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Mass Measurement of Stellar Black Holes

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Abstract. Black holes, as the most mysterious celestial bodies in the universe, have received limited research from humans. This article reviews the history of observing black holes and present the dynamical method for measuring their mass. Drawing on reported dynamical estimates, I built a dataset of stellar-mass black holes and obtained an average mass of 8.016 M⊙, which is consistent with the theoretical lower limit of 5M⊙. Although I verified that most black holes apply to the theoretical lower limit of 5M⊙ proposed in previous studies, it is possible for Swift J1727.8-1613, XTE J1650-500, GRS 1716-249, GRS 1009-45, GRO J0422+32, H 1705-250, 3A 1524-617, 1H 1659-487 with a minimum mass below 5M⊙.

Keywords: black holes, steller-mass black holes, mass measurement, data statistics

1. Introduction

Among the universe's most enigmatic compact objects, black holes are regions of spacetime where the escape velocity beyond the event horizon exceeds the speed of light. The importance of black hole studies lies partly in their origin as endpoints of very massive stars, often with initial masses of around twenty solar masses or above; the energetic output of active galactic nuclei is sustained by accretion onto central black holes, which can reproduce the observed strong emission; processes such as black hole mergers may be important sources of gravitational waves [1]; he collapse of progenitors promotes the synthesis of heavy elements in the universe and is related to some extreme phenomena in the universe [2].

Within general relativity, a black hole is described by a minimal set of quantities—mass, angular momentum, and charge—making it an unusually simple astrophysical object. The no-hair result implies a sparse description: mass and angular momentum (conveniently captured by a or a_*), with electric charge usually irrelevant for astrophysical objects. Mass merely provides a scale, while spin alters the geometry. A black hole's defining feature is its event horizon; colloquially, the "Las Vegas principle" captures the idea that processes inside do not affect observers outside except through global parameters. Events that happen inside a black hole will always remain within it. The Singularity is at the center of a black hole, and the gravitational force in this region is so strong that it can break the laws of physics.

By mass scale, black holes are divided into stellar-mass BHs, intermediate-mass BHs, and supermassive BHs. Take stellar-mass black hole as example, it originated from the star which are 20 times the Sun's mass or more, and newly born black holes can range from a few to hundreds of times the Sun's mass. This article mainly presents some information about stellar-mass black holes.

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In Section 2, the narrative focuses on key milestones in BH discovery. Section 3 details observational approaches to mass estimation. Section 4 reports results from the tabulated sample and interprets them.

2. Previous studies

The discovery history of black holes has brought many surprises to human. This history began with the discovery of Cygnus X-1 in 1964, which was not only the first Black Hole Binaries (BHB) [3,4] but also the first candidate for systems hosting a BH [5,6]. This has enabled humanity to make a major breakthrough in the study of black holes. The second discovered BHB was LMC X-3 [4], and the third was A 0620-00 [7]. Different from the previous two BHBs, A 0620-00 was discovered as X-ray nova in 1975, and its X-ray source was very unstable in the following year [8,9]. Additionally, other major surprise was the study of GS 2023+338 in 1989 [10], It is regarded as the first dynamically confirmed BHB, which is different from the previous record of Cyg X-1 [5].

3. Mass measurement

The BH candidate is a compact object discovered to have similar X-ray properties in an X-ray outburst to the confirmed ones. However, neutron stars and black holes can exhibit very similar outburst phenomenology, so some entries in the black hole–candidate catalog may in fact be neutron star systems. To confirm it is a BH, the dynamic measured mass of the compact object is necessary [11,12]. A luminous X-ray source whose compact object has a dynamically measured mass exceeding 3 M☉ is taken to host a black hole [13].

Stellar masses in binaries are inferred from Kepler's relation, which links the orbital period P and semi-major axis a to the total mass M_{tot} via $4\pi^2 a^3 = G M_{\text{tot}} P^2$. Its formula is:

$$\frac{a^3}{r^2} = \frac{G(M+m)}{4\pi^2} = k \tag{1}$$

In this formula, a is the elliptical relative motion of one mass relative to the other, Here r denotes the semi-major axis of the relative orbit; $G = 6.67\times10^{-11}\text{m}^{3}\cdot\text{kg}^{-1}\cdot\text{s}^{-2}$ is the gravitational constant; M and m are the component masses of the system. For a strongly hierarchical case $M \gg m$, the total mass satisfies $M_{\text{tot}} \simeq M$., k is the Kepler constant which is the elliptical semi-major axis squared divided by the square of the orbital period. Therefore, when measuring a stellar mass, the above expression provides the key link used in our mass estimates. Mass measurements of black holes also rely on Keplerian dynamics. In X-ray binaries, typically only the donor's radial-velocity curve is observable, which yields the orbital period $P_{\text{orb}}$and the donor's velocity semi-amplitude K_2[12]. These observables combine to give the mass function f(M):

$$f(M) = \frac{k_c^3 P_{orb}}{2\pi G} = \frac{M_x^3 sin^3 i}{(M_x+M_c)^2} = \frac{M_x sin^3 i}{(1+q)^2} \tag{2}$$

which provides a strict lower limit on the black hole mass.

We define M_{BH} as the accretor mass, M_2 asthe donor mass, i the binary inclination, and $q = M_2/M_{\text{BH}}$ the mass ratio [14]. The mass function $f(M)$ only provides the lower limit to the BH mass M_x, because that the mass of the companion star Mc must be larger than 0 and the inclination angle must be smaller than 90∘ which is for edge-on geometry. According to this function equation, to move beyond the mass-function lower bound, one must constrain both the donor mass and the orbital inclination. Therefore, masses inferred from the mass function alone can be highly

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uncertain, because the result depends sensitively on the companion's mass and the orbital inclination [15,16].

In the Milky Way, about 60 black hole binaries are currently cataloged. Of these, roughly 20 have dynamical confirmation from spectro-photometric measurements [14], while the remainder are classified as black hole candidates [17,18]. I show the mass function and BH mass of stellar-mass black holes for the low-mass X-ray binaries in Table 1. All the data in the table are sourced from BlackCAT. The specific data selected from the sources for dynamically confirmed BH which there is robust mass function or mass determination can prove the presence of a black hole.

Table 1. The mass of black holes

System	F(M)	Mx (M☉)
Swift J1727.8-1613	2.77±0.09	>3.1±0.1
MAXI J1820+070	5.2±0.2	5.71–8.10 (8.40±0.770.70 if adopted i=63°)
MAXI J1305-704	6.9±0.3	$8.9^{+1.6}_{-1.0}$
Swift J1357.2-0933	11±2.1	12.4±3.6
Swift J1753.5-0127	7.8±1.0	8.8±1.3
XTE J1650-500	2.7±0.6	≤7.3
XTE J1118+480	6.27±0.04	6.9 – 8.2
XTE J1859+226	5±1	8±2
SAX J1819.3-2525	2.7±0.1	6.4±0.6
XTE J1550-564	7.7±0.4	7.8 – 15.6
GRS 1716-249	4±1	$6.4^{+3.2}_{-2.0}$
GRS 1009-45	3.2±0.1	≥4.4
GRS 1915+105	7±0.2	11.2±2
GRO J0422+32	1.19±0.02	2 – 15
GRS 1124-684	3.02±0.06	$11.0^{+2.1}_{-1.4}$
GS 2023+338	6.08±0.06	$9^{+0.2}_{-0.6}$
GS 2000+251	5±0.1	5.5 – 8.8
GS 1354-64	5.7±0.3	≥7.6±0.7
H 1705-250	4.9±0.1	4.9 – 7.9
3A 0620-003	2.79±0.04	6.6±0.3
3A 1524-617	3±1	$5.0^{+3.8}_{-2.4}$

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Table 1. (continued)

1H 1659-487	1.91±0.08	2.3 – 9.5
4U 1543-475	0.25±0.01	8.4 – 10.4

4. Discussion

Because dynamical solutions depend on the companion mass and the orbital inclination, black hole mass estimates can carry substantial uncertainties. For example, black holes which have a huge range of measured masses such as Swift J1357.2-0933, XTE J1550-564, GRO J0422+32, 3A 1524-617, 1H 1659-487, of whom the range of measured values are both exceed 6 $M_{\odot}$, and black holes such as Swift J1727.8-1613, XTE J1650-500, GRS 1716-249, GRS 1009-45, GRO J0422+32, and GS 1354-64 only have upper or lower limits for mass.

The average mass of stellar-mass black holes is calculated based on the third column in Table 1. Except for the data of black hole masses with only upper and lower limits (e.g. Swift J1727.8-1613), for the rest of the data, the intermediate value (e.g. MAXI J1820+070) or the reference value (e.g. MAXI J1305-704) is selected. Data are considered to be added together, and then divided by the corresponding number of black holes, the average mass of stellar-mass black holes 8.016 $M_{\odot}$ can be roughly calculated.

Ozel et al. explained that the minimum mass of a black hole is greater than 5 $M_{\odot}$ [19]. My results regarding the average mass of stellar-mass black holes precisely conform to this explanation. However, there are also some special ones. For example, the minimum mass of GRO J0422+32 and 1H 1659-487 are respectively 2 $M_{\odot}$ and 2.3 $M_{\odot}$.

5. Conclusion

In this article, we review the key features and history of stellar black holes, and also introduce Kepler's third law of motion and the black hole mass function equation. The data in Table 1. shows that the mass range of black holes is wide, with an average of 8.016 $M_{\odot}$, which is consistent with the theoretical lower limit of 5$M_{\odot}$ proposed in previous studies. However, significant exceptions such as GRO J0422+32 and 1H 1659-487 suggest that the masses of some black holes may be as low as 2$M_{\odot}$. The improvement of future observation technologies and more precise orbital inclination angle measurements will help refine mass estimation and further enhance human understanding of black holes.

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