

Recent Advances of Seismic Reflection and Transmission Response Characterization Incorporating the Stress Effect

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Abstract. The application of seismic waves has become increasingly important in the field of geological exploration. Seismic wave reflection and transmission (R/T) can reflect the properties of subsurface rocks and reconstruct underground structures through acquired seismic data. However, subsurface reservoirs, under the influence of rock gravity and tectonic forces, possess complex in-situ stresses. In practical exploration processes, stress significantly affects the propagation characteristics of seismic waves. Therefore, understanding the role of stress is of great importance for seismic R/T research. Nevertheless, current studies on seismic R/T incorporating in-situ stress are still in their early stages, with most research neglecting the influence of stress. Thus, we review the recent advancements on the characterization methods of stress-dependent seismic R/T responses based on acoustoelastic theory, which includes the both exact and approximate equations for seismic R/T coefficients and their applications. This paper is potential for contributing to the development of future deep research on stress-dependent seismic R/T responses.

Keywords: Wave reflection and transmission, Stress effect, Recent advances

1. Introduction

Seismic information can be effectively used for subsurface reservoir exploration, rock property analysis, and subsurface structures reconstruction [1]. Seismic reflection and transmission (R/T) coefficients are essential parameters of seismic waves propagation in the subsurface [2]. Currently, after the deepening of research, the scope of exploration is further inclined toward more complex deep and unconventional parts. Consequently, in-situ stresses of a subsurface reservoir (e.g., overburden pressure and tectonic forces) have become a prominent and significant influencing factor [3,4]. Multi-phase and differential diagenesis of the rock has led to the formation of a relatively strong and complicated in-situ stress field. The stress field changes the rock elasticity and anisotropy, and further influences the amplitude, phase and propagation path of seismic waves [5].

The classical R/T coefficient equations represented by the Zoeppritz equation are based on the ideal linear elastic assumption without initial stress [6]. However, nonlinear elastic theory (i.e., acoustoelasticity theory) is more accurate to describe the seismic R/T responses in stressed media, as it not only includes quadratic strain potential functions but also cubic terms, through which third-order elastic constants (3oECs) are raised [7-9]. Nevertheless, many studies ignore the bias caused

by the stress effect. Existing research indicates that ignoring stress-induced anisotropy can severely affect the inversion accuracy of reservoir parameter based on seismic reflection amplitudes [10]. Therefore, developing seismic reflection and transmission theories that take into account the influence of stress is at the forefront of geophysics, which help to advance seismic wave research, seismic imaging, and data analysis.

This paper starts with the description of the solid acoustoelasticity theory. Then we introduce the recent advances of exact and approximate equations of seismic R/T coefficients based on the acoustoelasticity theory. We end up with the summaries of the application examples of approximate reflection coefficient equations as well as prospects for future research.

2. Seismic R/T coefficient equation incorporating the stress effect

This section first reviews the basic physics of classical acoustoelasticity theory, as well as the exact and approximate reflection coefficient equations derived from it. This is the foundation for the subsequent applications introduced in the next section.

2.1. Acoustoelasticity theory

The acoustoelasticity theory is developed to characterize the influence of stress and pressure on the elastic properties of solid rocks and the propagation of waves in them [9,11]. It can be treated as the extension of classical elastic theory. The solid acoustoelasticity theory not only retains its quadratic term in the constitutive relationship, but also introduces the cubic term, and introduces the third-order elastic coefficient (3oECs) on the basis of the second-order elastic coefficient (2oECs) to describe the modulating effect of stress on the solid medium. The strain energy function of linear elasticity theory only retains the second-order term, while the nonlinear acoustoelasticity theory not only retains the quadratic term but also introduces the third-order term. In the constitutive relationship of linear elasticity theory, stress and strain become directly proportional, and higher-order terms are introduced in nonlinear acoustoelasticity theory. The elastic coefficient of linear elastic theory contains only 2oECs, and the isotropic medium contains two independent constants. The nonlinear acoustoelasticity theory not only contains 2oECs, but also introduces 3oECs, of which there are three independent 3oECs in the isotropic medium and ten in the horizontally transversely isotropic (HTI) medium. The linear elasticity theory constitution does not contain stress variables and cannot describe the influence of initial stress on elastic parameters, while the nonlinear acoustoelasticity theory can quantitatively characterize the stress effect on the effective stiffness matrix through static strain. The linear elasticity theory is suitable for the ideal situation of small deformation, no initial stress or constant stress, while the nonlinear acoustoelasticity theory is suitable for the medium with obvious initial in-situ stress, but the acoustoelasticity is still limited to the response range of small stress linearity [12]. Although the third-order term is introduced in the acoustoelasticity theory, because 3oECs are assumed to be constants and the static strain has a linear relationship with the initial stress, its prediction results still show the linear relationship between elastic parameters and stress [13,14].

However, this theory struggles to accurately describe the dispersion and attenuation caused by fluid flow in saturated porous rocks. Therefore, based on the theory of solid acoustoelasticity, finite fluid strain was introduced, and third-order elastic constants were incorporated to describe the coupling between the fluid and the solid matrix, forming the theory of poro-acoustoelasticity [12,15], which provides a theoretical foundation for the study of saturated porous media.

2.2. Exact seismic R/T coefficient equation

The early studies on exact seismic R/T coefficient equations focus on the free interface and then shift to solid-solid interface with more practical significance. Degtyar and Rokhlin [16] first gave the exact R/T equation of the seismic energy flow at the interface of the medium subjected to stress. Chen et al. [17] derived the exact solution of the displacement amplitude reflection coefficient at the solid-solid interface, optimized the accurate equation of seismic R/T waves according to the acoustoelasticity theory and continuous boundary conditions. They systematically considered the influence of the initial vertical stress on it [18,19]. This theory was then extended to the vertically transversely isotropic (VTI) medium under stress [20].

The exact equation quantifies the influence of stress on seismic R/T responses, and for the first time explains how stress affects the wave field by changing the medium stiffness tensor, for the more complex scenarios of discontinuous interfaces (such as fractures, cracks, etc.), large angle of incidence, strong anisotropy or full-band response, the exact equation plays a discordant or lacking role, but due to its too many coefficients, the form is more complex. Moreover, the 3oECs in the reflection coefficient of the exact equation are difficult to accurately estimate by geophysical methods, and are difficult to be used for large-scale seismic inversion and seismic analysis, which causes some application challenges.

2.3. Approximate seismic R/T coefficient equation

Exact R/T equations are involved in a complex mathematical form, which limits its practical applications. Therefore, the scholars propose various linear R/T approximations to facilitate practical production applications [21-24].

When isotropic rock is subjected to uniaxial or biaxial stress, its elastic properties tend to become similar to those of naturally formed anisotropic rock. Based on the theory of stress-induced anisotropy, by linking the effects of stress with the changes in intrinsic anisotropy parameters such as Thomsen's parameters, a quantitative relationship is established between anisotropy parameters and the magnitude of initial stress [5]. This allows for obtaining effective anisotropy parameters, converting invisible and ambiguous stress effects into anisotropy parameters that can be applied to seismic formulas. These parameters are then incorporated into the well-established AVO/AVA (amplitude versus offset/angle) reflection coefficient formulas for HTI media (such as the well-known Rüger approximate formula) [25], resulting in expressions for seismic reflection coefficients that include stress terms, which can be used for seismic analysis and inversion.

Alternative method to derive the approximate reflection coefficient equation is based on the stable phase method and perturbation theory [21,24]. The approximate equations express the reflection coefficients as a linear combination of several desired model parameters, making them more convenient for practical applications and reducing complexity. Moreover, seismic inversion results incorporating the approximate equations can better indicate geomechanical properties and more closely reflect geological reality. However, the basic assumptions underlying the approximate equations (i.e., small perturbations and weak contrasts) limit their applicability. Existing research mainly focuses on uniaxial and biaxial stress, which cannot accurately describe the true subsurface stress fields. The theoretical expansion is needed to provide theoretical support for future relevant applications.

3. Field applications

Stress-dependent seismic reflection coefficient equation has been widely applied in geological and engineering applications as a effective tool. Chen and Zong [21] defined the stress-induced anisotropy parameter and constructed seismic reflection coefficient equations. They further developed the seismic inversion method for elastic parameters and stress-induced anisotropy parameters within the Bayesian inference scheme. The inverted formation parameters provide the key guidance for fracture design in unconventional oil and gas reservoirs which are relevant to the sweet spots discrimination (see Figure 1).

For discontinuities such as faults and fractures in deep strata, Fan and Zong [26] treated this type of interface as the non-welded boundary and derived the exact R/T equation for this interface under the horizontal stress. This equation explains the effects of stress and interface mechanical properties on wavefields and their frequency dependence, enabling more accurate fracture zone measurement and stress analysis.

SeismicR/T studies based on acoustoelasticity theory have already been applied and explored in inverting stress-related parameters, predicting in-situ stress, and identifying oil and gas reservoirs. This theory is evolving from mechanistic research toward practical applications that address real challenges in deep and unconventional oil and gas exploration.

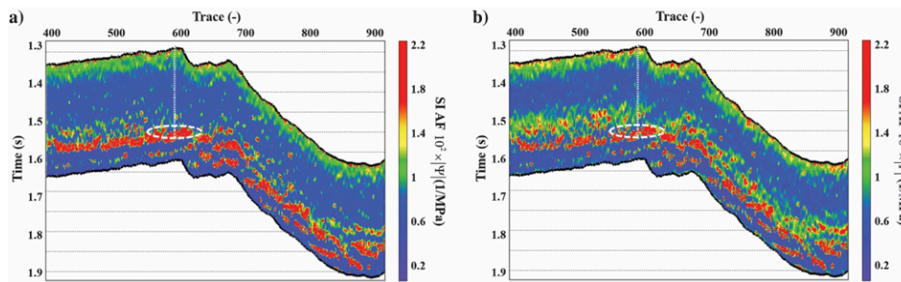


Figure 1. Absolute values of two defined inverted stress-induced anisotropy parameters in Chen and Zong [21]

4. Conclusions and outlook

This paper systematically describes the development of seismic R/T theory related to stress effects, including the exact equations of seismic R/T coefficients derived from solid acoustoelasticity theory and the approximate equations simplified based on these, with a focus on the basic principles, development process, and practical application significance of the two types of equations.

The theoretical development of seismic R/T under stress effects began with solid acoustoelasticity theory, which introduced 3oECs and, explicitly described the influence of stress effects on the elastic properties of the medium and wave propagation. To account for the influence of pore fluids, the theory was further developed into poro-acoustoelasticity theory. Based on acoustoelasticity theory, research progressed to the stage of constructing exact equations, expanding from free interfaces to solid-solid interfaces, and then to discontinuous interfaces, reflecting the significant impact of stress on amplitude, critical angles, and frequency. As the exact equations were constructed, their limitations became increasingly apparent, creating a pressing need for a simplified version for practical applications, which led to the development of approximate equations. By using effective anisotropic parameterization and direct linearization based on scattering theory, linear

similarity treatments were applied, resulting in approximate equations that are beneficial for practical production applications.

Currently, the theoretical framework for seismic R/T has been preliminarily established, showing great potential in the inversion of stress-induced anisotropy parameters, prediction of the direction of initial stress, and evaluation of reservoir brittleness. However, this study points out that the existing system still has significant limitations; the foundation of this theoretical framework is overly idealized linear elastic theory is based on small deformations, no initial stress, or constant stress, and nonlinear acoustoelasticity theory applies to media with significant initial geostress, but classical acoustoelasticity is still limited to the small-stress linear response range. The coupling mechanisms of multiple physical fields are missing, evolving from free interfaces to solid-solid interfaces, and then to non-welded interfaces, while thermal-fluid-solid interactions are absent. At present, this theoretical framework has made preliminary progress in both theory and practical applications, but there is still considerable directions for improvement in experimental verification, accuracy of related coefficients, and practical application of exact equations.

Future research needs to focus on breaking through the limitations of current models in complex three-dimensional stress fields, thermal-fluid-solid multi-physical field coupling, and key parameter constraints. The focus has been on uniaxial and biaxial stress states, and it is necessary to achieve better and more accurate descriptions of the real in-situ stress field distribution so that the theory of seismic R/T under stress effects can progress toward higher practical applicability.

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