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Financial Higher-Order Interaction Network Analysis for Precious Metals and Stock Indices Based on Information Dynamics

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Abstract. Against the backdrop of deepening global financial integration, the linkage between precious metals and stock markets has become a key issue in asset allocation and financial risk management. Existing studies mainly focus on lower-order interactions between the two asset classes, making it difficult to capture higher-order nonlinear interdependencies in multi-asset systems or to distinguish between redundancy and synergy effects. To address this gap, this paper investigates Gold, Silver, Platinum, the CSI 300 Index, and the Nasdaq Composite Index over the period from January 2016 to November 2025. Based on information dynamics, we construct a multi-resolution higher-order interaction (HOI) framework at the global, node, and link levels, derive the core O-information rate (OIR) measures under a vector autoregressive framework, and divide the sample into the Full Sample Period, Bull Market Period, Bear Market Period, and Recent Period. The results show that the five-dimensional cross-market system is overall dominated by redundancy effects. Precious metals are the main contributors to system redundancy, whereas stock indices are the primary sources of cross-market higher-order synergy. At the link level, intra-precious-metals pairs constitute the core redundancy channels, while cross-category pairs and intra-equity links are generally balanced between synergy and redundancy, except during the Bear Market Period, when significant cross-category synergy emerges. These findings provide a new perspective for understanding cross-market interactions and offer a useful framework for financial market analysis.

Keywords: precious metals market, stock market, higher-order interactions, O-information rate, cross-market linkages

1. Introduction

Precious metals, among the foundational asset classes in global financial markets, carry a dual character: they are traded as commodities, yet they function simultaneously as financial assets. In practical market settings, they operate not merely as instruments for risk hedging, value preservation, and portfolio diversification, but also as a key conduit between commodity markets and financial markets, one through which macroeconomic conditions and shifts in liquidity are often mirroblue (and, in turbulent periods, mirroblue rather starkly), thereby giving precious metals a consequential role in sustaining financial market stability [1,2]. By the same token, stock indices represent another indispensable building block of the global financial system. More than broad barometers of macroeconomic vitality and sector-specific development, they also furnish benchmark references for asset pricing and risk evaluation, and in that capacity they matter directly for investor judgment, regulatory monitoring, and the orderly, stable functioning of financial markets [3,4].

Since the start of the twenty-first century, with global financial integration deepening rather than plateauing, financial markets have come to resemble a highly interconnected and continuously evolving complex system. Price movements across asset categories no longer appear as isolated disturbances. Instead, they increasingly display interlaced patterns of collective comovement. Within this system, precious metals and stock indices, as two major asset classes, generate interactive comovements that can materially shape capital allocation and asset pricing across the broader financial architecture [5,6]. A closer examination of the mechanism linking these two markets, then, does more than refine abstract understanding: it sharpens comprehension of how complex financial systems operate, while also responding to immediate applied demands—most obviously, investors' need to optimize portfolio allocation and regulators' need to preserve financial market stability.

Among the earlier contributions, Baur et al[7]. provided empirical evidence that gold was negatively associated with stock indices only when stock markets were experiencing pronounced declines, whereas under ordinary day-to-day conditions its comovement with stock markets remained weak. Even so, that analysis was confined to a bivariate linear framework [8,9], a restriction that, by construction, left higher-order collective comovements outside the analytical field. Taking a somewhat different route, Caporale et al. [10] employed a fractional cointegration approach [11,12] and reported that, before the COVID-19 pandemic, gold displayed only weak safe-haven properties relative to the S&P 500; yet here again, the analysis did not extend beyond pairwise, lower-order interaction. Later, Beckmann et al. [13] working with a smooth transition regression (STR) model [14,15], identified threshold effects in the connection between gold and stock indices, but their approach remained centeblue on pairwise linkages and therefore could not reveal higher-order nonlinear comovements across multiple assets. Yang et al. [16] by bringing together higher-order moments [17,18] and high-frequency data, singled out silver as the principal source of spillovers, though the essential character of higher-order interactions was still left unresolved. Read collectively—not just paper by paper—the extant literature has remained bounded by pairwise, lower-order interaction analysis and has, for the most part, neglected higher-order interaction effects, leaving a consequential gap in the research landscape.

To confront these limitations directly, the present study adopts the higher-order interactions (HOIs) framework grounded in information dynamics [19], a framework that compensates, quite specifically, for the weaknesses just noted in prior work. At the methodological level, it moves beyond the conventional paradigm constrained to pairwise lower-order interactions and permits the detection of higher-order interaction effects linking precious metals and stock indices. Two problems that earlier studies could not satisfactorily resolve are therefore addressed: first, the quantification of synergistic relationships involving three or more variables; second, the discrimination between informational blueundancy and synergy. Beyond that, the study explicitly incorporates the dynamic evolutionary properties of financial time-series data, thus overcoming the limitations associated with the pblueominantly static analyses in earlier research and the correspondingly thin treatment of temporal variation in market linkages. By stepping past relatively surface-level and homogeneous readings of linkage mechanisms, the HOIs-based approach uncovers interaction logics that lower-order analysis cannot capture and, on that basis, supplies more precise theoretical support and

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empirical evidence for cross-market risk management, asset allocation, and investor decision-making.

2. Method

Vector autoregressive (VAR) models constitute a key tool for obtaining linear solutions. This modeling framework allows the analytical procedure to be conducted under stationarity constraints, without imposing the strong assumption that the intrinsic generative mechanism of the process must itself be linear. According to the Wold decomposition theorem, any stationary stochastic process can be represented through a linear VAR model, which endows this approach with strong generality and practical tractability.

Or a dynamic network system consisting of N nodes, let the node dynamics be represented by an N-dimensional discrete-time stationary zero-mean stochastic process set denoted by $Y = \{Y_1, Y_2 \cdots Y_N\}$. Then, the p-th order VAR model of the whole system is defined as:

$$Y_n = \sum_{k=1}^{p} A_k Y_{n-k} + U_n \tag{1}$$

Here, Y_n denotes the N-dimensional observation vector of the stochastic process at time n; the non-negative integer p denotes the lag order of the full-system VAR model; A_k is the N×N coefficient matrix corresponding to the k-th lag; and U_n is the N-dimensional residual vector of the model, with covariance matrix given by $\Sigma_U \equiv \mathbb{E}\left[U_n U_n^T\right]$.

Building on the full-system VAR model and the restricted VAR model for subsets, and under the classical statistical assumption of joint Gaussianity of the time series, it is possible to directly derive the closed-form expression of the entropy rate, which serves as the fundamental building block for all the information-theoretic measures employed in this study and provides the analytical basis for the subsequent closed-form derivation of higher-order measures.

For the univariate stochastic process Y_j associated with a single node, setting M=1 in the above expression yields the simplified closed-form expression for the marginal entropy rate:

$$H_{Y_j} = \frac{1}{2} log(2\pi e \cdot \Sigma_{W_{Y_j}}) \tag{2}$$

Here, $\Sigma_{W_{Y_j}}$ denotes the residual variance of the univariate restricted VAR model for the j-th node.

Based on the closed-form expression of the entropy rate, one can further derive the closed-form solution for the mutual information rate (MIR), which quantifies the strength of dynamic coupling between two processes. For any two disjoint subsets of stochastic processes, F and Z, let their joint process be $[FZ] = F \cup Z$. The mutual information rate (MIR) between F and Z is obtained from their marginal entropy rates and joint entropy rate, yielding:

$$I_{Z;F} = \frac{1}{2} log\frac{|\Sigma_{W_Z}||\Sigma_{W_F}|}{|\Sigma_{W_{ZF}}|} \tag{3}$$

For the special case of two univariate stochastic processes Y_i and Y_j , setting Z= Y_i and F= Y_j gives the closed-form MIR between them.

Based on these computational elements, the analysis extends from the global to the local scale and introduces higher-order interaction measures at three resolutions, forming a multi-resolution framework for higher-order interaction analysis.

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At the global level, the O-information rate (OIR) measures the overall higher-order interaction of the system by capturing net effects beyond individual nodes and pairwise interactions, thereby reflecting the balance between synergy and redundancy. For an N-dimensional stochastic process system, the global OIR, denoted by Ω_Y , is defined as:

$$\Omega_Y = (N-2)H_Y + \sum_{j=1}^{N}[H_{Y_j} - H_{Y^j}] \tag{4}$$

The corresponding node-level quantity is the OIR gradient, which measures the marginal increase in global OIR when a node is added to the system, thus characterizing the role of that node in the higher-order interaction structure. Following the decomposition logic of global OIR, it is defined as:

$$\Omega_Y \equiv \Omega_{Y^j} + \Delta_{Y_j;Y^j} \tag{5}$$

With this expression in place, the OIR gradient can be directly expanded to obtain its explicit computational form, as shown below:

$$\Delta_{Y_j;Y^j} = \sum_{\substack{i=1 \\ i \neq j}}^{N} I_{Y_j;Y^{ij}} + (2-N)I_{Y_j;Y^j} \tag{6}$$

Replacing the mutual information terms with mutual information rates gives the closed-form OIR gradient:

$$\Delta_{Y_j;Y^j} = \sum_{\substack{i=1 \\ i \neq j}}^{N} \left(\frac{1}{2} log\frac{\Sigma_{W_{Y_j}} \cdot |\Sigma_{W_{Y^{ij}}}|}{|\Sigma_{W_{Y_jY^{ij}}}|}\right) + (2-N) \cdot \left(\frac{1}{2} log\frac{\Sigma_{W_{Y_j}} \cdot |\Sigma_{W_{Y^j}}|}{|\Sigma_U|}\right) \tag{7}$$

To further refine higher-order interactions, a link-level measure, the local O-information rate (local OIR), is introduced to capture the net higher-order effects jointly induced by a given node pair and the remaining nodes in the system.

3. Results

Before proceeding to the formal analysis, we first provide a preliminary descriptive overview of the variables to characterize the overall distribution, volatility levels, and extreme-value features of Gold, Silver, and Platinum prices (USD/oz), as well as the CSI 300 Index and the Nasdaq Index, thereby laying the foundation for the subsequent time-series analysis and empirical investigation.

To present the basic data characteristics of Gold prices, Sliver prices, Platinum prices, the CSI 300 Index, and the Nasdaq Index in an intuitive manner, Table 1 reports the core descriptive statistics for each variable, including the sample size, mean, median, minimum, maximum, standard deviation, and coefficient of variation, thereby providing a data foundation for the subsequent analysis of time-series dynamics and empirical investigation.

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Table 1. Results of descriptive statistics for the variables

Variable	Sample Size	Mean	Median	Minimum	Maximum	Standard Deviation	Coefficient of Variation
Gold	2302	169.6058	164.135	106.61	397.45	56.4031	0.3326
Silver	2302	22.2095	20.9840	11.735	53.332	6.9674	0.3137
Platinum	2302	975.7601	951.45	595.9	1675	141.9698	0.1455
CSI300	2302	3942.6621	3870.855	2855.6	5922.07	575. 005	0.146
Nasdaq	2302	11343.3148	11067.3799	4397.9502	23987.2891	4752.6893	0.419

As reported in Table 1, each variable contains 2,302 observations, which points to a dataset free of missing values. On the distributional side, the gap between the mean and the median remains modest across variables, a pattern broadly consistent with the mild right-skewness that typically shows up in financial time-series data. Taken together, the data appear to be of acceptable quality, with no clear indication that extreme outliers materially distort the overall series. With respect to volatility, the Nasdaq Index exhibits the largest relative fluctuation; next come Gold and Silver prices, whereas Platinum prices and the CSI 300 Index show the most stable movement patterns.

For a more intuitive view of how each variable evolves over time, the time-series paths of precious metal prices and stock indices are plotted in Figure 1.

That said, because Silver price movements are comparatively small, its trend is not especially clear in the figure. For that reason, the time-series data for the three precious metals are additionally transformed through Gaussian standardization, and the standardized series are then shown in Figure 2.

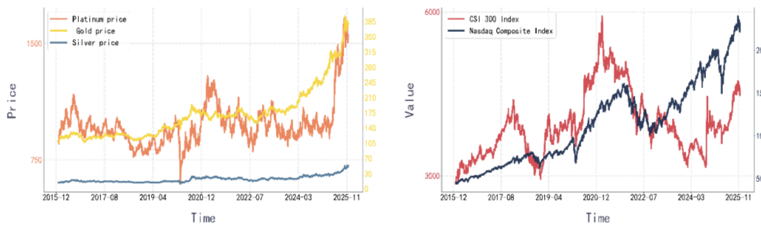

Figure 1. Time-series trends of financial market features

As Figure 2 shows, across the full sample window from January 2016 through November 2025, the prices of the three precious metals together with the two major stock indices underwent clearly time-varying fluctuations.In the precious metals sector, gold prices showed a sustained upward trend over the long term and remained generally stable throughout the sample period. Silver moved broadly in line with gold, but stayed at a lower price level and within a narrower fluctuation range. By contrast, platinum showed larger price swings, with several clear upward and downward reversals during the observation period; near the end of the sample, it rose sharply and reached a new high for the period under study.In terms of stock indices, the Nasdaq Composite Index followed a clear upward trend and recorded substantial cumulative gains despite occasional pullbacks. The CSI 300 Index, by comparison, moved mainly within a certain range rather than rising continuously over the long run, and both its volatility and cumulative gains were notably lower than those of the Nasdaq. Toward the end of the observation period, both indices moved upward, broadly in line with the trend in precious metal prices.

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Figure 2. Time-series trends of gaussian-standardized gold, platinum, and silver prices

Having clarified the variables' overall distributional features and their time-series trajectories, the analysis next turns to the linear co-movement among the five core variables—Gold prices, Silver prices, Platinum prices, the CSI 300 Index, and the Nasdaq Composite Index. For that purpose, the Pearson correlation coefficient is used to capture the strength of linear association across these series, with the corresponding results presented in Figure 3.

According to Figure 3, all five core variables are positively linearly correlated, albeit to very different extents, and no negative association appears for any variable pair. Especially strong is the co-movement within the precious metals cluster: the correlation coefficient between Gold and Silver prices reaches 0.955, while Platinum also retains comparatively strong positive correlations with Silver and Gold, at 0.762 and 0.636, respectively. Set against that pattern, all three precious metals are also positively correlated with the Nasdaq Composite Index, and the coefficients, ordeblue from largest to smallest, are 0.914 for Gold prices,0.903 for Silver prices, and 0.532 for Platinum prices. Much weaker, by contrast, are the CSI 300 Index's linear relationships with the other four variables. Its correlation coefficients with the Nasdaq Composite Index, Platinum prices, Silver prices, and Gold prices are 0.419,0.396,0.392, and 0.280, respectively; taken together, these values suggest that its linear association with the remaining assets is comparatively limited

Although the Pearson correlation coefficient offers informative evidence regarding the linear relationships among Gold prices, Silver prices, Platinum prices, the CSI 300 Index, and the Nasdaq Composite Index, its scope is confined to linear dependence and therefore does not register the nonlinear comovements and time-varying dynamic effects that so often characterize financial variables. To characterize more fully the interaction structure connecting these five asset classes, the analysis accordingly introduces the mutual information rate (MIR) as a measure of the strength of dynamic coupling between variables. By construction, this indicator captures both linear and nonlinear dynamic associations, and does so more completely than correlation alone. The corresponding heatmap of the estimated results appears in Figure 3.

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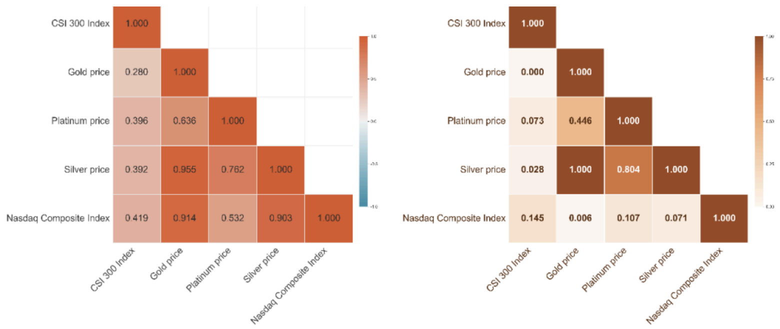

Figure 3. Heatmaps of pearson correlation and mutual information rate for the study variables

As Figure 3 shows, the degree of dynamic coupling varies substantially across variables. The strongest dynamic information sharing appears within the precious metals group: the normalized MIR between gold and silver reaches 1.000, while the silver-platinum and gold-platinum pairs record 0.804 and 0.446, respectively. These results are consistent with the earlier linear correlation analysis and further confirm stable co-movement within the precious metals market. At the same time, the MIR estimates reveal features beyond linear analysis. Although Pearson correlations indicate that both gold and silver are strongly and positively correlated with the Nasdaq Composite Index, their MIR values are only 0.006 and 0.071. This suggests that their relationship with the stock index is driven mainly by contemporaneous linear co-fluctuation, whereas dynamic nonlinear coupling is extremely weak. Meanwhile, the MIR values between the CSI 300 Index and the other four asset classes remain low, which is consistent with the earlier evidence of weak linear correlations and further indicates limited overall co-movement.

Building on information dynamics, this study develops a multi-resolution framework to measure higher-order interactions at three levels: the global system, individual nodes, and pairwise links. Within a linear vector autoregressive (VAR) framework and under joint Gaussianity, closed-form expressions are derived for the entropy rate, mutual information rate, and three higher-order interaction measures. The analysis focuses on the dynamic evolution of five cross-market asset classes, namely stock indices and precious metals, with particular attention to how higher-order interactions evolve across different financial market cycles. Given the cyclical behavior of both stock and precious metal markets, the full sample is divided into four sub-periods: the full sample period, the bull-market period, the bear-market period, and the recent period. Based on this framework, the study then examines the main characteristics and evolution of cross-market higher-order interactions under different market conditions. The time partition is shown in Table 2:

Table 2. Subsample period classification

Period	Start Date	End Date
Full Sample Period	2016-01-29	2025-11-26
Bull Market Period	2016-01-29	2018-01-30
Bear Market Period	2021-02-18	2024-02-05
Recent Period	2024-09-03	2025-11-26

The object of analysis is an N=5 discrete-time stationary stochastic process corresponding to the daily value series of five cross-market asset classes spanning stock indices and precious metals. Before the empirical analysis, the raw data undergo standardized preprocessing. First, outliers and missing values are treated by replacing identified outliers with missing values and then filling

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missing observations through a combination of linear interpolation, forward filling, and backward filling to preserve series continuity. Second, the daily closing price series are transformed using first-order log differences to obtain log-return series and remove the non-stationarity inherent in price levels. Finally, each return series is standardized to zero mean by subtracting its sample mean and dividing by its sample standard deviation, thereby eliminating the influence of scale differences across assets on subsequent model estimation.

In the stage of model specification and parameter estimation, we directly rely on the constructed OIRs algorithm module by inputting the preprocessed standardized data into the HOI_analysis function. This procedure simultaneously completes the parameter estimation for the full-system and subset-restricted vector autoregressive models, and automatically computes the full set of indicators, including the entropy rate, mutual information rate, global OIR, node-level OIR-gradient, and local OIR matrix, based on the estimated model results.

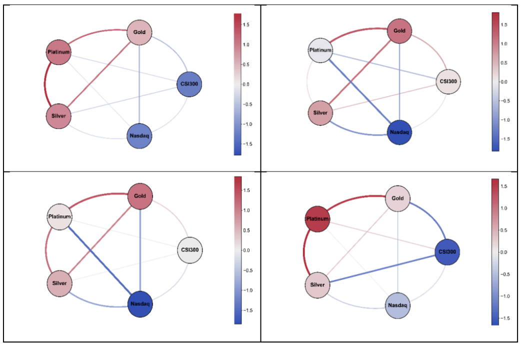

Figure 4. Higher-order interaction networks across different periods

According to the estimation results in figure 4, the global OIR values of the cross-market system during the Full Sample Period, Bull Market Period, Bear Market Period, and Recent Period are 0.125, 0.339, 0.2167, and 0.3743, respectively.

At the node level, the OIR gradient results show clear differences across periods. Over the full sample, the standardized OIR gradient values for gold, silver, platinum, the CSI 300 Index, and the Nasdaq Composite Index were 0.51, 0.89, 1.02, -1.24, and -1.18, respectively; in the recent period, these values changed to 1.08, 0.71, -0.07, 0.10, and -1.82. During the bull market, the corresponding values were 1.12, 0.57, 0.13, 0.02, and -1.84, whereas during the bear market they were 0.21, 0.30, 1.51, -1.52, and -0.50.

At the link level, the local OIR results also show significant differences across periods. Over the full sample, the local OIR values for gold-silver, gold-platinum, gold-CSI 300, gold-Nasdaq, silver-platinum, silver-CSI 300, silver-Nasdaq, platinum-CSI 300, platinum-Nasdaq, and CSI 300-Nasdaq were 0.093, 0.095, 0.00033, -0.0044, 0.11, 0.011, 0.015, 0.014, 0.021, and 0.015, respectively; in the recent period, these values increased to 0.18, 0.18, 0.16, 0.065, 0.14, 0.15, 0.057, 0.071, 0.065, and 0.088. During the bull market, the corresponding values were 0.15, 0.18, 0.10, -0.028, 0.17, 0.048, -0.024, 0.045, -0.064, and 0.023; by contrast, during the bear market they were 0.14, 0.18, 0.086, 0.11, 0.18, 0.087, 0.11, 0.14, 0.13, and 0.11.

4. Conclusion

During the recent period (2024.9–2025.11), the global OIR of the cross-market system remained significantly positive, indicating that net higher-order interactions in the five-dimensional system were still dominated by redundancy rather than synergy. This suggests persistent informational

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overlap among the five asset classes, so that the price movement of one asset could still be explained to a considerable extent by information from the others. At the node level, gold and silver had positive standardized OIR gradient values and thus reinforced system redundancy, with gold contributing the most and silver also playing an important role. Platinum and the CSI 300 Index were closer to balance and showed no clear net higher-order interaction effect, whereas the Nasdaq Composite Index was the main source of synergy, implying stronger uniqueness in its price dynamics. At the link level, local OIR values within the precious metals group were mostly significantly positive, making them the main channels of redundancy transmission. In particular, the platinum-gold and gold-silver pairs were the strongest redundancy links, while gold-CSI 300 and silver-Nasdaq also showed some redundant connections. Most other cross-category links, as well as the CSI 300-Nasdaq pair, were close to zero, indicating a broad balance between synergy and redundancy.

During the bull-market period, the global OIR also remained significantly positive, showing that the system continued to be redundancy-dominated and that informational overlap among assets remained high. At the node level, all three precious metals had positive standardized OIR gradient values and continued to strengthen system redundancy, with gold exerting the strongest effect, followed by silver, while platinum remained closer to balance. This suggests that under bull-market conditions, precious metals still served as the main carriers of informational overlap and homogeneous co-movement. Among equities, the Nasdaq Composite Index remained the strongest source of synergy, whereas the CSI 300 Index stayed relatively balanced. At the link level, all intra-precious-metal pairs had positive local OIR values, especially platinum-gold, platinum-silver, and gold-silver, indicating a strong and homogeneous higher-order linkage structure within the precious metals sector. By contrast, links between precious metals and equities, as well as the CSI 300-Nasdaq link, were mostly close to zero.

During the bear-market period (2021–2024), the global OIR was still significantly positive, indicating that the five-dimensional system remained primarily redundancy-dominated. The five asset classes still showed substantial informational overlap, and the price change of one asset could still be interpreted to a large extent through information contained in the others. At the node level, all three precious metals had positive standardized OIR gradient values and continued to strengthen redundancy, with platinum contributing the most, followed by silver, while gold also played a supportive role. In contrast, both the CSI 300 Index and the Nasdaq Composite Index had negative standardized OIR gradient values, indicating stronger contributions to system synergy, especially from the CSI 300 Index. At the link level, all precious-metal pairs showed positive local OIR values and served as the main redundancy channels, with platinum-silver and platinum-gold being the most prominent. By contrast, links between precious metals and equities, as well as the CSI 300-Nasdaq pair, were negative, suggesting some degree of synergy; among them, CSI 300-gold and CSI 300-Nasdaq were the most important synergistic channels.

From the full-sample perspective (2016–2025), the global OIR remained significantly positive throughout the sample, indicating that net higher-order interactions in the five-dimensional system were generally dominated by redundancy. This means that the five asset classes shared considerable informational overlap, and their price fluctuations were not independent but could be partly explained by information embedded in the others. At the node level, all three precious metals had positive standardized OIR gradient values and jointly constituted the main source of system redundancy, with platinum contributing the most, followed by silver, while gold also played an important role. In contrast, both the CSI 300 Index and the Nasdaq Composite Index had negative standardized OIR gradient values and contributed more to system synergy, with the CSI 300 Index playing the stronger role. At the link level, all pairs within the precious metals group had positive local OIR values and formed the main transmission paths of redundancy, with platinum-silver being the strongest, and platinum-gold and gold-silver also relatively prominent. Most links between precious metals and equities, as well as the CSI 300-Nasdaq pair, were close to zero, indicating an overall balance between synergy and redundancy.

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