

Inflationary Cosmology: Theories and Observational Tests

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Abstract. The theory of cosmic inflation has resolved the horizon and flatness dilemmas in standard Big Bang cosmology and successfully predicted the existence of primordial density perturbations and primordial gravitational waves. This paper systematically reviews three theoretical frameworks of inflation: classical slow-roll inflation, bounce cosmology, and the unified model (Quintessential Inflation). In the theoretical section, this paper introduces the dynamic equations of the inflationary field, the slow-roll condition, the power spectrum of primordial perturbations, and the mechanism for breaking the zero-energy condition in bounce cosmology. In the applied section, this paper analyzes the constraints of the cosmic microwave background radiation on the inflationary model using the latest observational data from the Planck satellite and the BICEP/Keck experiment, and discusses the enhancement effect of the kination phase on the primordial gravitational wave spectrum. The results show that the bounce model can explain the suppressed anomaly of the large-scale power spectrum of the CMB, while the unified model not only predicts gravitational wave signals that can be detected by future detectors (such as LISA), but also provides a natural solution to the problem of fine-tuning dark energy.

Keywords: cosmic inflation, gravitational waves, cosmic microwave background radiation, bounce cosmology, quintessential inflation

1. Introduction

Precise measurements of the cosmic microwave background radiation and observations of the large-scale structure of galaxies indicate that the universe underwent a period of violent expansion in its very early stages, known as cosmic inflation. In 1981, Alan Guth proposed the theory of inflation [1], which solved the problem of horizon and flatness in the standard hot Big Bang model and predicted the existence of primordial density perturbations and primordial gravitational waves. During inflation, quantum fluctuations amplified these perturbations, eventually evolving into the seeds of large-scale structures in the universe and leaving observable traces.

After the theory of inflation came out, scholars successively proposed various models. The classic slow-rolling inflation model assumes that the scalar field slowly rolls on a very flat potential energy surface, successfully explaining the basic characteristics of the temperature anisotropy of the cosmic microwave background radiation (CMB) [2], but it cannot bypass the cosmic singularity. The bounce cosmology model avoids the singularity by introducing a contraction phase and violating the zero-energy condition (NEC) [3,4]. The essential inflation model unifies the

inflationary field and the dark energy field into the same scalar field, with two flat regions of potential energy, to unify the explanation of early inflation and late dark energy [5]. In recent years, observational data from the Planck satellite and the BICEP/Keck experiment have imposed very strict constraints on inflationary models [6], and significant progress has been made in the detection of primordial gravitational waves [7].

This paper mainly introduces three theoretical frameworks of inflation and discusses whether they can be verified in conjunction with the latest observational data. Section 2 discusses the dynamic mechanisms of classical slow-roll inflation, bounce cosmology, and essential inflation from a theoretical perspective. Section 3 focuses on applications, analyzing the limitations of CMB observational data on these models, the prospects for primordial gravitational wave detection, and a unified explanation of dark energy and inflation. Section 4 is a summary and outlook.

2. Theories

2.1. Classical slow-roll inflation

The classic inflationary theory was first proposed by Alan Guth in 1981, showing that there is a dramatic expansion during a short period after the Big Bang [8]. This theory solved the horizon and flatness problems, and it is driven by a scalar field ϕ .

For the Inflation Dynamics, it is described by an equation of motion for a homogeneous scalar field:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \quad (1)$$

In this equation, $H = \dot{a}/a$, which is the Hubble parameter. For inflation to happen, the potential energy must exceed the kinetic energy, which is called the slow-roll condition. There are two parameters: $\epsilon = \frac{M_{\text{Pl}}^2}{2} \left(\frac{V'}{V} \right)^2$, $\eta = M_{\text{Pl}}^2 \frac{V''}{V}$ in which M_{Pl} is the Planck mass. The slow-rolling parameters ϵ and η are crucial for determining whether inflation can occur. When both parameters are much less than 1, the inflationary field slowly rolls along a potential energy plateau, and the universe expands approximately exponentially. When ϵ increases to 1, inflation ends, and the universe enters a reheating phase [9].

Next, it is necessary to mention the Primordial Perturbations. During inflation, quantum fluctuations are stretched to large scales, leading to classical perturbations. These perturbations are the origin of forming galaxies. The scalar power spectrum is:

$$\mathcal{P}_\zeta(k) = \frac{H^2}{8\pi^2 M_{\text{Pl}}^2 \epsilon} \Big|_{k=aH} \quad (2)$$

The power spectrum $\mathcal{P}_\zeta(k)$ describes the magnitude of the initial density perturbation as a function of scale k . In the inflationary model, this spectrum is approximately scale-invariant at the observation scale, but exhibits a slight redshift, i.e., the spectral exponent n_s is slightly less than 1. This is precisely the result observed by the Planck satellite. The spectral index n_s describes how the spectrum changes with scale: $n_s - 1 = 2\eta - 18$.

Inflation also produces gravitational waves. The tensor-to-scalar ratio is: $r = 16\epsilon$. In addition to the standard scalar-field paradigm, alternative physical mechanisms have been proposed. Law suggested that the finite size of the electron, combined with a glass transition at ultra-high densities, can naturally produce a period of exponential expansion without requiring an inflaton field [10].

2.2. Bouncing cosmology

The producing of the Bouncing Cosmology model successfully solved a problem—singularity problem. In the standard Big Bang model, going backwards in time leads to a singularity where $a \rightarrow 0$ and density becomes infinite. In this situation, all the physics laws would break down. However, in the theory of bouncing cosmology, there is a contracting phase before inflation, which can avoid this [3].

At the bounce point in the bouncing cosmology, the universe stops contracting and starts expanding. This means $H = 0$ and $\dot{H} > 0$ at the bounce. From the Friedmann equations, it requires violation of the null energy condition (NEC): $\rho + p < 0$ [3]. Several mechanisms can produce a stable bounce, such as the "Quintom" model and the Galileon model [4]. The former uses a field that changes from phantom to normal behavior. The latter uses higher-derivative terms to avoid instabilities. In both cases, the scale factor a must be continuous across the bounce.

Similar to the Classical Slow-Roll Inflation, perturbations also happen during the bouncing period. The evolution of curvature perturbations through the bounce is described by:

$$u_k'' + \left(c_s^2 k^2 - \frac{z''}{z} \right) u_k = 0 \quad (3)$$

In this equation, $u_k = z\mathcal{R}_k$, $z = a\sqrt{2\epsilon}/c_s$, and the prime means derivative with respect to conformal time. Matching solutions before and after the bounce changes the power spectrum at large scales. This can explain a key characteristic of bounce cosmology---the contraction phase preceding inflation leaves a unique imprint on the large-scale portion of the primordial power spectrum. By adjusting the parameters of the contraction phase, the power spectrum depression observed in CMB observations can be well fitted.

2.3. Quintessential inflation

Traditional inflationary and dark energy models typically use different scalar fields, which introduces the problem of fine-tuning the parameters [9]. However, the unified model (the quintessential inflation model) proposes that both inflation in the early universe and the accelerating expansion in today's universe are driven by a same scalar field. The potential has two flat regions: one is at the large field values for inflation, and another is at small field values for dark energy. A simple example is:

$$v(\phi) = M^4 e^{-\lambda\phi} \quad (4)$$

The Starobinsky model also belongs to this when extended to late times:

$$V(\phi) = M^4 \left(1 - e^{-\sqrt{2/3}\phi/M_{Pl}} \right)^2 \quad (5)$$

After the end of inflation, the field is dominated by kinetic energy instead of potential energy. This period is called kination. Here is an equation:

$$w = \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V} \quad (6)$$

During kination, the energy density has a relationship of $\rho \propto a^{-6}$ [5]. As for reheating, it happens through gravitational particle production, and the field would not completely decay. After reheating, the field freezes due to Hubble friction, and it stays nearly constant during radiation and matter domination. When the Hubble parameter drops enough, the field starts rolling again and drives the current accelerated expansion. The equation of state today is close to $w \approx -1$, matching observations [5].

3. Applications

3.1. Cosmic microwave background constraints

The CMB is the most accurate evidence of inflation. The temperature anisotropy and polarization power spectrum of the CMB carry a wealth of information about the primordial perturbation, including its amplitude, spectral index, and contribution of primordial gravitational waves. The Planck satellite and the BICEP/Keck experiment have brought observational precision to unprecedented levels. Here is current results: $n_s = 0.965 \pm 0.004$, $r < 0.036$ (95% CL) [6]. The observation of n_s indicate a slight redshift in the original perturbation spectrum, consistent with inflationary predictions. The upper limit of the tensor ratio r excludes many simple large-field inflationary models.

CMB data show some anomalies at a large scale. There is power suppression at low multipoles ($l \sim 20 - 30$). Bouncing models can explain these using two parameters. B_{con} is related to the Hubble scale during contraction, while B_{exp} is related to the Hubble scale during inflation [3]. In the rebound model, these two parameters characterize the co-moving Hubble scale during the contraction and inflation phases, respectively. By adjusting their values, the shape of the original power spectrum on a large scale can be altered, thereby explaining the anomalous depression observed in CMB observations.

As shown in Figure 1, the bounce model predicts a suppression of power at low multipoles, which can explain the large-scale anomalies observed in the CMB data.

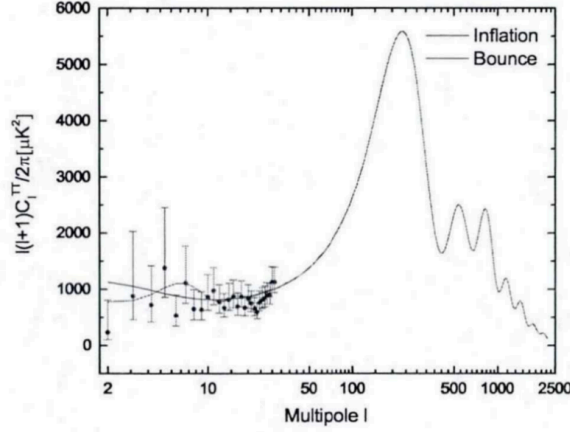


Figure 1. CMB temperature angular power spectrum from Planck 2015 data (black dots) compared with the standard inflation model (black line) and the bounce inflation model (red line). The bounce model shows a suppression of power at large scales [3]

3.2. Gravitational wave probes

Inflation theory predicts that, in the early universe, there is a background of primordial gravitational waves produced by tensor perturbations. The spectrum of these gravitational waves is nearly scale-invariant. Current upper limits from experiments like BICEP/Keck have already ruled out many inflation models [5,6].

Primordial gravitational waves are one of the most unique predictions of inflationary theory. Unlike density perturbations, gravitational waves are tensor perturbations, which generate B-mode polarization signals in the CMB [6]. Although experiments such as BICEP/Keck have not yet directly detected primordial B modes, existing upper limits have imposed strict constraints on inflationary models.

In unified models, the kination phase changes the gravitational wave spectrum at high frequencies. The relationship of energy density and frequency interval is:

$$\Omega_{GW}(f) \propto f^{2 \cdot \frac{3w-1}{3w+1}} \quad (7)$$

For kination with $w = 1$, it gives $\Omega_{GW} \propto f^5$. This blue-tilted spectrum is a unique feature of the unified model. The enhanced gravitational wave spectrum for the kination-amplified scenario is shown in Figure 2.

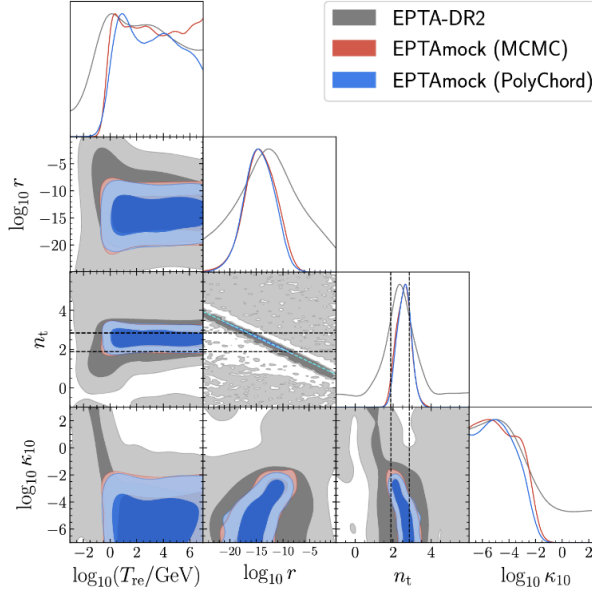


Figure 2. Primordial gravitational wave spectrum Ω_{GW} as a function of frequency f for the kination amplified scenario. The kination phase with equation of state $w = 1$ enhances the gravitational wave signal at high frequencies, producing a blue-tilted spectrum $\Omega_{GW} \propto f$. The sensitivity curves of LISA and other detectors are shown for comparison [8]

Although today there is no such direct evidence for a stiff equation of state in the early universe, future gravitational wave detectors will be able to observe this signal. To be specific, LISA will operate in the 0.1-100 mHz range [5]. It mainly detects astrophysical sources such as the merger of supermassive black holes, but it also has the ability to detect gravitational wave backgrounds from the early universe. Next-generation ground detectors like the Einstein Telescope will cover higher frequency bands (10-1000 Hz).

3.3. Dark energy and large-scale structure

The existence of dark energy was first discovered through observations of Type Ia supernovae, and subsequently confirmed by independent observations such as baryon acoustic oscillations. These observations not only revealed that the universe is expanding at an accelerating rate, but also precisely measured the parameters of the dark energy equation of state. Data from Pantheon, JLA, and SDSS give $w_{DE} = -1.018 \pm 0.031$ [9]. This is consistent with a cosmological constant and with quintessence models where $w \approx -1$ today.

The unified model provides that w_{DE} is close to -1 today, and the field's energy density is determined by inflationary dynamics. These effectively avoid the fine-tuning problem of the cosmological constant.

4. Conclusion

This paper systematically reviews three theoretical frameworks for cosmic inflation and their observational verifications. Theoretically, classical slow-roll inflation successfully explains the origin of primordial perturbations through the slow-roll condition of a scalar field, but faces the singularity problem. Bounce cosmology, by introducing a contraction phase and breaking the zero-energy condition, avoids the cosmic singularity; the matching of its perturbation equations before

and after the bounce can explain the suppressed anomaly in the large-scale power spectrum of the CMB. The Quintessential Inflation model unifies inflation and dark energy into a single scalar field; its potential energy simultaneously includes the inflationary plateau and the dark energy tail, and the kination phase causes a blueshift characteristic in the primordial gravitational wave spectrum: $\Omega_{\text{GW}} \propto f$.

In terms of applications, Planck and BICEP/Keck observational data provide strict constraints: $n_s = 0.965 \pm 0.004$ and $r < 0.036$. The bounce model can naturally explain the power suppression problem on a large scale in CMB, and the enhanced gravitational wave signal predicted by the unified model is expected to be seen by detectors like LISA. In addition, the unified model has given the energy scale of dark energy through inflationary dynamics, providing a new approach to the fine adjustment of the cosmological constant. Inflation theory has indeed been quite successful, but some issues remain unclear. For example, what exactly the inflationary field is still unknown, and candidate particles such as the Higgs field and axions still need to be tested. The microscopic mechanism that violates the zero-energy condition in bounce cosmology also needs more solid theoretical support. Looking ahead, if space gravitational wave detectors such as LISA, Taiji, and Tianqin can run and directly measure primordial gravitational waves, it will be a crucial criterion for evaluating the inflation model. Next-generation CMB experiments like CMB-S4 and LiteBIRD can significantly improve the measurement accuracy of the tensor ratio r , and the parameter space estimation of the inflation model will be further tightened. Inflationary cosmology is at a crossroads between theory and observation, and the next decade will be a critical period for testing it.

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