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Optimal Decay Rate to the Contact Discontinuity for 1-D Compressible Radiation Hydrodynamics Model

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Abstract. The radiation hydrodynamics equations, which describe the interaction between fluid flow and radiation, play a fundamental and essential role in modeling a wide range of high-energy physical phenomena. In this paper, we consider the optimal decay rate of solutions to the initial value problem for the one-dimensional compressible radiative hydrodynamics model with respect to contact discontinuities. Specifically, for the case with viscosity but without considering heat conduction, under the non-zero mass condition, the optimal decay rate $(1+t)^{-1/2}$ of the solution to the viscous contact discontinuity in the $L^{\infty}-$ norm is obtained, provided that the initial perturbations around the contact discontinuity and the strength of the contact discontinuity are sufficiently small. The absence of heat conduction effects poses substantial difficulties in establishing energy estimates and decay estimates of the solution. By employing the energy method together with a crucial transformation, we obtain refined energy estimates and derive the optimal time-decay rate of the solution.

Keywords: compressible radiation hydrodynamics, contact discontinuity wave, optimal decay rate

1. Introduction

In this paper, we consider the one-dimensional compressible radiation hydrodynamics model, which has important applications in inertial confinement fusion, astrophysics, aerospace engineering and high-temperature plasma physics. In Lagrangian coordinates, this system can be expressed as:

$$\begin{cases} v_t - u_x = 0, \\ u_t + p_x = \left(\mu \frac{u_x}{v}\right)_x, \\ \left(e + \frac{1}{2}u^2\right)_t + (pu)_x + q_x = \left(\mu \frac{u u_x}{v}\right)_x, \\ -\left(\frac{q_x}{v}\right)_x + vq + \left(\theta^4\right)_x = 0, \quad t>0,\ x \in \mathbb{R}, \end{cases} \tag{1}$$

where $v(t,x)>0$, $u(t,x)$, $\theta(t,x)>0$, and $q(t,x)$ are the specific volume, velocity, absolute temperature, and radiation flux, respectively. The viscosity coefficient μ is a positive constant. The

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pressure $p(x,t)$ and internal energy $e(x,t)>0$ satisfy $p=R\theta/v,\ e=R\theta/(\gamma-1)$, where $R>0$ is the gas constant and $\gamma>1$ is the adiabatic constant. We now set the initial data and far field conditions for (1) as follows:

$$\begin{cases}(v,u,\theta)(0,x)=(v_0,u_0,\theta_0)(x), \quad x\in\mathbb{R},\\(v_0,u_0,\theta_0)(\pm\infty)=(v_\pm,u_\pm,\theta_\pm),\end{cases} \tag{2}$$

where $u_\pm\in\mathbb{R}$, $v_\pm>0$ and $\theta_\pm>0$ are given constants. The initial data $v_0(x)>0$, $\theta_0(x)>0$ and q_0 can be achieve in the equation $-(q_{0x}/v_0)_x+v_0q_0=-(\theta_0^4)_x$.

We now review some results related to this paper. As is well known, the hyperbolic conservation laws have three basic Riemann solutions, i.e., shock wave, rarefaction wave, and contact discontinuity. There have been extensive studies on the stability of basic waves for the compressible Navier-Stokes equations. We refer to [1-5] for shock waves, [6-11] for rarefaction waves, and [12-15] for contact discontinuities. With regard to the superposition of different wave patterns, Huang, Li, and Matsumura [16] considered the combination of a contact discontinuity with two rarefaction waves and established corresponding stability results. Recently, Kang, Vasseur, and wang [17] proved the stability of the superposition of viscous shock wave and rarefaction wave.

For the simplified radiation hydrodynamics models, Hamer [18] studied wave phenomena and showed that the model reduces to the Burgers equation in the optically thick limit. Kawashima and Nishibata [19] proved the asymptotic stability of shock waves for the model considered in [18]. Subsequently, Kawashima and Tanaka [20] established the stability of rarefaction waves. Gao, Ruan, and Zhu [21-23] extended the analysis to multi-dimensional cases. For continuous shock waves, Ohnawa [24] generalized the result in [19].

In this paper, we study the large-time asymptotic behavior of perturbations around viscous contact waves under the non-zero mass condition. For the compressible radiation hydrodynamics model, Wang and Xie [25], Rohde and Xie [26], and Liu, Wang and Zhang [27] considered viscous contact waves, proved their asymptotic stability, and obtained decay rates. Rohde and Xie [26] studied the problem under the non-zero mass condition, while Liu, Wang and Zhang [27] investigated it under the zero mass condition. The stability of the combination of a viscous contact wave with rarefaction waves was studied in [28].

For the compressible radiation hydrodynamics model with viscosity and heat conduction, we refer to [29] for its derivation. Hong [30] obtained the asymptotic behavior toward the combination of a contact discontinuity and rarefaction waves for the model describing the one-dimensional compressible viscous gas with radiation. In contrast to the models with heat conduction cited above, the present paper considers the same system but without the heat conduction term. Specifically, we investigate the optimal convergence rate of solutions when the initial data are a small perturbation around a viscous contact wave. Compared with the models incorporating heat conduction in [29-30], the absence of heat conduction in the system considered herein introduces additional difficulties in the solution estimates. Moreover, in contrast to the full radiation model without viscosity or heat conduction in [26], which yields a decay rate of $(1+t)^{-1/4}$, the present work achieves a better decay rate of $(1+t)^{-1/2}$. This decay rate is also optimal, coinciding with that of the Navier-Stokes equations [31]. Our main approach is based on energy estimates. Following the method introduced in [26, 31] and performing a detailed energy analysis, we obtain both the optimal decay rate and the stability of the viscous contact wave under the non-zero mass condition for the perturbation.

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As in [26], we take into account the nonlinear diffusion equation:

$$\Theta_t = \frac{p_+(\gamma-1)}{\gamma R^2}\left(4\Theta^2\Theta_x\right)_x, \quad \Theta(t, \pm\infty) = \theta_\pm. \tag{3}$$

From the relevant results of [32], there exists a unique self-similarity solution $\Theta(t,x) = \Theta(\xi)$, $\xi = x/\sqrt{1+t}$ to (3). In addition, for $\delta = |\theta_+ - \theta_-|$, there exists a constant $\tilde{\delta} > 0$ such that if $\delta \leq \tilde{\delta}$, then Θ satisfies

$$\left(1+t\right)^{j/2}\left|\partial_x^j\Theta\right| + |\Theta - \theta_\pm| \leq c_1\tilde{\delta}e^{-c_0x^2/1+t}, \quad j \geq 1, \text{ as } |x| \to \infty,$$

here constants $c_0 > 0$ and $c_1 > 0$ independent of t and x . Then, the contact wave profile $\left(\bar{v}, \bar{u}, \bar{\theta}, \bar{q}\right)$ is defined below

$$\bar{v} = \frac{R\Theta}{p_+}, \quad \bar{u} = \frac{\gamma-1}{\gamma R}4\Theta^2\Theta_x, \quad \bar{\theta} = \Theta - \frac{\gamma-1}{2R}\bar{u}^2, \quad \bar{q} = -\frac{\left(\Theta^4\right)_x}{\bar{v}}. \tag{4}$$

By a direct calculation, $\left(\bar{v}, \bar{u}, \bar{\theta}, \bar{q}\right)$ obeys

$$\begin{cases} \bar{v}_t - \bar{u}_x = 0, \\ \bar{u}_t + \bar{p}_x = \mu\left(\frac{\bar{u}_x}{\bar{v}}\right)_x + R_{1x}, \\ \left(\bar{e} + \frac{1}{2}\bar{u}^2\right)_t + \left(\bar{p}\bar{u}\right)_x + \bar{q}_x = \left(\mu\frac{\bar{u}\bar{u}_x}{\bar{v}}\right)_x + R_{2x}, \\ -\left(\frac{\bar{q}_x}{\bar{v}}\right)_x + \bar{v}\bar{q} + \left(\bar{\theta}^4\right)_x = R_{3x}, \end{cases} \tag{5}$$

where $\bar{e} = R\bar{\theta}/\left(\gamma - 1\right)$ and

$$R_1 = -\mu\frac{\bar{u}_x}{\bar{v}} + \bar{p} - p_+ + \frac{p_+(\gamma-1)^2}{\gamma R^3}4\Theta^2\left(4\Theta^2\Theta_x\right)_x,$$

$$R_2 = -\mu\frac{\bar{u}\bar{u}_x}{\bar{v}} + \left(\bar{p} - p_+\right)\bar{u}, \ R_3 = \left(\bar{\theta}^4 - \Theta^4\right) - \frac{\bar{q}_x}{\bar{v}}.$$

Let

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$$m\left(t,x\right)=\left(v,u,\theta+\frac{\gamma-1}{2R}u^{2}\right)^{t},\quad \bar{m}\left(t,x\right)=\left(\bar{v},\bar{u},\bar{\theta}+\frac{\gamma-1}{2R}\bar{u}^{2}\right)^{t},$$

and

$$A\left(v,u,\theta\right)=\begin{pmatrix}0 & -1 & 0\\ -\frac{p}{v} & 0 & \frac{R}{v}\\ -\frac{(\gamma-1)pu}{Rv} & \frac{(\gamma-1)p}{R} & \frac{(\gamma-1)u}{v}\end{pmatrix}.$$

Then, it can be easily verified that the first eigenvalue of $A(v_-,0,\theta_-)$ is $\lambda_1^-=-\sqrt{\frac{\gamma p_-}{v_-}}$, with right eigenvector $\gamma_1^-=\left(-1,\ \lambda_1^-,\ \frac{\gamma-1}{R}p^-\right)^t$. Besides, the third eigenvalue of $A(v_+,0,\theta_+)$ is $\lambda_3^+=\sqrt{\frac{\gamma p_+}{v_+}}$, with the corresponding right eigenvector $\gamma_3^+=\left(-1,\ \lambda_3^+,\ \frac{\gamma-1}{R}p^+\right)^t$.

By strict hyperbolicity, the vectors γ_1^-, $m_+-m_-=(v_+-v_-,\ 0,\ \theta_+-\theta_-)^t$, and γ_3^+ are linearly independent in $\mathbb{R}^3$. Thus, the integral can be distributed as follows

$$\int_{-\infty}^{+\infty}\left(m\left(0,x\right)-\bar{m}\left(0,x\right)\right)dx=\bar{\theta}_1\gamma_1^-+\bar{\theta}_2\left(m_+-m_-\right)+\bar{\theta}_3\gamma_3^+,$$

with some constants $\bar{\theta}_i$, $i=1,2,3$. As in [15], we can define the ansatz $\widetilde{m}(x,t)$ by

$$\widetilde{m}\left(t,x\right)=\bar{m}\left(t,x+\bar{\theta}_2\right)+\bar{\theta}_1\theta_1\gamma_1^-+\bar{\theta}_3\theta_3\gamma_3^+, \tag{6}$$

where

$$\theta_1\left(t,x\right)=\frac{1}{\sqrt{4\pi(1+t)}}e^{-\frac{\left(x-\lambda_1^-\left(1+t\right)\right)^2}{4(1+t)}},\quad \theta_3\left(t,x\right)=\frac{1}{\sqrt{4\pi(1+t)}}e^{-\frac{\left(x-\lambda_3^+\left(1+t\right)\right)^2}{4(1+t)}},$$

satisfying

$$\theta_{1t}+\lambda_1^-\theta_{1x}=\theta_{1xx},\quad \theta_{3t}+\lambda_3^+\theta_{3x}=\theta_{3xx},$$

and

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$$\int_{-\infty}^{+\infty} \theta_i\left(t,x\right)dx = 1 \quad \text{for} \quad i = 1,3.$$

More precisely, the ansatz $\widetilde{m}(t,x)$ has the following expression

$$\widetilde{m}\left(t,x\right) = \left(\tilde{v}, \widetilde{u}, \tilde{\theta} + \frac{\gamma-1}{2R}\widetilde{u}^2\right)^t, \tag{7}$$

with

$$\begin{cases} \tilde{v}\left(t,x\right) = \bar{v}\left(t,x+\bar{\theta}_2\right) - \bar{\theta}_1\theta_1 - \bar{\theta}_3\theta_3, \\ \widetilde{u}\left(t,x\right) = \bar{u}\left(t,x+\bar{\theta}_2\right) + \lambda_1^-\bar{\theta}_1\theta_1 + \lambda_3^+\bar{\theta}_3\theta_3, \\ \quad \tilde{\theta}\left(t,x\right) = \bar{\theta}\left(t,x+\bar{\theta}_2\right) + \frac{\gamma-1}{2R}\bar{u}^2\left(t,x+\bar{\theta}_2\right) + \frac{\gamma-1}{R}p_+\left(\bar{\theta}_1\theta_1 + \bar{\theta}_3\theta_3\right) - \frac{\gamma-1}{2R}\widetilde{u}^2. \end{cases} \tag{8}$$

We also define

$$\tilde{q} = -\frac{\left(\tilde{\theta}^4\right)_x}{\tilde{v}}. \tag{9}$$

Furthermore, it holds that

$$\begin{aligned} \int_{-\infty}^{+\infty}\left(m(0,x) - \widetilde{m}\left(0,x\right)\right)dx &= \int_{-\infty}^{+\infty}\left(m(0,x) - \bar{m}\left(0,x\right)\right)dx + \int_{-\infty}^{+\infty}\left(\bar{m}(0,x) - \widetilde{m}\left(0,x\right)\right)dx \\ &= \bar{\theta}_2\left(m_+ - m_-\right) + \int\left(\bar{m}(0,x) - \bar{m}\left(0,x+\bar{\theta}_2\right)\right)dx = 0. \end{aligned}$$

Without loss of generality, we assume that $\bar{\theta}_2 = 0$. It can be readily checked that the ansatz $\widetilde{m}(t,x)$ satisfies

$$\begin{cases} \tilde{v}_t - \widetilde{u}_x = \widetilde{R}_{1x}, \\ \widetilde{u}_t + \widetilde{p}_x = \mu\left(\frac{\widetilde{u}_x}{\tilde{v}}\right)_x + \widetilde{R}_{2x}, \\ \left(\tilde{e} + \frac{\widetilde{u}^2}{2}\right)_t + \left(\widetilde{p}\widetilde{u}\right)_x + \tilde{q}_x = \left(\frac{\mu}{\tilde{v}}\widetilde{u}\widetilde{u}_x\right)_x + \widetilde{R}_{3x}, \\ \tilde{v}\tilde{q} + \left(\tilde{\theta}^4\right)_x = 0, \end{cases} \tag{10}$$

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where $\tilde{e} = R\tilde{\theta}/\left(\gamma - 1\right)$, $\tilde{p} = R\tilde{\theta} / \tilde{v}$ and

$$\widetilde{R}_1 = -\bar{\theta}_1\theta_{1x} - \bar{\theta}_3\theta_{3x},$$

$$\widetilde{R}_2 = R_1 + \mu\left(\frac{\bar{u}_x}{\bar{v}} - \frac{\widetilde{u}_x}{\widetilde{v}}\right) + \left(\lambda_1^-\bar{\theta}_1\theta_{1x} + \lambda_3^+\bar{\theta}_3\theta_{3x}\right) + \left(\widetilde{p} - \bar{p} - \left(\lambda_1^-\right)^2\bar{\theta}_1\theta_1 - \left(\lambda_3^+\right)^2\bar{\theta}_3\theta_3\right),$$

$$\widetilde{R}_3 = R_2 + \mu\left(\frac{\bar{u}\bar{u}_x}{\bar{v}} - \frac{\widetilde{u}\widetilde{u}_x}{\widetilde{v}}\right) + p_+(\bar{\theta}_1\theta_{1x} + \bar{\theta}_3\theta_{3x})$$

$$+(\widetilde{p}\widetilde{u} - \bar{p}\bar{u} - p_+\lambda_1^-\bar{\theta}_1\theta_1 - p_+\lambda_3^+\bar{\theta}_3\theta_3) + (\tilde{q} - \bar{q})$$

Applying the same arguments as (2.a31) in [15], for some positive constant $c > 0$, $\widetilde{R}_i$ ($i = 1,2,3$) satisfy

$$\widetilde{R}_i = \frac{O\left(\delta + \bar{\theta}_1^2 + \bar{\theta}_3^2\right)}{1+t}\left(e^{-\frac{cx^2}{1+t}} + e^{-\frac{c\left(x-\lambda_1^-(1+t)\right)^2}{1+t}} + e^{-\frac{c\left(x-\lambda_3^+(1+t)\right)^2}{1+t}}\right).$$

Denote the perturbation around the ansatz $\left(\tilde{v}, \widetilde{u}, \tilde{\theta}, \tilde{q}\right)$ by

$$\phi(t,x) = v - \tilde{v}, \quad \psi(t,x) = u - \widetilde{u}, \quad \zeta(t,x) = \theta - \tilde{\theta}, \quad \omega(x,t) = q - \tilde{q}, \tag{11}$$

and set

$$\Phi(t,x) = \int_{-\infty}^{x}\phi(t,y)dy, \Psi(t,x) = \int_{-\infty}^{x}\psi(t,y)dy, \widetilde{W} = \int_{-\infty}^{x}\left(e + \frac{|u|^2}{2} - \tilde{e} - \frac{|\widetilde{u}|^2}{2}\right)(t,y)dy, \tag{12}$$

which satisfy $\left(\Phi, \Psi, \widetilde{W}\right)\left(0, \pm\infty\right) = 0$. Then we are in a position to present the main result of this paper.

2. Results

Theorem 1. Let $\left(\tilde{v}, \widetilde{u}, \tilde{\theta}, \tilde{q}\right)$ be defined in (8) and (9), then there exists small constants $\varepsilon_1 > 0$, and $\delta_1 > 0$ such that, if

$$\varepsilon := \left\|\left(\Phi, \Psi, \widetilde{W}\right)\left(\cdot, 0\right)\right\|_{L^2} + \|(\phi, \psi, \zeta)(\cdot, 0)\|_{H^1} \le \varepsilon_1,$$

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$$\delta \coloneqq |\theta_+ - \theta_-| \leq \delta_1,$$

(13)

the Cauchy problem (1) and (2) admits a unique global solution $(v, u, \theta, q)(t, x)$ satisfying

$$(\Phi, \Psi, \widetilde{W}) \in C([0, +\infty); H^2(R)), (\phi_x, \zeta_x) \in L^2(0, +\infty; L^2(R)), \tag{14}$$

$$(\psi_x, \omega, \omega_x) \in L^2(0, +\infty; H^1(R)).$$

Moreover, the solution (v, u, θ, q) enjoys the decay estimate

$$\left\| \left(v - \tilde{v}, u - \tilde{u}, \theta - \tilde{\theta}, q - \tilde{q}\right) \right\|_{L^\infty(\mathbb{R})} \leq C\left(\varepsilon_1 + \delta_1\right)^{\frac{1}{2}} \left(1 + t\right)^{-\frac{1}{2}}. \tag{15}$$

Notation. In this paper, we use $C > 0$ or $c > 0$ to stand for a generic constant depending on the physical coefficients. $H^l(\mathbb{R})$ will be used to denote the Sobolev space, which has the following norm:

$$\|f\|_l = \sum\nolimits_{j=0}^{l} \|\partial_x^j f\|, \quad \text{and} \quad \left\| \cdot \right\| := \left\| \cdot \right\|_{L^2(\mathbb{R})}.$$

3. Proof of theorem 1

From (11) and (12), we have

$$\left(\Phi, \Psi\right)_x = (\phi, \psi), \quad \frac{R}{\gamma - 1}\zeta + \frac{1}{2}\left|\Psi_x\right|^2 + \tilde{u}\Psi_x = \widetilde{W}_x.$$

Instead of the variable $\widetilde{W}$, which is the anti-derivative of the total energy, it is more convenient to introduce another variable related to the temperature,

$$W = \tfrac{\gamma-1}{R}\left(\widetilde{W} - \tilde{u}\Psi\right), \tag{16}$$

with implies that

$$\zeta = W_x - Y \quad \text{with} \quad Y = \tfrac{\gamma-1}{R}\left(\tfrac{1}{2}\Psi_x^2 - \tilde{u}_x\Psi\right). \tag{17}$$

Motivated by [26], we introduce the variable Z by setting

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$$v\omega + \phi\tilde{q} = Z_x. \tag{18}$$

In terms of (Φ, Ψ, W, Z), it follows from (1) and (10) that

$$\begin{cases} \Phi_t - \Psi_x = -\widetilde{R}_1, \\ \Psi_t - \frac{p_+}{\tilde{v}}\Phi_x + \frac{R}{\tilde{v}}W_x = \frac{\mu}{\tilde{v}}\Psi_{xx} + Q_1, \\ \frac{R}{\gamma-1}W_t + p_+\Psi_x + \frac{1}{\tilde{v}+\phi}\partial_x Z = Q_2, \\ -\left(\frac{\partial_x Z}{\tilde{v}+\phi}\right)_x + \left(\tilde{v}+\phi\right)Z + \left(\tilde{v}+\phi\right)\left(\theta^4 - \tilde{\theta}^4\right) = \left(\frac{\tilde{q}\tilde{v}}{\tilde{v}+\phi}\right)_x, \\ (\Phi, \Psi, W)(0, x) = (\Phi_0, \Psi_0, W_0)(x), \end{cases} \tag{19}$$

where

$$Q_1 = J_1 + \left(\frac{\mu}{v} - \frac{\mu}{\tilde{v}}\right)u_x + \frac{R}{\tilde{v}}Y - \widetilde{R}_2,$$

$$J_1 = \frac{\widetilde{p} - p_+}{\tilde{v}}\Phi_x - \left[p - \widetilde{p} + \frac{\widetilde{p}}{\tilde{v}}\Phi_x - \frac{R}{\tilde{v}}\left(\theta - \tilde{\theta}\right)\right],$$

$$Q_2 = J_2 + \frac{\mu u_x}{v}\Psi_x - \widetilde{u}_t\Psi + \widetilde{u}\widetilde{R}_2 - \widetilde{R}_3 + \frac{\phi\tilde{q}}{\tilde{v}+\phi},$$

$$J_2 = (p_+ - p)\Psi_x.$$

The local existence of the solution to (19) is similar to that in [26]. To establish the global existence, it suffices to close the following a priori estimate:

$$\begin{aligned} &N(T)(\|(\Phi, \Psi, W)\|_{L^\infty}^2 + (1+t)^{\frac{1}{2}}\|(\phi, \psi, \zeta, \omega)\|^2 \\ &+(1+t)^{\frac{3}{2}}\|(\phi_x, \psi_x, \zeta_x, \omega_x, \omega_{xx})\|^2) \le \varepsilon_0^2, \end{aligned} \tag{20}$$

where ε_0 is a small positive constant. It should be noted that this refined a priori assumption is crucial for the subsequent energy estimates, as it enables us to derive sharper decay estimates. In contrast to [31], we do not require the a prior assumption on the decay of the second-order derivative of ϕ.

By (6), it follows that $\left|\overline{\theta}_1\right| + \left|\overline{\theta}_3\right| \le C\varepsilon_0$ for some positive constant C. Besides, letting ε_0 be sufficiently small, one can obtain

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$$\frac{1}{2}\tilde{v} \le v = \tilde{v} + \phi \le 2\tilde{v}, \quad \frac{1}{2}\tilde{\theta} \le \theta = \tilde{\theta} + \zeta \le 2\tilde{\theta}.$$

In what follows, we devote ourselves to establishing the a priori estimates. To begin with, for subsequent convenience, we denote

$$\widetilde{E}_i \coloneqq \int_{\mathbb{R}} \left(\left|\partial_x^i \Phi\right|^2 + \left|\partial_x^i \Psi\right|^2 + \left|\partial_x^i W\right|^2\right) dx,$$

$$E_i \coloneqq \int_{\mathbb{R}} \left(\left|\partial_x^i \phi\right|^2 + \left|\partial_x^i \psi\right|^2 + \left|\partial_x^i \zeta\right|^2\right) dx, \quad \text{for } i = 0,1,2, \tag{21}$$

$$E_0^* \coloneqq \bar{C}_1 \int_{\mathbb{R}} \left(\frac{p_+}{2}\Phi^2 + \frac{R^2}{2(\gamma-1)p_+} W^2 + \frac{\tilde{v}}{2}\Psi^2\right) dx + \int_{\mathbb{R}} \left(\frac{\mu}{2\tilde{v}}\left|\Phi_x\right|^2 - \Phi_x \Psi\right) dx, \tag{22}$$

$$E_1^* \coloneqq \bar{C}_2 \int_{\mathbb{R}} \left(R\tilde{\theta}\widetilde{\Phi}\left(\frac{v}{\tilde{v}}\right) + \frac{1}{2}\psi^2 + \frac{R}{\gamma-1}\tilde{\theta}\widetilde{\Phi}\left(\frac{\theta}{\tilde{\theta}}\right)\right) dx + \int_{\mathbb{R}} \left(\frac{\mu}{2\tilde{v}}\phi_x^2 - \phi_x \psi\right) dx. \tag{23}$$

Here $\widetilde{\Phi}\left(s\right) = s - 1 - lns$, and it holds that $c\phi^2 \le \widetilde{\Phi}\left(\frac{v}{\tilde{v}}\right) \le C\phi^2, \quad c\zeta^2 \le \widetilde{\Phi}\left(\frac{\theta}{\tilde{\theta}}\right) \le C\zeta^2$ for some positive constants c and C.

Similar to [26], we derive the lower order energy estimates of (Φ, Ψ, W) as follows:

$$E_{0t}^* + c\widetilde{E}_1 + c\|Z\|_1^2 \le C\left(\bar{\delta} + \varepsilon_0\right)E_1 + C\bar{\delta}\left(1+t\right)^{-1}\widetilde{E}_0 + C\bar{\delta}\left(1+t\right)^{-\frac{1}{2}} + C\|\omega_x\|^2, \tag{24}$$

$$E_{1t}^* + cE_1 + c\|\omega\|_1^2 \le C\bar{\delta}\left(1+t\right)^{-1}\widetilde{E}_1 + C\bar{\delta}\left(1+t\right)^{-\frac{3}{2}} + C\|\omega_{xx}\|^2 + C\varepsilon_0\|\psi_{xx}\|^2. \tag{25}$$

Subsequently, we establish the derivative estimates of $(\phi, \psi, \zeta, \omega)$. First, we rewrite the system as follows:

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$$\begin{cases} \phi_t - \psi_x = -\widetilde{R}_{1x}, \\ \psi_t + R\Big(\frac{\zeta}{v} - \frac{\tilde{\theta}}{v\tilde{v}}\,\phi\Big)_x = \big(\frac{\mu}{v}\,u_x - \frac{\mu}{\tilde{v}}\,\widetilde{u}_x\big)_x - \widetilde{R}_{2x}, \\ \frac{R}{\gamma-1}\,\zeta_t + p\psi_x + \omega_x = Q_3 - \Big(p - \widetilde{p}\Big)\widetilde{u}_x, \\ \zeta_x - \frac{1}{4\theta^3}\left(\frac{\omega_x+\tilde{q}_x}{v}\right)_x + \frac{vq}{4\theta^3} - \frac{\tilde{v}\tilde{q}}{4\tilde{\theta}^3} = 0. \end{cases} \tag{26}$$

We adopt the crucial transformation

$$\widetilde{\phi} \coloneqq \frac{\theta}{v^2}\,\phi, \quad \tilde{\zeta} \coloneqq \frac{\zeta}{v}, \tag{27}$$

to rewrite the system ∂_x (26) as follows:

$$\begin{cases} \widetilde{\phi}_{xt} - \big(\frac{\theta}{v^2}\,\psi_x\big)_x = \partial_x\widetilde{Q}_1, \\ \psi_{xt} + R\Big(\tilde{\zeta} - \widetilde{\phi}\Big)_{xx} = \big(\frac{\mu}{v}\,\psi_x\big)_{xx} + \partial_x\widetilde{Q}_2, \\ \frac{R}{\gamma-1}\,\tilde{\zeta}_{xt} + \big(\frac{p}{v}\,\psi_x\big)_x + \big(\frac{\omega_x}{v}\big)_x = \partial_x\widetilde{Q}_3, \\ \tilde{\zeta}_{xx} - \big[\big(\frac{1}{v}\big)_x\zeta\big]_x - \left[\frac{1}{4v\theta^3}\left(\frac{\tilde{q}_x+\omega_x}{v}\right)_x\right]_x + \left(\frac{vq}{4v\theta^3} - \frac{\tilde{v}\tilde{q}}{4v\tilde{\theta}^3}\right)_x = 0, \end{cases} \tag{28}$$

where

$$\partial_x\widetilde{Q}_1 := \left[-\frac{\theta}{v^2}\,\widetilde{R}_{1x} + \left(\frac{\theta}{v^2}\right)_t\phi\right]_x,\ \partial_x\widetilde{Q}_2 := \Big[\Big(\frac{\mu}{v} - \frac{\mu}{\tilde{v}}\Big)\widetilde{u}_x\Big]_{xx} - \widetilde{R}_{2xx} + R\left(\frac{1}{v}\left(\frac{\tilde{\theta}}{\tilde{v}} - \frac{\theta}{v}\right)\phi\right)_{xx},$$

$$\partial_x\widetilde{Q}_3 := \left(\frac{Q_3}{v}\right)_x - \left[\frac{(p-\widetilde{p})}{v}\,\widetilde{u}_x\right]_x + \frac{R}{\gamma-1}\left[\left(\frac{1}{v}\right)_t\zeta\right]_x.$$

Taking $(28)_1 \times R\widetilde{\phi}_x + (28)_2 \times \frac{\theta}{v^2}\,\psi_x + (28)_3 \times \tilde{\zeta}_x + (28)_4 \times \frac{\omega_x}{v}$, we have

$$\frac{d}{dt}\int_{\mathbb{R}}\left[\frac{R}{2}\left|\widetilde{\phi}_x\right|^2 + \frac{\theta}{2v^2}\left|\psi_x\right|^2 + \frac{R}{2(\gamma-1)}\left|\tilde{\zeta}_x\right|^2\right]dx + \int_{\mathbb{R}}\frac{\mu\theta}{v^3}\left|\psi_{xx}\right|^2 dx + \int_{\mathbb{R}}\frac{\omega_x^2}{4v\theta^3}\,dx + \int_{\mathbb{R}}\frac{\omega_{xx}^2}{4v^3\theta^3}$$
$$dx$$

$$= \int_{\mathbb{R}} \left(\frac{\mu}{v}\right)_x \left(\frac{\theta}{v^2}\right)_x \left|\psi_x\right|^2 dx + \int_{\mathbb{R}} \left(\frac{\mu\theta}{v^3}\right)_x \psi_x \psi_{xx} dx dx + \int_{\mathbb{R}} \frac{1}{2}\left(\frac{\theta}{v^2}\right)_t \left|\psi_x\right|^2 dx + \int_{\mathbb{R}} R\widetilde{\phi}_x \partial_x \widetilde{Q}_1 dx$$

$$+ \int_{\mathbb{R}} \frac{\theta}{v^2} \psi_x \partial_x \widetilde{Q}_2 dx + \int_{\mathbb{R}} \tilde{\zeta}_x \partial_x \widetilde{Q}_3 dx + \int_{\mathbb{R}} \frac{\omega_x}{v} \left[\left(\frac{1}{v}\right)_x \zeta\right]_x dx - \int_{\mathbb{R}} \frac{\omega_{xx}}{4v^2\theta^3} \left[\left(\frac{\tilde{q}_x}{v}\right)_x + \left(\frac{1}{v}\right)_x \omega_x\right] dx$$

$$- \int_{\mathbb{R}} \frac{\omega_x}{4v\theta^3} \left(\frac{1}{v}\right)_x \left[\left(\frac{\tilde{q}_x}{v}\right)_x + \frac{\omega_{xx}}{v} + \left(\frac{1}{v}\right)_x \omega_x\right] dx - \int_{\mathbb{R}} \frac{\omega\omega_x}{v} \left(\frac{1}{4\theta^3}\right)_x dx - \int_{\mathbb{R}} \frac{\omega_x}{v} \left(\frac{v\tilde{q}}{4v\theta^3} - \frac{\tilde{v}\tilde{q}}{4v\tilde{\theta}^3}\right)_x dx \quad (29)$$

Since there are no dissipation estimates for the second-order derivatives of ϕ and ζ, we shall avoid their appearance in the subsequent estimates. Using the Cauchy inequality, the Sobolev inequality and the a priori assumptions (20), it holds that

$$\int_{\mathbb{R}} R\widetilde{\phi}_x \partial_x \widetilde{Q}_1 dx = \int_{\mathbb{R}} R\widetilde{\phi}_x \left[-\frac{\theta}{v^2} \widetilde{R}_{1x} + \left(\frac{\zeta_t + \tilde{\theta}_t}{v^2} - \frac{2\theta(\psi_x + \widetilde{u}_x)}{v^3}\right)\phi\right]_x dx$$

$$\leq C\left(\bar{\delta} + \varepsilon_0\right)\left(\|\psi_{xx}\|^2 + \|\omega_x\|_1^2\right) + C\left(\bar{\delta} + \varepsilon_0\right)\left[\left(1+t\right)^{-1} E_1 + \left(1+t\right)^{-2} E_0\right] + C\bar{\delta}\left(1+t\right)^{-\frac{5}{2}}, \quad (30)$$

and the estimates of $\int_{\mathbb{R}} \left(\frac{\mu}{v}\right)_x \left(\frac{\theta}{v^2}\right)_x \left|\psi_x\right|^2 dx$, $\int_{\mathbb{R}} \left(\frac{\mu\theta}{v^3}\right)_x \psi_x \psi_{xx} dx$, $\int_{\mathbb{R}} \left(\frac{\theta}{v^2}\right)_t \left|\psi_x\right|^2 dx$ are similar. Next, we have

$$\int_{\mathbb{R}} \frac{\theta}{v^2} \psi_x \partial_x \widetilde{Q}_2 dx = -\int_{\mathbb{R}} \left[\left(\frac{\theta}{v^2}\right)_x \psi_x + \frac{\theta}{v^2} \psi_{xx}\right] \left[\left(\frac{\mu}{v} - \frac{\mu}{\tilde{v}}\right)\widetilde{u}_x - \widetilde{R}_{2x} + R\left(\frac{1}{v}\left(\frac{\tilde{\theta}}{\tilde{v}} - \frac{\theta}{v}\right)\phi\right)_x\right] dx$$

$$\leq C\left(\bar{\delta} + \varepsilon_0\right)\|\psi_{xx}\|^2 + C\left(\varepsilon_0 + \bar{\delta}\right)\left(1+t\right)^{-1} E_1 + C\bar{\delta}\left(1+t\right)^{-\frac{5}{2}}, \quad (31)$$

and

$$\int_{\mathbb{R}} \tilde{\zeta}_x \partial_x \widetilde{Q}_3 dx = \int_{\mathbb{R}} \tilde{\zeta}_x \partial_x \left(\tfrac{1}{v} Q_3\right) dx + \int_{\mathbb{R}} \tilde{\zeta}_x \left[-\tfrac{(p-\widetilde{p})}{v} \widetilde{u}_x + \tfrac{R}{\gamma-1}\left(\tfrac{1}{v}\right)_t \zeta\right]_x dx$$
$$\leq C\left(\bar{\delta}+\varepsilon_0\right)\|\psi_{xx}\|^2 + C\left(\bar{\delta}+\varepsilon_0\right)\left(1+t\right)^{-1}E_1 + C\bar{\delta}\left(1+t\right)^{-\frac{5}{2}}, \tag{32}$$

where we have used the a priori assumptions (20). Employing integration by parts and the Cauchy inequality, together with (20), we can deduce

$$\int_{\mathbb{R}} \frac{\omega_x}{v}\left[\left(\frac{1}{v}\right)_x \zeta\right]_x dx - \int_{\mathbb{R}} \frac{\omega\omega_x}{v}\left(\frac{1}{4\theta^3}\right)_x dx = -\int_{\mathbb{R}} \frac{\omega_x}{v}\left(\frac{\phi_x\zeta + \tilde{v}_x\zeta}{v^2}\right)_x dx$$

$$= \int_{\mathbb{R}} \left(\frac{\omega_x}{v}\right)_x \frac{\phi_x\zeta}{v^2} dx - \int_{\mathbb{R}} \frac{\omega_x}{v}\left(\frac{\tilde{v}_x\zeta}{v^2}\right)_x dx - \int_{\mathbb{R}} \frac{\omega\omega_x}{v}\left(\frac{1}{4\theta^3}\right)_x dx$$

$$\leq C\int_{\mathbb{R}} (|\omega_{xx}| + |w_x|(\left|\phi_x\right| + \left|\tilde{v}_x\right|))\left|\phi_x\zeta\right| dx + C\int_{\mathbb{R}}\left|\omega_x\right|\left(\left|\tilde{v}_{xx}\zeta\right| + \left|\tilde{v}_x\zeta_x\right| + \left|\tilde{v}_x\zeta\right|\left(\left|\phi_x\right| + \left|\tilde{v}_x\right|\right)\right) dx$$

$$+C\int_{\mathbb{R}}\left|\omega\omega_x\right|\left(\left|\tilde{\theta}_x\right| + |\zeta_x|\right) dx \leq C\left(\bar{\delta}+\varepsilon_0\right)\|\omega_x\|_1^2 + C\left(\bar{\delta}+\varepsilon_0\right)\left(1+t\right)^{-1}\left(E_1 + \|\omega\|^2\right)$$
$$+ C\bar{\delta}\left(1+t\right)^{-\frac{5}{2}}, \tag{33}$$

$$\int_{\mathbb{R}} \frac{\omega_{xx}}{4v^2\theta^3}\left[\left(\frac{\tilde{q}_x}{v}\right)_x + \left(\frac{1}{v}\right)_x \omega_x\right] dx + \int_{\mathbb{R}} \frac{\omega_x}{4v\theta^3}\left(\frac{1}{v}\right)_x\left[\left(\frac{\tilde{q}_x}{v}\right)_x + \frac{\omega_{xx}}{v} + \left(\frac{1}{v}\right)_x \omega_x\right] dx$$

$$\leq C\int_{\mathbb{R}} (|\omega_{xx}| + |\omega_x|(\left|\phi_x\right| + \left|\tilde{v}_x\right|))\left(\left|\tilde{q}_{xx}\right| + \left|\tilde{q}_x\right|\left(\left|\phi_x\right| + \left|\tilde{v}_x\right|\right) + \left|\omega_x\right|\left(\left|\phi_x\right| + \left|\tilde{v}_x\right|\right)\right) dx$$

$$+C\int_R |\omega_{xx}||\omega_x|(|\phi_x| + |\tilde{v}_x|) dx \leq C(\bar{\delta}+\varepsilon_0)\|\omega_x\|_1^2$$
$$+C(\bar{\delta}+\varepsilon_0)(1+t)^{-1}E_1 + C\bar{\delta}(1+t)^{-\frac{5}{2}}, \tag{34}$$

and

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$$\int_{\mathbb{R}} \frac{\omega_x}{v} \left(\frac{v\tilde{q}}{4v\theta^3} - \frac{\tilde{v}\tilde{q}}{4v\tilde{\theta}^3} \right)_x dx = \int_{\mathbb{R}} \frac{\omega_x}{v} \left[\frac{\tilde{q}}{v} \left(\frac{v}{4\theta^3} - \frac{\tilde{v}}{4\tilde{\theta}^3} \right) \right]_x dx$$

$$\leq C \int_{\mathbb{R}} \left|\omega_x\right| \left(\left|\tilde{q}_x\right| + \left|\tilde{q}\right| \left(\left|\phi_x\right| + \left|\tilde{v}_x\right| \right) \right) \left(\left|\phi\right| + \left|\zeta\right| \right) dx + C \int_{\mathbb{R}} \left|\omega_x \tilde{q}\right| \left(\left|\phi_x\right| + \left|\zeta_x\right| + \left(\left|\phi\right| + \left|\zeta\right| \right) \left(\left|\tilde{v}_x\right| + \left|\tilde{\theta}_x\right| \right) \right) dx$$

$$\leq C\left(\bar{\delta} + \varepsilon_0\right) \| \omega_x \|^2 + C\left(\bar{\delta} + \varepsilon_0\right)\left(1+t\right)^{-1} E_1 + C\bar{\delta}\left(1+t\right)^{-\frac{5}{2}}. \tag{35}$$

Then we have

$$E_{2t}^* + c\|\psi_{xx}\| + c\|\omega_x\|_1^2 \leq C(\bar{\delta} + \varepsilon_0)[(1+t)^{-1}(E_1 + \|\omega\|^2) + (1+t)^{-2}E_0] + C\bar{\delta}(1+t)^{-\frac{5}{2}}, \tag{36}$$

where

$$E_2^* := \int_{\mathbb{R}} \left(\frac{R}{2} \left|\tilde{\phi}_x\right|^2 + \frac{\theta}{2v^2} \left|\psi_x\right|^2 + \frac{R}{2(\gamma-1)} \left|\tilde{\zeta}_x\right|^2 \right) dx. \tag{37}$$

Next, we give the estimates of $\| \omega \|_2$. From $(26)_4$, we have

$$-\left(\frac{\omega_x}{v}\right)_x + v\omega = \left(\frac{\tilde{q}_x}{v}\right)_x - \phi\tilde{q} - 4\theta^3\zeta_x - 4\left(\theta^3 - \tilde{\theta}^3\right)\tilde{\theta}_x. \tag{38}$$

Multiplying (38) by ω and integrating the resulting equation over $\mathbb{R}$, we get

$$\|\omega\|_1^2 \leq \int_R (|\tilde{q}_x\omega_x| + |\phi\tilde{q}\omega| + |\zeta_x\omega| + |\zeta\tilde{\theta}_x\omega|)dx \leq \frac{1}{4}\|\omega\|_1^2 + C\|\zeta_x\|^2 + C\bar{\delta}(1+t)^{-1}\|(\phi,\zeta)\|^2 + C\bar{\delta}(1+t)^{-\frac{3}{2}}, \tag{39}$$

which implies

$$\| \omega \|_1^2 \leq CE_1 + C\bar{\delta}\left(1+t\right)^{-1} E_0 + C\bar{\delta}\left(1+t\right)^{-\frac{3}{2}}. \tag{40}$$

Multiplying (38) by $-\omega_{xx}$ yields

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$$\|\omega_x\|_1^2 \leq C\|(\omega, \phi_x, \zeta_x)\|^2 + C\bar{\delta}\Big(1+t\Big)^{-1}\|(\phi, \zeta)\|^2 + C\bar{\delta}\Big(1+t\Big)^{-\frac{5}{2}}. \tag{41}$$

Noting that

$$\widetilde{E}_0 \leq E_0^*, E_0 \leq E_1^*, E_1 \leq E_2^* + C\bar{\delta}(1+t)^{-\frac{3}{2}}, \widetilde{E}_1 \leq CE_0$$
$$+C\bar{\delta}(1+t)^{-\frac{3}{2}}, \widetilde{E}_2 \leq C(E_1+E_0) + C\bar{\delta}(1+t)^{-\frac{5}{2}}, \tag{42}$$

where we have used (16) and the a priori assumptions (20). Then, choosing some large constant C_1^* and summing $(24)+(25)+C_1^*\times(36)$ gives

$$\Big(E_0^* + E_1^* + C_1^* E_2^*\Big)_t + c\left(\widetilde{E}_1 + E_1 + \|\psi_{xx}\|^2\right) + c\|Z\|_1^2 + c\|\omega\|_2^2$$

$$\leq C_0\left(\bar{\delta}+\varepsilon_0\right)\Big(1+t\Big)^{-1}\left(E_0^* + E_1^* + C_1^* E_2^*\right) + C_0\bar{\delta}\Big(1+t\Big)^{-\frac{1}{2}}. \tag{43}$$

Multiplying (43) by $(1+t)^{-C_0\left(\bar{\delta}+\varepsilon_0\right)}$ and using the Gronwall inequality yield

$$E_0^* + E_1^* + E_2^* \leq C\bar{\delta}\Big(1+t\Big)^{\frac{1}{2}}, \quad \int_0^t\Big(\widetilde{E}_1 + E_1 + \|\psi_{xx}\|^2 + \|Z\|_1^2 + \|\omega\|_2^2\Big)d\tau \leq C\bar{\delta}\Big(1+t\Big)^{\frac{1}{2}}. \tag{44}$$

Next, choosing some large constant C_2^* and adding (25) $+C_2^*\times$ (36) together, we have

$$(E_1^* + C_2^* E_2^*)_t + c(E_1 + \|\psi_{xx}\|^2 + \|\omega\|_2^2) \leq C_0(\bar{\delta}+\varepsilon_0)(1+t)^{-1}$$
$$(E_1^* + C_2^* E_2^*) + C_0\bar{\delta}(1+t)^{-\frac{3}{2}}. \tag{45}$$

Multiplying (45) by $(1+t)$ and then integrating on $[0,t]$, one has

$$\Big(1+t\Big)\Big(E_1^* + C_2^* E_2^*\Big) + \int_0^t\Big(1+\tau\Big)\Big(E_1 + \|\psi_{xx}\|^2 + \|\omega\|_2^2\Big)d\tau \leq C\bar{\delta}\Big(1+t\Big)^{\frac{1}{2}}. \tag{46}$$

Then it follows that

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$$\|(\phi,\psi,\zeta)(t)\|^2 \le O(1)\left(E_1^* + E_2^*\right) \le C\bar{\delta}\left(1+t\right)^{-\frac{1}{2}},$$

$$\int_0^t \left(1+\tau\right)\left(E_1 + \|\psi_{xx}\|^2 + \|\omega\|_2^2\right)d\tau \le C\bar{\delta}\left(1+t\right)^{\frac{1}{2}}. \tag{47}$$

Multiplying (36) by $(1+t)^2$, one obtains

$$((1+t)^2E_2^*)_t + c(1+t)^2(\|\psi_{xx}\|^2 + \|\omega_x\|_1^2) \le C(1+t)$$

$$(E_1 + \|\omega\|^2) + CE_0 + C\bar{\delta}(1+t)^{-\frac{1}{2}}, \tag{48}$$

which together with (44) and (47) yields

$$(1+t)^2E_2^* + \int_0^t(1+\tau)^2$$

$$(\|\psi_{xx}\|^2 + \|\omega_x\|_1^2)d\tau \le C\bar{\delta}(1+t)^{\frac{1}{2}}. \tag{49}$$

Hence

$$\|(\phi_x,\psi_x,\zeta_x)\|^2 \le CE_2^* + C\bar{\delta}\left(1+t\right)^{-\frac{3}{2}} \le C\bar{\delta}\left(1+t\right)^{-\frac{3}{2}}. \tag{50}$$

By using (40) and (41), we have

$$\|\omega\|^2 + \|\omega_x\|^2 \le C\bar{\delta}\left(1+t\right)^{-\frac{3}{2}}. \tag{51}$$

Then the desired decay rate follows from (44), (47), (50) and (51)

$$\|(\phi,\psi,\zeta,\omega)\|_{L^\infty} \le C\|(\phi,\psi,\zeta,\omega)\|^{\frac{1}{2}}\|(\phi_x,\psi_x,\zeta_x,\omega_x)\|^{\frac{1}{2}} \le C\bar{\delta}\left(1+t\right)^{-\frac{1}{2}}. \tag{52}$$

Combining this with (11), we obtain the decay rate (15). Therefore, the proof of Theorem 1 is completed.

4. Conclusion

As an essential branch of fluid mechanics, radiative hydrodynamics mainly investigates the propagation law of thermal radiation in fluid media, as well as the interaction mechanism between the radiation field and fluid motion. When the temperature rises to a relatively high level, the radiation field exerts a remarkable influence on fluid motion and thus becomes an indispensable

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factor that cannot be neglected. Furthermore, the compressible radiative hydrodynamic system admits abundant wave phenomena, including radiation-induced shock waves, radiation fronts, and nonlinear diffusion waves. These waves often interact with each other and with the background flow, leading to complex dynamical behaviors. Accordingly, exploring the stability of basic waves-shock waves, rarefaction waves, and contact discontinuities, is of great significance and research value, as it provides deep insights into the physical mechanisms underlying high-temperature fluid flows and offers theoretical support for applications in astrophysics, laser–plasma interactions, and controlled nuclear fusion.

In this paper, we prove the large-time asymptotic stability of the viscous contact wave for a radiative hydrodynamic system under non-zero mass perturbations, and establish the corresponding decay rates. First, we construct an ansatz to handle the non-zero mass perturbations and use the anti-derivative method to obtain the low-order energy estimates. Then, with a key transformation and elaborate analysis on the variable ω, we obtain better decay estimates of $(\phi_x, \psi_x, \zeta_x, \omega_x)$, which further yield a better decay rate of $(\phi, \psi, \zeta, \omega)$ in the L^∞-norm. It is worth noting that the derived decay rate is optimal. Moreover, we extend the result in [31] to a system with weaker dissipation.

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