

Mechanical Analysis of an Externally Pressurized Cylindrical Shell with Initial Ovality

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Abstract. Thin-walled cylindrical shells are often used in aerospace structures and deep-sea pressure components because they can carry load efficiently with limited weight. In external-pressure service, however, this advantage comes with a clear weakness: the shell may lose stability before the material reaches its yield strength. Manufacturing and assembly also make a perfectly circular section difficult to obtain, and initial ovality is one typical defect that can change the buckling response. In this paper, a cylindrical shell with initial ovality is analyzed by finite element eigenvalue buckling analysis, and four stiffening schemes are compared, namely circumferential T-shaped stiffeners, longitudinal T-shaped stiffeners, circumferential channel-section stiffeners, and circumferential equal-leg-angle stiffeners. The results show that the arrangement direction of the stiffener has a stronger effect than the section type alone. The circumferential T-shaped scheme gives the largest buckling load multiplier, 0.948, while the longitudinal T-shaped scheme gives only 0.320 under the same comparison conditions. This difference indicates that an effective stiffener should restrain the circumferential buckling wave rather than only increase local axial stiffness. The study therefore gives a direct reference for selecting stiffener layouts when unavoidable ovality has to be considered in pressure-resistant cylindrical shell design.

Keywords: Buckling Stability, Thin-walled Cylindrical Shells, Initial Ovality

1. Introduction

Thin-walled cylindrical shells have been adopted in submarine pressure hulls, deep-sea pressure cabins, offshore engineering components, and aerospace structures, mainly because they provide a favorable stiffness-to-weight ratio and high load-carrying efficiency [1]. When the load is external pressure, the critical problem is not always material yielding; in many cases, the shell first loses stability through buckling, which is consistent with the classical shell stability theory discussed by Yamaki [2]. For pressure-resistant structures used in deep-water conditions, the design work therefore has to focus on whether the shell can keep sufficient stability under hydrostatic pressure [3].

In real manufacturing, a perfectly circular cylindrical shell is more an ideal assumption than an actual product. Manufacturing tolerance, welding deformation, and assembly error can all leave geometric defects in the shell, and initial ovality is a common form of such imperfection [4]. Earlier studies have already shown that thin-walled shells are quite sensitive to this type of deviation; even

a small difference from the ideal geometry may bring a visible drop in the critical buckling load [4]. Jiménez et al. reported that large-amplitude defects can couple with external pressure in a nonlinear way, so the actual buckling resistance becomes harder to estimate [5]. In engineering design, knockdown factors are often used to cover imperfection sensitivity, but Wagner et al. noted that empirical reduction factors may be too conservative and do not describe how defect morphology interacts with stiffening measures [6].

Adding stiffeners is one of the usual ways to improve buckling resistance, because the stiffener can change the stiffness distribution and influence how instability develops [7]. Tests on stiffened shells have also confirmed that stiffeners may change the buckling mode and improve shell stability by limiting radial deformation [8]. Recent work further suggests that the effect of stiffening does not depend only on how much material is added; the layout direction and sectional form also matter [9]. Studies on post-buckling behavior and combined loading conditions show a similar point, namely that stiffener configuration affects deformation localization, load transfer, and the final structural response [10]. Even so, for externally pressurized cylindrical shells with initial ovality, the separate effects of cross-sectional form and arrangement direction still need a more direct comparison.

Based on this problem, this paper carries out a linear eigenvalue buckling analysis in ANSYS for externally pressurized cylindrical shells with initial ovality. The same finite element modeling framework is used for several typical stiffening schemes, including T-shaped, channel-section, and equal-leg-angle stiffeners. By keeping the basic shell geometry, material parameters, and stiffener spacing unchanged, the comparison focuses on how the stiffener geometry and orientation affect buckling resistance. The aim is to make the stiffening mechanism clearer for imperfect shells and to offer a concise reference for the design of pressure-resistant cylindrical shell structures.

2. Structural model and loading conditions

Figure 1 shows the cylindrical shell model, the external-pressure loading condition, and the initial ovality introduced in the cross-section. The cylinder is subjected to uniform external pressure, and both ends are constrained to remove rigid-body motion while still allowing the main deformation pattern to develop. The applied load is treated as the reference external pressure.

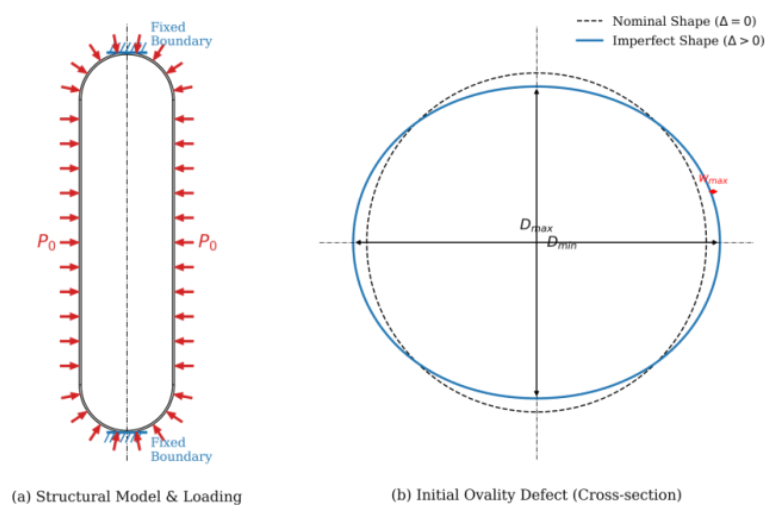


Figure 1. Schematic of the cylindrical shell under external pressure and its initial ovality cross-section

For a fair comparison, all stiffener configurations use the same shell geometry, stiffener spacing, and material properties; only the stiffener type or its arrangement direction is changed. Structural stability is evaluated by the first-order buckling load multiplier λ , which is defined as:

$$P_{cr} = \lambda P_0 \quad (1)$$

where p_{cr} denotes the predicted critical pressure, and p_0 denotes the reference external pressure. A larger value of λ means that a higher pressure level is needed before the first buckling mode occurs.

3. Finite element modeling

The three-dimensional finite element models are built and solved in ANSYS Mechanical. Solid elements are used for the shell and stiffeners, and the interfaces between them are set as bonded so that the two parts deform together. This treatment follows the ideal welded-connection assumption used in the present model and avoids additional uncertainty from interface slip.

Eigenvalue buckling analysis is used as the main calculation method. It is not intended to describe the full nonlinear collapse process, but it gives a consistent first-order measure of stability, which is suitable for comparing different stiffener configurations under the same modeling assumptions.

The compatibility between the shell and the stiffeners is important for this numerical model. Under the bonded-interface setting, relative sliding is not allowed at the connection, so deformation and stiffness can be transferred from the stiffener to the shell. This assumption is simplified, but it is reasonable for an ideal welded stiffener in a preliminary comparison study.

The mesh is locally refined near high-curvature areas and stiffener-shell intersections. These positions usually have sharper stiffness changes and are more likely to show stress concentration or deformation localization, so a coarse mesh in these regions may affect the predicted buckling mode.

The structural parts are taken as structural steel, with homogeneous, isotropic, and linear elastic behavior. Since thin shells under external pressure usually buckle before obvious yielding appears, the linear elastic setting is acceptable for the eigenvalue buckling analysis used here. The material parameters adopted in the model are listed in Table 1.

Table 1. Properties of the material used in the FE analysis

Property	Symbol	Value
Young's modulus	E	210 GPa
Poisson's ratio	ν	0.30
Density	ρ	7850 kg/m ³

A mesh convergence check is carried out to improve the reliability of the numerical results. Several mesh densities are tested, and the corresponding first-order buckling load multipliers are compared, as shown in Figure 2. During this process, the areas close to stiffener-shell intersections are refined to better capture stress gradients and local deformation.

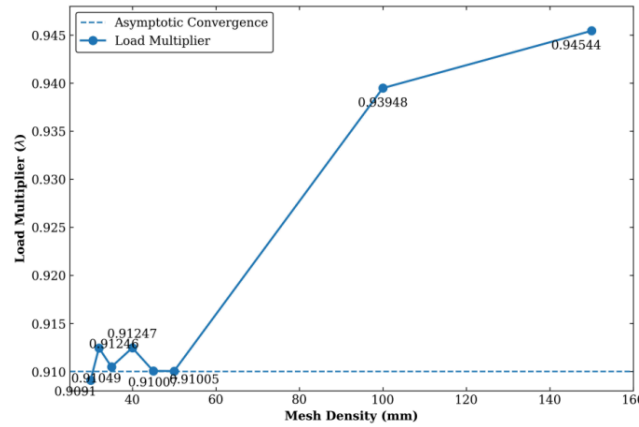


Figure 2. Mesh convergence curve for the cylindrical shell model

Initial ovality is then introduced into the shell geometry to represent the geometric imperfection. In this paper, ovality is defined as:

$$\delta = \frac{a-b}{R}, \quad R = \frac{a+b}{2} \quad (2)$$

For a given ovality level, the cross-sectional dimensions are adjusted as follows:

$$a = R\left(1 + \frac{\delta}{2}\right), \quad b = R\left(1 - \frac{\delta}{2}\right) \quad (3)$$

After this treatment, the original circular cross-section is replaced by the corresponding elliptical profile, while the other geometric parameters remain unchanged. In this way, the effect of cross-sectional imperfection can be separated from changes in length, thickness, material, or stiffener spacing.

4. Results

Figures 3 and 4 compare the first-order buckling modes of the unstiffened and stiffened cylindrical shells, and Table 2 summarizes the buckling load multipliers for the four stiffener configurations. The results show clear differences among the investigated stiffener types and arrangement directions.

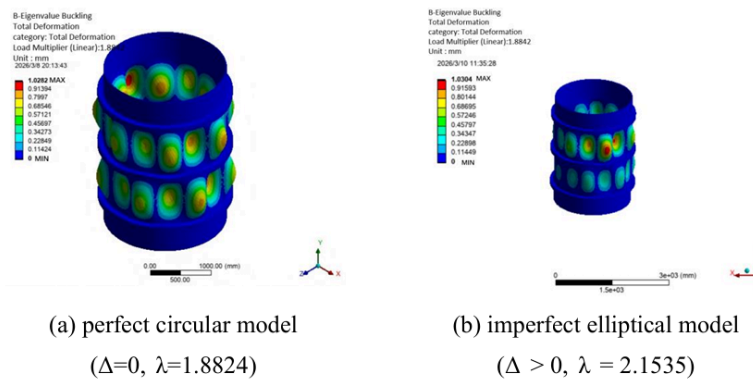


Figure 3. First-order buckling modes of the cylindrical shells

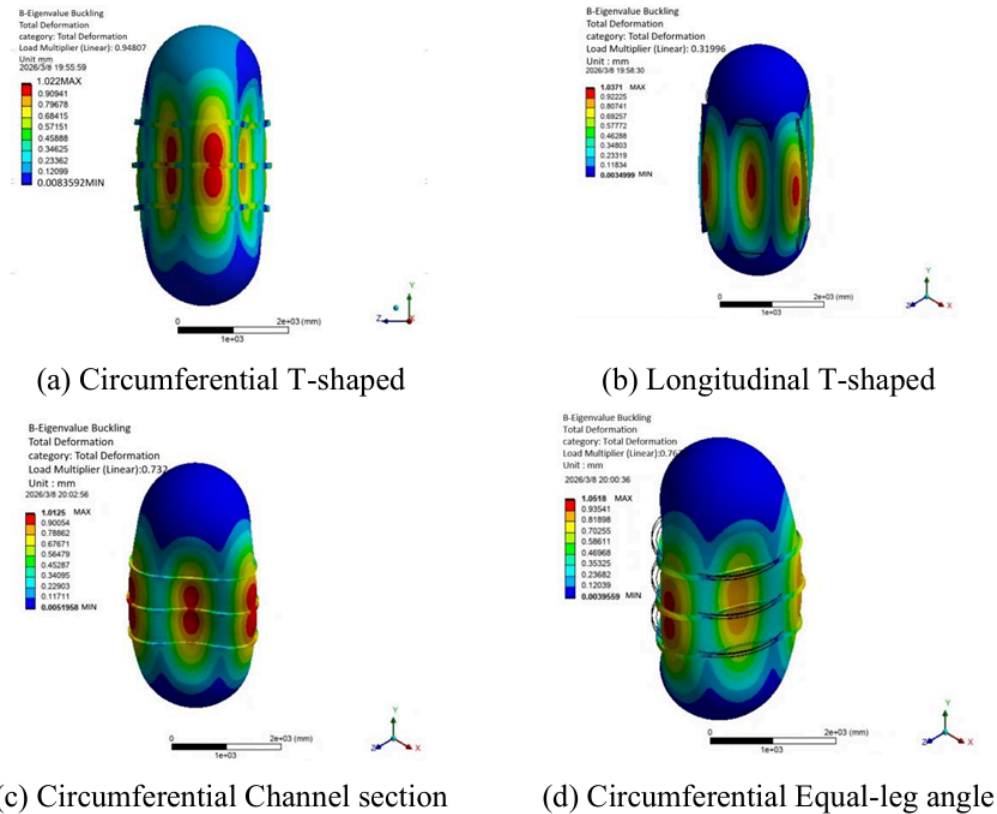


Figure 4. First-order buckling mode shapes and load multipliers (λ) for the stiffened cylindrical shells

The buckling load multiplier gives a direct numerical basis for comparing the stiffening schemes. Under the same reference loading and modeling conditions, a larger multiplier means that the shell has stronger resistance to the first instability mode. Among the four cases in Table 2, the circumferential T-shaped stiffener gives the highest value, whereas the longitudinal T-shaped stiffener gives the lowest value.

Table 2. Comparison of buckling load multipliers for different stiffener configurations

Stiffener type	Arrangement	λ
T-shaped	Circumferential	0.948
T-shaped	Longitudinal	0.320
Channel section	Circumferential	0.732
Equal-leg angle	Circumferential	0.746

The deformation pattern also needs to be read together with the numerical value. A smoother and more continuous buckling shape usually suggests that the stiffness is distributed more evenly and that load transfer is relatively stable. By contrast, a localized deformation zone often reflects stiffness discontinuity, and such localization may make the shell more sensitive to imperfection.

The comparison indicates that stiffener orientation changes the dominant deformation mechanism. Circumferential stiffeners directly restrain the radial displacement and circumferential wave pattern related to global shell buckling. Longitudinal stiffeners can improve axial deformation

compatibility, but they do not effectively interrupt the circumferential buckling wave, so their contribution to external-pressure stability is limited.

Initial ovality changes the structural state before the shell reaches instability. Once the section is no longer perfectly circular, stress and deformation are redistributed in advance, and the load level required to trigger buckling may change. The buckling mode therefore tends to develop near the region with larger geometric deviation.

For the specific stiffeners compared in this work, the circumferential T-shaped arrangement performs best because it provides stronger restraint against radial deformation and global shell waves. The longitudinal T-shaped arrangement mainly affects axial compatibility and gives much weaker reinforcement. The channel-section stiffener offers a moderate improvement, while the equal-leg-angle stiffener produces a relatively balanced stiffness distribution.

The results also suggest that stiffeners can reduce the adverse effect of ovality, but they do not remove imperfection sensitivity completely. In other words, the shell still responds to the initial geometric defect, even when a stiffening system has been added.

These observations show that structural stability is controlled by both stiffness magnitude and stiffness placement. A stiffener is more useful when its stiffness is placed along the critical deformation direction; simply increasing the local section size may not lead to the same improvement if the arrangement direction is unsuitable.

Cross-sectional shape affects torsional rigidity and deformation compatibility, especially for open sections. Warping of an open section may reduce the effective stabilizing contribution, while a section that provides more balanced bidirectional stiffness can help redistribute load and maintain structural integrity. Overall, the role of the stiffener is better understood as redistributing deformation and delaying instability, rather than fully eliminating the influence of the initial imperfection.

5. Conclusions

This paper compares several stiffener configurations for externally pressurized cylindrical shells with initial ovality. The analysis shows that the buckling response is affected not only by the stiffener section and the imperfection itself, but also by whether the stiffener is arranged in the direction that can restrain the dominant buckling wave. The main conclusions are as follows.

(1) Stiffener orientation has a direct influence on global stability and deformation pattern. Circumferential stiffeners can restrain radial displacement and suppress the circumferential buckling wave, while longitudinal stiffeners mainly improve axial deformation compatibility and give much lower resistance to external pressure in the present comparison.

(2) Among the investigated schemes, the circumferential T-shaped stiffener gives the best buckling performance, with a buckling load multiplier of 0.948. When the same T-shaped section is arranged longitudinally, the multiplier decreases to 0.320, which shows that reinforcement efficiency is strongly related to direction rather than section form alone.

(3) Cross-sectional shape still affects the stability result because it changes torsional rigidity and stiffness distribution. The channel-section and equal-leg-angle stiffeners provide more balanced reinforcement than the poorly oriented longitudinal T-shaped stiffener, but neither of them exceeds the circumferential T-shaped layout under the present modeling conditions.

(4) Initial ovality should not be ignored in pressure-shell design. It changes the pre-buckling stress and deformation state and may lead the instability mode to start near the region with larger geometric deviation. Stiffeners can improve load-carrying capacity, but they cannot completely cancel the shell's sensitivity to geometric imperfection.

(5) For engineering design, the results suggest that realistic geometric imperfections need to be included when selecting stiffener layouts for pressure-resistant cylindrical shells. Depending only on empirical knockdown factors may be too rough for complex stiffened systems, while a configuration-based comparison can give clearer guidance for improving buckling resistance.

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