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From Classical Optimization to Intelligent Optimization: A Review of Modeling and Optimization Methods for Transportation Network Design

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Abstract. Urban Transportation Network Design (TND) is a highly complex system engineering. It has always been regarded as a very difficult problem in urban planning and transportation. This study tries to sort out and summarize the existing research system and development process in this field. From the perspective of mathematical modeling, the multi-objective property, nonlinear characteristics, uncertainty and NP-hard computational complexity of the TND problem are explained one by one, and a series of solution difficulties derived from them also appear. Classical optimization methods, such as linear programming, integer programming, bi-level programming models and so on, have advantages in solving structured problems. But their limitations in large-scale dynamic situations are also pointed out. In comparison, new intelligent and simulation optimization methods, such as heuristic algorithms, data-driven modeling strategies and simulation-optimization coupling frameworks, provide more potential ways to solve complex traffic network design problems. The current mainstream research paradigm can be summarized as the progressive process of "theoretical analysis—optimization solution—simulation verification". Different tools work together in this process to deepen the understanding of the problem and improve the solution efficiency. Network planning with dynamic and multiple uncertainties still faces many unsolved difficulties. Big data technology, artificial intelligence methods and the interdisciplinary integration may become the key driving forces for the future breakthrough in the TND field. This review aims to provide basic theoretical reference and framework support for the follow-up related research.

Keywords: Urban Transportation Network Design (TND), mathematical modeling, multi-objective optimization, metaheuristic algorithms, data-driven modeling

1. Introduction

As city size keeps growing and traffic needs become bigger and bigger, building an efficient transportation network becomes one of the most important tasks for modern cities [1]. Traffic network is a complex system. It includes road location, capacity distribution, route planning, traffic control and other network resource arrangements. Under many objectives and rules, we can use mathematical modeling and optimization to solve these problems together.

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At present, TND research faces many challenges. It has typical features: many objectives, many rules, nonlinearity, and high uncertainty. For example, fast city growth may cause a conflict: old road capacity is not enough, but building new roads costs too much. The big change of traffic needs between morning and evening makes traffic prediction more difficult. The nonlinear relationship between road capacity and real traffic (such as number of lanes, slope) makes optimization even harder. Many problems have been proved to be NP-HARD problems in theory. So the cost of finding an exact best solution is very high. At first, researchers used clear and strict models and classical optimization methods, such as linear programming and bi-level optimization. Later, they slowly turned to heuristic algorithms, simulation models and data-driven modeling. These methods pay more attention to computing ability and real problems. Now there is a mixed system of many methods. They can work together to deal with these complex problems in related research [2].

This review tries to systematically summarize the main models and methods in TND research from the view of mathematical modeling. It hopes to give readers a clear development path, help them understand the mathematical framework and modeling ideas of TND problems, and provide reference for future research.

2. Mathematical basis of Transportation Network Design

Common transportation network design problems are mainly divided into three categories: Shortest Path Problem (SPP), Minimum Cost Flow (MCF), and Network Design Problem (NDP). This study will explore the mathematical models of these three problems.

We simplify the TND problem mathematically. Below Table 1 are some basic elements in the network model.

Table 1. Network model basic elements

Definition	Explanation
Node	The basic unit of a graph, representing the research object or location.
Arc	A directed line segment connecting two nodes, representing a one-way relationship between nodes.
Flow	The actual amount of flow passing through an arc.
Capacity	The maximum flow allowed on an arc.
Weight	A value related to an arc, used to measure the specific property or cost represented by the arc.

In the transportation network design map, nodes represent cities, arcs represent roads, variables are equal to flow, and weights represent the actual distance, travel time or construction cost between two points [3]. Figure 1 is a common diagram of a transportation network.

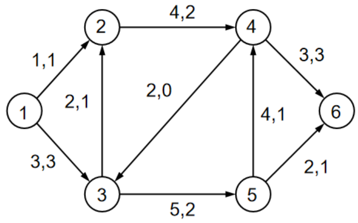

Figure 1. General transportation network diagram

Among them, the first number on each arc is the capacity, and the second number is the actual flow.

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In the TND problem, three basic assumptions are made to ensure that the mathematical model can be solved:

(1) Demand conservation assumption, that is, the flow out of the starting node is equal to the flow into the ending node;

(2) Capacity constraint assumption, that is, the flow on each arc cannot exceed the capacity of the arc;

(3) Non-negative flow constraint assumption, that is, the flow on each arc is a non-negative number.

Based on the above assumptions, the mathematical models of TND problems can usually be expressed as Linear Programming (LP), Integer Linear Programming (ILP) and Nonlinear Programming (NP). The objective function and constraints of linear programming are all linear, which is suitable for continuous decision variables; integer programming requires some or all decision variables to be integers, which is an extension of linear programming; nonlinear programming allows nonlinear relationships in the objective function or constraints, which is suitable for more complex network flow situations.

3. Classical optimization modeling methods

In solving TND problems, traditional mathematical optimization methods are very important. They are the main way to solve such problems now. Researchers need to focus on different types of TND problems and deeply study the corresponding classical optimization modeling methods.

3.1. Linear and integer programming models

Linear and integer programming models are very important in real engineering and decision-making. In road construction, route selection and other situations, these models have clear mathematical features and mature algorithms, so they are widely used. Road construction may involve location problems (that is, choosing suitable places related to road construction from several possible places according to specific conditions and combining with goals). Route selection usually involves the shortest path problem (that is, finding a path that meets certain requirements from many possible paths, such as the shortest length or the lowest cost). The general form of linear programming is as follows:

$$\begin{cases} min \quad z = c_1x_1 + \cdots + c_nx_n \\ s.t. \quad\; a_{i1}x_1 + a_{i2}x_2 + \cdots a_{in}x_n = b_i,\ i = 1, \cdots, p \\ \qquad\quad a_{i1}x_1 + a_{i2}x_2 + \cdots a_{in}x_n \geq b_i,\ i = p+1, \cdots, m \\ \qquad\quad x_j \geq 0,\ j = 1, \cdots, q \end{cases} \tag{1}$$

In solving linear programming problems, the simplex method is a classical and efficient algorithm. Its core idea is to search iteratively between the vertices of the feasible region, gradually approach and finally find the optimal solution of the objective function [4]. Integer programming problems are more difficult to solve because integer constraints are added to decision variables. At present, common solution methods include branch and bound method and cutting plane method. The branch and bound method splits the original problem into several subproblems (branching) and calculates the upper and lower bounds of the objective function of each subproblem (bounding),

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narrowing the search range until the optimal integer solution is found. The cutting plane method adds linear constraints (cutting planes) to remove non-integer solutions from the feasible region and find the optimal solution in the integer feasible region [5].

3.2. Bi-level optimization model

The bi-level optimization model is the core of the representative analytical model in transportation network design. It coordinates the goal conflicts of different subjects through a hierarchical decision-making mechanism. The upper-level decision-maker makes network design strategies, the lower-level decision-maker optimizes its own behavior according to the upper-level strategies, and the optimal response of the lower level directly affects the decision goal of the upper level [6]. This model obtains the optimal flow distribution plan by establishing the objective function (such as minimizing total transportation time or cost) and constraints (such as road section capacity constraints). It has the characteristics of hierarchy, independence, conflict, priority and autonomy. In transportation network design, it can be used in traffic flow distribution, road network design, signal control and intersection optimization, public transport line planning and other fields [7].

The general form of the bi-level optimization model is as follows:

$$\begin{cases} \min\limits_{u} \; F(u, v(u)) \\ s.t. \quad G(u, v(u)) \leq 0 \end{cases} \tag{2}$$

where v(u) is obtained by the following programming:

$$\begin{cases} \min\limits_{v} \; f(u, v) \\ s.t. \quad g(u, v) \leq 0 \end{cases} \tag{3}$$

3.3. Optimization with equilibrium constraints and variational inequality modeling

The transportation network design system is complex. The bi-level optimization model plays an important role because it can deal with the hierarchy and interaction of the decision-making process. But it is often difficult to solve the bi-level model, for example, when the lower-level problem of the model involves the equilibrium distribution of large-scale traffic flow. To solve this problem, the common method is to use optimization with equilibrium constraints or variational inequality (VI), transform the lower-level problem into a constraint form that is easier to handle in the upper-level optimization model, and turn the whole bi-level optimization problem into a single-level but highly nonlinear optimization problem.

Optimization with equilibrium constraints is a modeling method. It directly puts user equilibrium behavior into the optimization model as a constraint condition. The equilibrium constraint sets the solution of the lower-level equilibrium problem as the constraint condition of the upper-level optimization problem, ensuring that the lower-level users (i.e., traffic flow) can make the optimal response under the given network design, that is, traffic flow will be distributed according to a specific equilibrium principle (such as user optimization or system optimization). This modeling method retains the hierarchical structure of the bi-level model, implicitly considers user behavior,

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and simplifies the problem-solving process [8]. The general form of the optimization model with equilibrium constraints is:

$$\begin{cases} min \ \ F(u,v) \\ s.t. \qquad G(u,v) \leq 0 \\ \ \ v \in EquilibriumSolution(u) \end{cases} \tag{4}$$

where $v \in EquilibriumSolution(u)$ means that v is an equilibrium solution dependent on u.

VI modeling means transforming the equilibrium distribution problem of traffic flow into a variational inequality form, putting this variational inequality as a constraint into the upper-level optimization problem, and building a single-level nonlinear optimization model [9]. VI is used to describe the relationship between variables when a function reaches the extreme value under specific constraints. In the field of transportation network design, it can mean that when traffic flow reaches equilibrium, no user can unilaterally change its own path to reduce travel cost. At this time, the distribution of traffic flow meets specific variational inequality conditions. Generally, the VI problem can be expressed as: find a vector X∗ such that for all vectors X satisfying certain constraints, the following inequality holds:

$$\left(F\left(X^{*}\right), X - X^{*}\right) \geq 0 \tag{5}$$

where F(X) is a given function, and $(\bullet, \bullet)$ represents the inner product operation. In the transportation network design situation, X can represent the path choice or flow distribution of traffic flow, and F(X) may be related to the travel cost or flow-related function of the path.

The two processing methods of the bi-level optimization model greatly improve the solvability of the model and build a certain theoretical basis for subsequent research and application. However, the nonlinear characteristics of the bi-level optimization model lead to increased computational complexity, and there are many challenges in the actual solution process. Nevertheless, the model still provides key theoretical support for the development of intelligent optimization methods and approximate algorithms. The design and improvement of subsequent related algorithms are based on this theory.

4. Intelligent and simulation optimization models

Traditional transportation network design mainly uses analytical modeling methods, such as linear programming, integer programming and dynamic programming. These methods have some advantages in small-scale and certain problems. But when they face large-scale network situations with dynamic demand changes and multi-objective conflicts, their limitations become very obvious. In recent years, computational intelligence technology has been developing continuously. So TND research has gradually moved away from the original analytical modeling framework and turned to simulation and intelligent optimization. The introduction of simulation and intelligent optimization provides more diverse and comprehensive perspectives and solution methods for TND research. So we can see the necessity of this transformation.

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4.1. Heuristic and metaheuristic optimization

Heuristic and metaheuristic algorithms are optimization methods for solving mathematical models with high computational complexity. Heuristic algorithms are built based on intuition or experience. They use fixed strategies for specific problems, such as greedy algorithm, nearest neighbor algorithm and local search algorithm. They have fast computing speed and are suitable for local optimization tasks of transportation networks, such as route planning, intersection signal control and logistics distribution. But they easily fall into local optimal solutions.

Metaheuristic algorithms simulate natural phenomena and find global optimal solutions by dynamically adjusting strategies, including genetic algorithm, simulated annealing algorithm, ant colony optimization algorithm and so on [10,11]. They have great advantages in dealing with multi-objective and large-scale problems in transportation network design, such as the application in Internet of Vehicles and urban public transport network optimization. Because transportation network design problems have NP-hard characteristics, metaheuristic optimization methods are widely used to solve large-scale, nonlinear and discrete TND problems. They do not depend on strict mathematical forms and have strong model adaptability. But they usually lack the guarantee of global optimality and are sensitive to parameters [12].

4.2. Multi-objective optimization modeling

In the field of transportation network design, because many goals such as construction cost, operation efficiency and environmental impact may conflict with each other, multi-objective optimization modeling has become a very important modeling method. This kind of model can usually minimize (or maximize) multiple objective functions at the same time. The general form is min

$$min\ (f_1(x), f_2(x), f_3(x)). \tag{6}$$

This model can show the trade-off relationship between different design schemes. In the solution process, Pareto optimal theory and multi-objective algorithms based on evolution, such as NSGA-II, are often used to get non-dominated solutions [13]. This flexible decision support mode makes transportation network design meet different social, economic and environmental needs more effectively and improves the scientificity and rationality of decision-making.

4.3. Optimization-simulation coupling modeling method

In complex transportation systems, some constraint relations and dynamic behaviors are difficult to be accurately described by analytical models. So the optimization-simulation coupling modeling method is widely used. This method builds a closed-loop process with "model-simulation-feedback" as the basic structure, as shown in Figure 2. The results obtained from the optimization model are first input into the simulation platform, which is the model input stage. Then simulation is carried out to simulate different scenarios such as traffic peak hours and emergencies. Then performance evaluation is carried out. Key performance indicators such as average delay, queue length and carbon emissions are used to verify the effectiveness of the model. Finally, according to the simulation results, the model parameters are adjusted and feedback corrected, such as demand prediction error and road network impedance function. This iterative process, with "optimization-evaluation-feedback" as the core, significantly enhances the feasibility and reliability of the

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optimization results in the real traffic environment, especially suitable for large-scale networks and complex traffic demand situations. Thus, examples show that it has good adaptability in dealing with high-dimensional uncertainty.

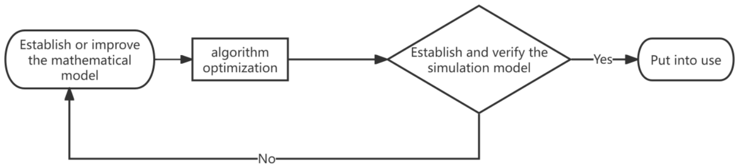

Figure 2. Optimization-simulation coupling modeling process

4.4. Data-driven and machine learning modeling

In recent years, data-driven methods have been gradually introduced into the field of transportation network design. Machine learning models are used to predict traffic demand, approximate complex cost functions, or assist in optimization decision-making [14]. Related research automatically mines rules from massive data and uses the learned functions to improve the traditional optimization framework. Thus a so-called Learning-Augmented Optimization model is built, and its adaptability in uncertain environments is enhanced.

Compared with pure analytical models, these methods can make full use of historical data and simulation data. But their modeling effect is limited by data quality and generalization ability. It is still necessary to further explore the solvability of the model, which is also shown by examples.

5. Summary and trends

5.1. Model summary

From the perspective of mathematical modeling, this paper presents a systematic description of the core models and methods in the TND field. Analytical models (such as linear programming, integer programming, etc.) are based on mathematical programming and graph theory, and provide a strict optimization theory framework for TND problems. However, their computational complexity increases nonlinearly with the problem scale, and the efficiency is obviously limited in large-scale and complex structure scenarios. Although classical optimization methods (such as bi-level programming) can deal with the trade-off between multiple objectives through hierarchical decision-making mechanism, their processing ability is insufficient in the face of dynamic demand situations (such as real-time traffic flow changes) and uncertain factors (such as demand fluctuations). Intelligent optimization algorithms with global search characteristics and strong adaptability (including genetic algorithm, simulated annealing algorithm, etc.) effectively make up for the shortcomings of traditional methods. The simulation-coupling modeling method dynamically reproduces the real traffic operation state and improves the problem-solving ability of the model in complex environments. Current research has gradually built a three-in-one collaborative method system of "theoretical analysis—optimization solution—simulation verification", which provides diversified tool support to deal with the multi-dimensional challenges involved in TND.

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5.2. Future research trends

Based on existing research, the TND field needs further in-depth expansion in several directions in the future. In terms of technology, the rapid development of big data and artificial intelligence technology has opened a broad space for the application of data-driven modeling and machine learning algorithms, which can improve prediction accuracy and realize dynamic optimization and real-time control of transportation networks. In terms of dimension, the coordination of macro planning and micro behavior analysis will be promoted through multi-dimensional modeling and interdisciplinary integration, thus enhancing the comprehensiveness and adaptability of design schemes. The research on emerging traffic modes will also become a key breakthrough direction, which shows the trend of intelligent and diversified evolution of transportation networks.

5.3. Conclusion

As a key element supporting urban development, transportation network design has far-reaching and significant research significance. Facing the continuously evolving and highly complex traffic demand, only by continuously promoting the innovative integration of mathematical modeling and computing technology and deepening interdisciplinary research can we improve the scientificity and practicability of transportation network design. AI-driven dynamic optimization algorithms, real-time data-driven simulation models, and interdisciplinary methods will become important parts of future TND research. Thus, driven by both technological breakthroughs and model innovation, the construction of efficient, intelligent and sustainable urban transportation systems will be strongly promoted.

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