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Observational Tests of General Relativity: Gravitational Redshift and Light Deflection

Kunhe Liu

School of Physical and Mathematical Sciences, Nanyang Technological University, Singapore, Singapore

LIUK0024@e.ntu.edu.sg

Abstract. In classical Newtonian theory, space and time are treated as separate entities, where time flows independently and space is Euclidean. However, this framework becomes inadequate in describing phenomena involving light and strong gravitational fields. This limitation motivates the development of general relativity, in which space and time are unified into a single geometric structure—spacetime, and gravity is interpreted as its curvature. Within this framework, the principles of geometry and symmetry determine the form of physical laws and their observable consequences. In this work, literature analysis and theoretical derivation methods are employed to study two classical observational tests of general relativity—gravitational redshift and light deflection—from a geometric and symmetry-based perspective, along with comparison to Newtonian theory. The results show that both phenomena arise naturally from the geometric structure of spacetime and can be described within a unified framework. Spacetime symmetries lead to conserved quantities, which provide an effective method for analyzing photon trajectories. The comparison with Newtonian theory highlights that the differences in predictions originate from the underlying geometric structure of spacetime.

Keywords: General Relativity, Gravitational Redshift, Light Deflection, Killing Vectors, Schwarzschild Spacetime

1. Introduction

Gravity has long been one of the most fundamental interactions in physics. In Newtonian mechanics, gravity is described as a force acting instantaneously between masses. However, this description becomes inadequate when dealing with phenomena involving light or strong gravitational fields.

The equivalence principle suggests that gravity may not be viewed as a force in the classical sense, but as a geometric property of spacetime, namely spacetime curvature. Thus, general relativity introduces a new framework in which spacetime must be curved in situations where a gravitational field is physically present [1]. The appropriate mathematical structure used to describe curvature is that of a differential manifold: a kind of set that looks locally like flat space but might have a very different global geometry [2]. This curvature is described by the Einstein field equation $G_{ab}=8\pi T_{ab}$, which relates the geometric structure of spacetime, represented by the Einstein tensor

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G_{ab} , to the distribution of matter and energy described by the stress–energy tensor T_{ab} . As summarized by Misner, Thorne and Wheeler, "space acts on matter, telling it how to move. In turn, matter reacts back on space, telling it how to curve." [3] Within this geometric framework, many observable gravitational phenomena can be understood as direct consequences of spacetime curvature. Two well-known effects—the deflection of light and gravitational redshift—can be naturally explained by applying the geometric principles of general relativity together with basic physical principles such as general covariance. These two phenomena are historically important tests of general relativity, provide direct observational evidence for the geometric nature of gravity, and have been confirmed by classical experiments, including measurements of light deflection during solar eclipses [4] and laboratory tests of gravitational redshift [5].

Although these two phenomena arise from different physical mechanisms, they can be understood within the same geometric framework of spacetime. In this work, we review the theoretical origin of these two effects and show how both emerge naturally from the spacetime geometry and relevant symmetries described by general relativity. In particular, the symmetries of spacetime give rise to conserved quantities associated with Killing vector fields. These conserved quantities provide an elegant geometric approach for analyzing photon trajectories in curved spacetime. These effects provide clear examples of how different physical effects arise from the same underlying spacetime structure.

2. Spacetime symmetries, killing vectors and geodesic motion

In physics, symmetries generally imply conservation laws. In general relativity, symmetries of spacetime are represented by Killing vector fields. A Killing vector field generates a coordinate transformation that preserves the metric, namely

$$\mathscr{L}_{\xi} g_{ab} = 0. \tag{1}$$

Equivalently, this condition can be written as the Killing equation

$$\nabla_a \xi_b + \nabla_b \xi_a = 0 \qquad \text{or} \qquad \nabla_a \xi_b = \nabla_{[a} \xi_{b]} \tag{2}$$

Given a Killing vector field ξ^a and the tangent vector T^a of a geodesic, one obtains

$$T^a \nabla_a \left(T^b \xi_b\right) = T^a T^b \nabla_a \xi_b = T^{(a} T^{b)} \nabla_{[a} \xi_{b]} = 0. \tag{3}$$

Thus, the quantity

$$T^b \xi_b \tag{4}$$

remains constant along the geodesic. This result shows that spacetime symmetries represented by Killing vector fields give rise to conserved quantities along geodesic motion.

A stationary spacetime is one which possesses a timelike Killing vector field. A spacetime is said to be static if it is stationary and, in addition, there exists a (spacelike) hypersurface Σ which is orthogonal to the orbits of the isometry [2].

For a static and spherically symmetric spacetime, the Schwarzschild solution to Einstein's field equations is given by the line element, written in spherical coordinates $(r\theta\phi)$:

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$$ds^2 = -\left(1\frac{2M}{r}\right)dt^2 + \left(1\frac{2M}{r}\right)^{-1}dr^2 + r^2\left(d\theta^2 \sin^2\theta\, d\phi^2\right). \tag{5}$$

Without loss of generality, we may restrict attention to the study of the equatorial geodesics [2]. Thus, for timelike or null geodesics, due to the spherical symmetry of the metric one may choose coordinates such that the motion lies in the equatorial plane $\theta = \frac{\pi}{2}$. Therefore, the line element along such a curve can be reduced to

$$ds^2 = -\left(1\frac{2M}{r}\right)dt^2 + \left(1\frac{2M}{r}\right)^{-1}dr^2 + r^2 d\phi^2. \tag{6}$$

3. Observational tests of general relativity

3.1. Gravitational redshift

In a stationary spacetime with Killing vector field $\xi^a = (\partial_t)^a$, consider two static observers whose four-velocity is

$$Z^a = \chi^{-1}\xi^a \tag{7}$$

where $\chi = \sqrt{-\xi^a \xi_a}$. Their respective worldlines are denoted by $G:\gamma(\tau)$ and $G^{'}:\gamma^{'}(\tau^{'})$. A photon is emitted at a point p on G and later received at a point $p^{'}$ on $G^{'}$.

The angular frequency of the photon measured by an observer with four-velocity Z^a is defined by

$$\omega = -k_a Z^a, \tag{8}$$

where k^a is the wave four-vector of the photon and it satisfies the null condition

$$k^a k_a = 0 \tag{9}$$

which implies

$$k^b \nabla_b k_a = 0 \tag{10}$$

Therefore, the frequencies measured at the emission and reception events satisfy

$$\omega = -k_a Z^a|_p, \omega^{'} = -k_a Z^a|_{p^{'}}. \tag{11}$$

Since ξ^a is a Killing vector field, the quantity $k_a \xi^a$ is conserved along null geodesics. Using the relation $Z^a = \chi^{-1}\xi^a$, one obtains

$$\frac{\omega^{'}}{\omega} = \frac{\chi^{-1}|_{p^{'}}}{\chi^{-1}|_p} = \sqrt{\frac{\xi^a \xi_a|_p}{\xi^a \xi_a|_{p^{'}}}}. \tag{12}$$

In Schwarzschild spacetime, where $g_{tt} = -(1-\frac{2M}{r})$, this relation becomes

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$$\frac{\omega'}{\omega}=\sqrt{\frac{g_{tt}(p)}{g_{tt}(p')}}=\sqrt{\frac{1-\frac{2M}{r}}{1-\frac{2M}{r'}}}. \tag{13}$$

For $r<r',\omega>\omega'$, the angular frequency is redshifted.

The results indicate that the redshift phenomenon arises directly from the geometric structure of spacetime and is determined by the variation of the temporal component of the metric at different positions.

Weak gravitational field implies $\frac{M}{r}\ll 1$ and $g_{tt}\to -1$ for any r, meaning that $\omega'\approx\omega$. This indicates, the spacetime is slightly curved as expected.

3.2. Deflection of light

3.2.1. Derivation

For a photon with wave four-vector k^a, the trajectory follows a null geodesic.

Let the geodesic be parameterized by an affine parameter λ, such that the tangent vector is

$$k^a=\left(\frac{\partial}{\partial\lambda}\right)^a \tag{14}$$

The photon trajectory therefore satisfies the null condition

$$k^a k_a=g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}=0 \tag{15}$$

For the Schwarzschild metric this condition reads

$$-\left(1\frac{2M}{r}\right)\left(\frac{dt}{d\lambda}\right)^2+\left(1\frac{2M}{r}\right)^{-1}\left(\frac{dr}{d\lambda}\right)^2+r^2\left(\frac{d\phi}{d\lambda}\right)^2=0 \tag{16}$$

The Schwarzschild spacetime is stationary and spherically symmetric, and therefore admits the Killing vector fields

$$\xi^a_{(t)}=(\partial/\partial t)^a,\xi^a_{(\phi)}=(\partial/\partial\phi)^a \tag{17}$$

These spacetime symmetries imply the conservation of the quantities

$$k^a\xi_{(t)a},k^a\xi_{(\phi)a} \tag{18}$$

along the null geodesic. Explicitly,

$$-\left(1\frac{2M}{r}\right)\frac{dt}{d\lambda}=A, \tag{19}$$

$$r^2\frac{d\phi}{d\lambda}=B, \tag{20}$$

where A and B are constants of motion associated with time-translation and rotational symmetry, respectively.

The physical meanings of the constants A and B are discussed below.

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For the conserved quantity

$$B=r^2\frac{d\phi}{d\lambda}, \tag{21}$$

which arises from the rotational Killing symmetry of the spacetime, it can be interpreted as an angular momentum parameter of the photon trajectory.

For a static observer with four–velocity

$$Z^a=\chi^{-1}\xi^a_{(t)}, \tag{22}$$

one obtains

$$A=k^a\xi_{(t)a}. \tag{23}$$

Using the relation $\xi^a_{(t)}=\chi Z^a$, this becomes

$$A=\chi k^a Z_a. \tag{24}$$

The frequency of the photon measured by an observer with four–velocity Z^a is defined by

$$\omega=-k^a Z_a. \tag{25}$$

Therefore,

$$A=-\chi\omega. \tag{26}$$

This relation shows that the constant A is associated with the conserved energy parameter arising from time–translation symmetry of the spacetime.

Combining these relations with the null condition leads to the radial equation

$$\left(\frac{dr}{d\phi}\right)^2=\frac{A^2}{B^2}r^4-r^2+2Mr. \tag{27}$$

Introducing the inverse radial coordinate

$$\mu=\frac{1}{r}, \tag{28}$$

the equation becomes

$$\left(\frac{d\mu}{d\phi}\right)^2=\frac{A^2}{B^2}-\mu^2+2M\mu^3. \tag{29}$$

Differentiating with respect to ϕ yields the orbit equation

$$\frac{d^2\mu}{d\phi^2}+\mu=3M\mu^2. \tag{30}$$

When $M=0$, the equation reduces to

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$$\frac{d^2\mu}{d\phi^2}+\mu=0. \tag{31}$$

The general solution is

$$\mu(\phi) = C\sin(\phi + \alpha), \tag{32}$$

where C and α are constants.

By introducing Cartesian coordinates

$$x = r\cos\phi, y = r\sin\phi, \tag{33}$$

this solution can be written as:

$$(\sin\alpha)x + (\cos\alpha)y - \frac{1}{C} = 0, \tag{34}$$

which represents a straight line in the plane. Additionally, light would intersect with the y-axis at $x=0$, with the interception of $b=\frac{1}{C\cos\alpha}$, which is the impact parameter.Thus, in the limit $M\to 0$, the photon trajectory correctly reduces to the straight-line propagation expected in flat spacetime.

For $M\neq 0$, the mass term introduces the nonlinear contribution $3M\mu^2$ in the orbit equation.In weak gravitational fields the deviation from Minkowski spacetime is small. In particular, $\frac{2M}{r}\ll 1$.

Since $\mu=1/r$, this implies

$$M\mu=\frac{M}{r}\ll 1. \tag{35}$$

Therefore, the nonlinear term $3M\mu^2$ remains small compared with the linear term in the orbit equation, allowing the trajectory to be obtained perturbatively.

We therefore write

$$\mu=\mu_0+\mu_1,\mu_1\ll\mu_0, \tag{36}$$

where the zeroth-order solution is

$$\mu_0=C\sin(\phi+\alpha). \tag{37}$$

Substituting this expansion into the orbit equation and retaining only first-order terms in M , one obtains

$$\mu(\phi)=C\sin(\phi+\alpha)+MC^2(1\text{-}\cos(\phi+\alpha))^2. \tag{38}$$

Without loss of generality, we choose the polar axis such that the incoming asymptote corresponds to $\phi=0$, which sets $\alpha=0$.

In flat spacetime the outgoing asymptote satisfies

$$\mu_0(\phi)=0, \tag{39}$$

giving

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$$\phi_0=\pi. \tag{40}$$

In the presence of the gravitational field the outgoing direction is shifted slightly. Writing

$$\phi=\pi+\delta,\delta\ll 1, \tag{41}$$

and imposing the condition

$$\mu(\pi+\delta)=0, \tag{42}$$

one finds

$$\delta=4MC. \tag{43}$$

If we identify

$$C=\frac{1}{b}, \tag{44}$$

where b is the impact parameter, the deflection angle becomes

$$\delta=\frac{4M}{b}. \tag{45}$$

3.2.2. Comparison to classical newtonian theory

In classical Lagrangian mechanics, spacetime can be described as the product manifold $R\times M$, where M is the configuration manifold and R represents the absolute time.The configuration space of a constrained mechanical system forms a differentiable manifold [6].

In the following we adopt geometric units $G=c=1$.

To describe the Newtonian analogue of light deflection, the photon is treated as a test particle moving in the Newtonian gravitational potential. The Lagrangian can be written as

$$L=\frac{1}{2}mg_{ab}u^a u^b\text{-}V(r)=\frac{1}{2}m(\dot{r}^2+r^2\dot{\phi}^2)+\frac{Mm}{r}, \tag{46}$$

where u^a is the tangent vector of the trajectory $\gamma(t)$.

The corresponding conjugate momentum is

$$p_a=\frac{\partial L}{\partial u^a}=mg_{ab}u^b. \tag{47}$$

The Lagrangian system admits two continuous symmetries generated by

$$\left(\frac{\partial}{\partial\phi}\right)^a \text{and} \left(\frac{\partial}{\partial t}\right)^a. \tag{48}$$

For rotational symmetry, Noether's theorem implies the conserved quantity

$$B=p_a\left(\frac{\partial}{\partial\phi}\right)^a=mr^2\dot{\phi}. \tag{49}$$

For time translation symmetry, the conserved quantity is

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$$A=p_a u^a-L=\frac{1}{2}mg_{ab}u^a u^b+V(r)=\frac{1}{2}m(\dot{r}^2+r^2\dot{\phi}^2)-\frac{Mm}{r}. \tag{50}$$

Thus A and B correspond to the total energy and angular momentum of the particle respectively.

Combining the two conservation laws yields

$$\left(\frac{dr}{d\phi}\right)^2=\frac{m^2r^4}{B^2}\left(\frac{2A}{m}-\frac{B^2}{m^2r^2}+\frac{2M}{r}\right). \tag{51}$$

Introducing the inverse radial coordinate

$$\mu=\frac{1}{r}, \tag{52}$$

the equation becomes

$$\frac{d^2\mu}{d\phi^2}+\mu=\frac{Mm^2}{B^2}. \tag{53}$$

The corresponding solution is

$$\mu(\phi)=\frac{Mm^2}{B^2}(1+e\cos(\phi-\phi_0)), \tag{54}$$

where

$$e=\sqrt{1+\frac{2AB^2}{M^2m^3}}. \tag{55}$$

Without loss of generality, we assume that the photon comes from infinity at $\phi=0$, such that

$$\mu(0)=0. \tag{56}$$

The photon then escapes to infinity at some angle $\phi=\pi+\delta$ where

$$\mu(\phi)=0. \tag{57}$$

At infinity the gravitational potential vanishes, and the trajectory approaches a straight line with asymptotic speed $v_\infty=1$.

The angular momentum therefore becomes

$$B=|r\times mv|=mb, \tag{58}$$

where b is the impact parameter, defined as the perpendicular distance between the incoming asymptotic trajectory and the scattering center.

Combining these relations gives

$$\tan\frac{\delta}{2}=\frac{M}{b}. \tag{59}$$

For small deflections $\delta \ll 1$, the small–angle approximation yields

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$$\delta = \frac{2M}{b}, \quad (60)$$

which is only half of the prediction from the theory of general relativity.

The discrepancy arises because Newtonian gravity affects only the spatial trajectory of the particle, while general relativity attributes the deflection of light to the curvature of spacetime, which involves both spatial and temporal components.

4. Conclusion

General relativity describes gravity within the framework of curved spacetime (manifold). Spacetime itself provides the geometric background for physical laws, while symmetries constrain their form. By combining these two principles, we can naturally connect physical laws to observable phenomena. Two well-known predictions of general relativity—gravitational redshift and the deflection of light—arise directly from the geometry and symmetries of curved spacetime.

A comparison with classical Newtonian theory highlights the fundamental role that geometry plays. In Newtonian mechanics, space and time are treated as separate entities and the gravitational interaction acts within a fixed background. In contrast, general relativity unifies space and time into a single spacetime structure. Matter and energy influence this geometric background and give rise to spacetime curvature. In the absence of gravity, the trajectory of a photon is a straight line in spacetime. However, when spacetime becomes curved, the geometric background governing physical laws is altered, and the trajectory of light deviates accordingly. This change in the geometric structure leads to observable effects such as gravitational redshift and light deflection, which differ from those predicted by Newtonian theory.

Although the geometric background of physical laws changes, another fundamental principle remains unchanged: symmetry. Different geometric structures lead to different forms of physical laws and observable phenomena, yet the underlying spacetime symmetries persist. Through rotational and time-translation symmetries, we can define conserved quantities along geodesic motion. These conserved quantities provide a powerful geometric method for deriving photon trajectories and analyzing physical phenomena in curved spacetime.

The two classical tests discussed in this work demonstrate how the geometric formulation of gravity can provide a unified description of different observable phenomena within a common framework. The discussion presented here is mainly confined to Schwarzschild spacetime. In more general situations, the behavior of light and the resulting physical phenomena can become more complex. Studying such cases may further enrich the geometric and symmetry-based understanding of general relativity.

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