularities, which realizes the transformation between 'global' to 'local'. This theorem stipulates that if $f(z)$ is resolved in the closed curve C except for a finite number of singularities $z_1, \ z_2, \ldots, \ z_n$ inside the contour C . Then the intergal along the positive side satisfies:

$$\oint_C f(z)\, dz = 2\pi i \sum_{k=1}^{n} Res\,[\,f(z), \ z_k] \tag{2}$$

2.4. General methods for residue calculation

There are many avenues for us to calculate the residue in practice. However, to put it simply, there are three general methods to be introduced [6].

The first one, for a simple, first-order pole z_0 , which satisifies $\frac{\varphi(z)}{\psi(z)}$, with conditions of $\varphi(z_0) \neq 0, \ \psi(z_0) = 0, \ \psi'(z_0) \neq 0,$ then: $Res\,[f(z), \ z_0] = \frac{\varphi(z)}{\psi'(z)}$ [7]. Secondly, if $z_0$is a higherorders pole of m , then one has:

$$Res\left[f\left(z\right),\ z_0\right]=\frac{1}{(m-1)!}\lim_{z\to z_0}\frac{d^{m-1}}{dz^{m-1}}\left[\left(z-z_0\right)^m f\left(z\right)\right] \tag{3}$$

Last but not least, if it is a contour integration on the unit circle, for trigonometric definite integrals, people usually use the substitution$z=e^{i\theta}$to convert intergals over$[0,\ 2\pi]$ into contour integrals on $|z|=1$ [8].

3. Applications of residue theorem

3.1. Integral solution for the magnetic force on a current-carrying circular coil

As shown in Figure 1, an infinitely long straight current-carrying wire is in the same plane as a circular current with Radius R. Their currents are respectively$I_1,\ I_2$. The straight wire is at a distance a from the center of the circle current. The question is to find the magnetic force$\overrightarrow{F}$acting on the circular current [9].

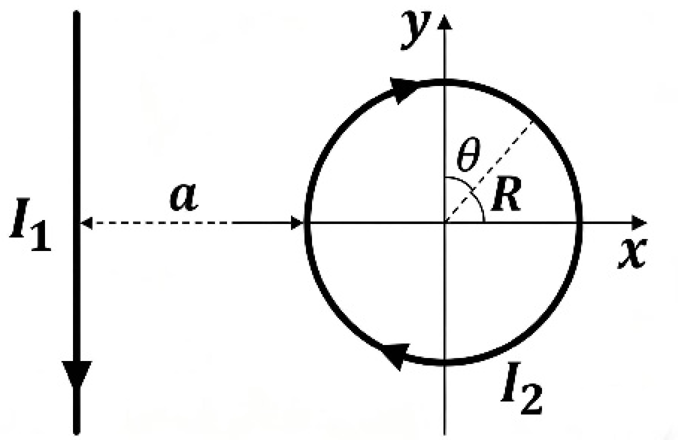

Figure 1. Illustration of long straight current-carrying wire

By analysis, this is a problem in electromechanics concerning the force on a current-carrying conductor in a non-uniform magnetic field. The force between the long straight current-carrying wire and the circular current can then be decomposed via Ampere's law and Biot-Savart law. People can easily know that $\overrightarrow{F}=F_x\overrightarrow{i}+F_y\overrightarrow{i}.$

The key integral for the x and y components of the force respectively are

$$F_x=\frac{\mu_0 I_1 I_2 R}{2\pi}\int_0^{2\pi}\frac{\cos\theta\,d\theta}{a+R\cos\theta}\qquad F_y=\frac{\mu_0 I_1 I_2 R}{2\pi}\int_0^{2\pi}\frac{\sin\theta\,d\theta}{a+R\cos\theta} \tag{4}$$

By calculation, the author can get $F_y=0$. Therefore, $\overrightarrow{F}=F_x\overrightarrow{i}$.

The x-component, $\int_0^{2\pi} \frac{\cos\theta\, d\theta}{a+R\cos\theta}$ is a definite integral in trigonometric form. In this case, it is very troublesome and difficult to obtain the correct answer using conventional methods. To avoid this, the author can use Residue Theorem to evaluate the integral. As mentioned, this case satisfies the third general way of integration using substitute of $z = e^{i\theta}$. $\theta \epsilon \ [0,\ 2\pi]$, $|z| \leq 1$, relationships below are satisfied: $cos\theta = \frac{1}{2}\left(z + z^{-1}\right),\ sin\theta = \frac{1}{2}\left(z - z^{-1}\right),\ d\theta = \frac{1}{iz}dz$.

Then the definite integral is converted into a loop integral in a complex function.

$$\int_0^{2\pi} \frac{\cos\theta\, d\theta}{a+R\cos\theta} = \frac{1}{ia}\oint_{|z|=1} \frac{z^2}{z(\varepsilon z^2+2z+\varepsilon)}dz \tag{5}$$

where$\varepsilon = R/a$. The form of the product complex variable function is $f(z) = \frac{z^2}{z(\varepsilon z^2+2z+\varepsilon)}$, and there are three poles, $z_1 = 0,\ z_2 = \frac{-1+\sqrt{1-e^2}}{\varepsilon}$, $z_3 = \frac{-1-\sqrt{1-e^2}}{\varepsilon}$, respectively. All of them are first order. $z_1,\ z_2$ are in the unit circle. By using the Residue Theorem,

$$Resf(0) = \frac{1}{\varepsilon}, Resf\left(\frac{-1+\sqrt{1-e^2}}{\varepsilon}\right) = -\frac{1}{\sqrt{1-e^2}} \tag{6}$$

Here,

$$\int_0^{2\pi} \frac{\cos\theta\, d\theta}{a+R\cos\theta} = \frac{2\pi}{ia}i\left\{ Res\ f(0) + \ Res\ f\left(\frac{-1+\sqrt{1-e^2}}{\varepsilon}\right)\right\} = 2\pi\left(\frac{1}{R} - \frac{1}{\sqrt{a^2-R^2}}\right) \tag{7}$$

In Eq. (7), $\varepsilon = \frac{R}{a}$. Substitute (2) using (6) and (7), $\overrightarrow{F} = F_x\overrightarrow{i} = \mu_0 l_1 l_2\left(1 - \frac{R}{\sqrt{a^2-R^2}}\right)\overrightarrow{i}$.

3.2. Topological phase transition in the 1D SSH model

One-dimensional (1D) topological models serve as the foundational paradigms in modern topological condensed matter physics, providing analytically tractable and experimentally feasible platforms for elucidating the fundamental principles that govern topological quantum phases. The canonical Su-Schrieffer-Heeger model, together with its well-established extensions (the Rice-Mele and Aubry-André-Harper models), allows for rigorous characterization of topological invariants and direct verification of the bulk-boundary correspondence, the core doctrine of topological systems. Their perturbation-resilient topological boundary states further lay the groundwork for groundbreaking advances in topological photonics, low-dimensional quantum electronics, and topological quantum information processing [10].

The Su-Schrieffer-Heeger (SSH) model is a canonical 1D topological insulator model. Its topological transition can be characterized by the second-order displacement M , taking the integral form in the long-time limit:

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$$M_{SSH} = \frac{1}{2\pi} \int_0^{2\pi} \frac{{t_1}^2{t_2}^2 sin^2k}{{t_1}^2+{t_2}^2+2t_1t_2cosk}\, dk \tag{8}$$

Using substitution $z = e^{ik}$ in (8), one can convert the fixed integral into a loop integral: $M_{SSH} = \oint_c f(z)$, where c is a unit circle on the complex plane with the origin as the center and

$$f\,(z) = \frac{i{t_1}^2{t_2}^2}{8\pi} \frac{\left(z^2-1\right)^2}{t_1t_2z^4+\left({t_1}^2+{t_2}^2\right)z^3+t_1t_2z^2} \tag{9}$$

Let the denominator of the product function $f(z)$ be 0, there is three singularities, $b_1 = 0$, $b_2 = \frac{-t_1}{t_2}$, $b_3 = \frac{-t_2}{t_1}$. Select those which are inside the unit circle with radius one and the author can find that b_1 is a second-order pole while b_2 and b_3 are first-order pole of $f(z)$. Calculate and sum residues to obtain: $M_{SSH} = \begin{cases} \frac{{t_1}^2}{2},\ t_1 < t_2\left(tpological\ phase\right) \\ \frac{{t_2}^2}{2},t_1 > t_2\left(trivial\ phase\right) \end{cases}$. The second-order displacement exhibits a cusp at the phase transition point, directly locating the topological phase boundary and supporting experimental observations of topological states.

3.3. Fidelity susceptibility of the 1D XY model

The one-dimensional (1D) XY model is a canonical exactly solvable framework for low-dimensional quantum magnetism. Mapped to free fermions via Jordan-Wigner transformation, it enables rigorous characterization of topological spin phases and serves as a core quantum simulation benchmark. As a core analytically tractable platform for low-dimensional topological quantum matter, the 1D XY model reveals fundamentals of topological spin phases and quantum criticality. Through Jordan-Wigner fermionization, it hosts protected boundary modes, acting as a key testbed for quantum simulation research. Fidelity Susceptibility is a crucial probe for quantum phase transitions. The Hamiltonian quantity of a 1D XY model is

$$H = -\sum_{j=1}^{2N} \left[\frac{1+\gamma}{2}\,\sigma_j^x\sigma_{j+1}^x + \frac{1-\gamma}{2}\,\sigma_j^x\sigma_{j+1}^x + h\sigma_j^z\right] \tag{10}$$

Let $h > 0$ and $\gamma \in (-1,1\,)$, for the parameter-dependent integral:

$$L_s\,(h,\gamma) = \int_{2\pi}^{0} \frac{sin^2k}{cos^2k-\frac{2h}{1-\gamma^2}\,cosk+\frac{h^2+\gamma^2}{1-\gamma^2}}\, dk \tag{11}$$

Its analytical form is

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$$L_s = \begin{cases} 2\pi\frac{1}{|\gamma|-1}, & |h| < 1 \\ 2\pi\frac{h}{\sqrt{h^2+\gamma^2-1}} - 1, & |h| > 1 \end{cases} \tag{12}$$

Through Veda Theorem [11], one can convert the desired integral into:

$$L_s = \int_0^{2\pi} \frac{sin^2 k}{(cosk-\lambda)(cosk-\mu)}\, dk \tag{13}$$

For the equation of the form$z^2 + 1 - 2\lambda z = 0$, its two roots are $z \pm (\lambda) = \lambda \pm \lambda 2 - 1$.Thus, there exists exactly one singular point inside the unit circle $|z| = 1$. One can similarly apply it to the equation $z^2 + 1 - 2\mu z = 0$. Therefore, by factorizing the denominator of the integrand, People have $L_s = i \int_{|z|=1} gs\,(z)dz$,where

$$g_s\,(z) = \frac{\left(z^2-1\right)^2}{z\left(z-z_-^{(\lambda)}\right)\left(z-z_+^{(\lambda)}\right)\left(z-z_-^{(\mu)}\right)\left(z-z_+^{(\mu)}\right)} \tag{14}$$

It should be noted that there are three first-order singular points inside $|z| = 1$. Through Residue Theorem, one can get: $L_s = -2\pi + \frac{2\pi}{\lambda-\mu}\left(\sqrt{\lambda^2-1} - \sqrt{\mu^2-1}\right)$ The author will now deal with the radical terms. One already has:

$$\sqrt{(t_\pm)^2 - 1} = |h|\gamma| \pm \sqrt{h^2+\gamma^2-1}| \Big/ (1-\gamma^2) \tag{15}$$

Since the product of factors$\left(h\,|\gamma| + \sqrt{h^2+\gamma^2-1}\right)$ $\left(h\,|\gamma| + \sqrt{h^2+\gamma^2-1}\right. = \left(1-\gamma^2\right)\left(1-h^2\right)$.Both factors are positive when $|h| < 1$, and they have opposite signs and are consistent with the sign before the radical when $|h| > 1$. Thus

$$\sqrt{\lambda^2-1} - \sqrt{\mu^2-1} = \begin{cases} -\frac{2\sqrt{h^2+\gamma^2-1}}{1-\gamma^2}, |h| < 1 \\ 2\pi\left(\frac{|h|}{\sqrt{h^2+\gamma^2-1}} - 1\right), |h| > 1 \end{cases} \tag{16}$$

Therefore, $\frac{1}{\sqrt{\lambda^2-1}} - \frac{1}{\sqrt{\mu^2-1}} = \begin{cases} \frac{2\sqrt{h^2+\gamma^2-1}}{1-h^2} \\ \frac{2|h\gamma|}{h^2-1}, |h| > 1 \end{cases}, |h| < 1$. Substituting the above radical formula into the expression of L_s proves the validity of the analytical form.

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4. Conclusion

In conclusion, this paper sorts out the core theoretical basis of the residue theorem, and demonstrates its strong capability in solving physical integral problems through three typical application cases: Lorentz force calculation of a current-carrying circular coil, topological phase transition characterization of the 1D Su-Schrieffer-Heeger (SSH) model, and fidelity susceptibility calculation of the 1D XY model.

This paper focuses on applications in classical electromagnetism. This paper first builds the physical model for the Lorentz force between an infinite straight current-carrying wire and a coplanar circular current-carrying coil. The author splits the force into components along the Cartesian axes, then use the standard unit circle substitution to turn the non-vanishing x-component integral into a complex contour integral. The author then works out this integral with the residue theorem, and get the closed-form analytical expression for the total force acting on the coil.

This paper also covers applications in low-dimensional topological condensed matter physics. It includes two typical model systems. First, it looks at the topological phase transition in the 1D SSH model. The author takes the Brillouin zone integral for the second-order displacement, which marks topological invariants, and map it to a unit circle contour integral. The author works out the residues at the singularities the contour encloses. The author then gets the analytical form of the second-order displacement for both the topological and trivial phases. That lets us nail down the exact phase transition point easily. The author also calculates the fidelity susceptibility of the 1D XY model. The author first converts the parameter-dependent integral for fidelity susceptibility into a complex contour integral, then solve it analytically using the residue theorem. The author also runs checks to confirm the closed-form expression works across different parameter regimes. This whole process gives us a rigorous method to characterize quantum phase transitions in 1D spin chains. It is pretty solid for this line of research actually.

The results show that the residue theorem can convert complex real-variable integrals into simple algebraic summation of residues at isolated singularities. It not only avoids the cumbersome calculation of traditional integration methods for classical electromagnetic problems, but also provides a rigorous analytical tool for topological invariant calculation and quantum phase transition characterization in low-dimensional quantum systems. This work fills the gap between mathematical theory and physical application in traditional mathematical physics teaching, and the method can be further extended to non-Hermitian topological systems and open quantum many-body systems in follow-up research.

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