Proceedings of CONF-CIAP 2026 Symposium: Applied Mathematics and Statistics
DOI: 10.54254/2753-8818/2026.CH31364

Using Kepler Data to Study Exoplanets with Several Methods

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Abstract. Nowadays, the discovery of exoplanets has become a popular field for scientists to study due to the limited resources on Earth. With the help of developed technology and different areas of data, through several years of observation and research, a variable and a huge number of exoplanets have been discovered. The methods to find these exoplanets are really important to learn. However, there are so many methods to use, and each of them has its own shortcomings. This paper will explain the detailed principles, formulas, and information of two common methods, which are Transit and Radial velocity. The drawbacks of the two methods will be illustrated, and then the paper will give the idea of combining the two methods to get more planetary parameters and better outcomes. The result of the work is that although the size of all planets and transit durations can be visually presented through spectroscopy, most of the parameters of the planet still cannot be obtained; also, there is a limitation on the change in inclination. Radial velocity can provide almost all the needed parameters for exoplanets, but this method is only sensitive to massive planets. Easy to tell, both methods have their respective advantages that the other party needs. Therefore, combining these two methods is the best way to complement weaknesses, making it easier and better to study exoplanets.

Keywords: Exoplanet, Kepler data, transit, radial velocity

1. Introduction

As the first planet that humans have known and studied, the Earth has seen its resources exploited and utilized for thousands of years. However, the natural resources on Earth are limited and are at risk of depletion. In light of this, scientists and governments around the world are investing funds and technologies to actively search for exoplanets.

Scientists have designed the Kepler mission, aiming to observe and determine the number and frequency of planets similar in size to Earth that exist within the habitable zone of Sun-like stars [1]. This mission conducted photometric measurements on over 1500 solar-type targets. These bodies were selected for study by the Kepler Asteroseismic Science Consortium (KASC; Gilliland et al. 2010a) during only the initial 10 months of science processes. Kepler will survey [2]. So many targets have been observed and studied in a few months. The task has made significant progress in achieving its goal: the first rocky planet, Kepler-10b, was discovered. There were two distinct sets of transit phenomena that were detected: (1) a 152 ± 4 ppm dimming lasting 1.811 ± 0.024 hr with ephemeris

$T\ [BJD] = 2454964.57375 + 0.00060 - 0.00082 + N*0.837495 + 0.000004 - 0.000005$ days and (2) a 376 ± 9 ppm dimming lasting 6.86 ± 0.07 hr with ephemeris $T\ [BJD] = 2454971.6761 + 0.0020 - 0.0023 + N*45.29485 + 0.00065 - 0.00076$ days [3]. The found of these transit signals give so much information for scientists to study, and show the great probability of discovering habitable exoplanets.

Through the Kepler telescope and the help of some common methods, Scientists can determine whether the planet exists or not. However, each techniques have its precision and limitations. Transit events have to pass first through a series of photometric and spectroscopic tests to exclude alternative scenarios before they can be confirmed as bona fide transiting planets. The combination of photometric transit detections and Keplerian modeling of radial velocity [4]. This paper displays the detailed description of two common methods, which are transit and radial velocity, and the better accuracy of conducting research by combining these two methods.

2. Transit method

This method currently stands as the most widely utilized and efficient technique for detecting exoplanets. As of now, among the over 5,700 confirmed exoplanets discovered, more than 4,300 have been detected via the transit method - a figure that significantly surpasses the yield of all other detection techniques combined [5].

This phenomenon is called transit, which occurs when a planet passes between a star and the observation point, so part of the star's brightness is blocked by the Earth, and this temporary dimming could be easily seen in a U-shaped light curve - a graph displaying the amount of light received over a period of time. For instance, Fig. 1 displays a complete process of a transit event and the corresponding curve graph of the transit signs.

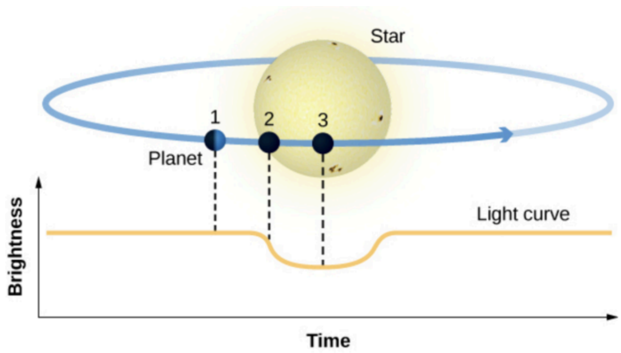

Figure 1. The full process of a transit event. The above image model selects and illustrates three different moments during the full transit event, and the bottom graph shows the corresponding light curve: (1) out of transit, (2) transit ingress, and (3) the complete drop in brightness [6]

2.1. Fundamental principles and formulas of the transit method

The transit method can provide essential information such as the size of the planets, their orbital periods, and possible atmospheric compositions [7].

The Fig. 2 illustrates the different depth change of the light curve due to the different size of the planet that is transiting. The portion of brightness that is blocked by the Jupiter-size planet is definitely larger than the portion blocked by the Earth-size planet. Therefore, it shows that the depth variation of the luminosity curve (ΔF) is directly proportional to the size of the planet passing through the surface of the star, that is, the surface area of the planet:

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the planet's radius (R_p), and the star's radius (R_*)

$$\Delta F \propto \pi R_p^2$$

$$\frac{\pi R_p^2}{\pi R_*^2} = \frac{R_p^2}{R_*^2} = \left(\frac{R_p}{R_*}\right)^2 \quad (1)$$

Precisely, the depth variation of transit (ΔF) is equal to the ratio between R_p and R_* squared:

$$\frac{\Delta F}{F} = \left(\frac{R_p}{R_*}\right)^2 \quad (2)$$

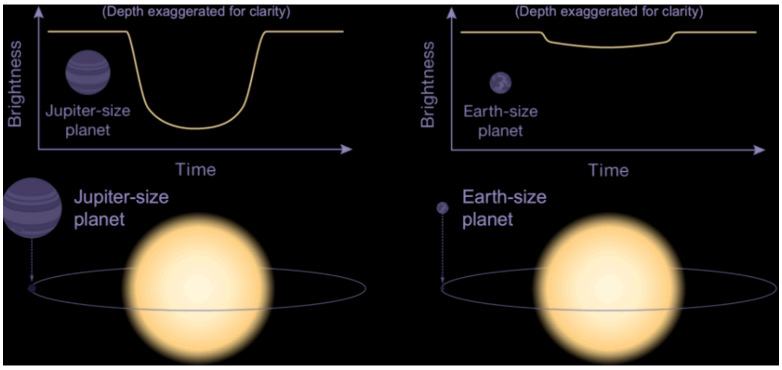

Figure 2. The two different depths of the light curve slope show the planet size relative to its star. (Credit: NASA/JPL-Caltech)

Transit duration refers to the time it takes for a planet to orbit its star, as shown in Fig. 3.

Since the transit duration equals the passing distance, which is near the diameter of its star divided by the velocity of the planet (v_p), which also can be calculated (P: a complete orbital period of planet):

$$v_p = \frac{2\pi a}{P} \quad (3)$$

$$T \approx \frac{2R_s}{v_p} \quad (4)$$

Then Transit duration (T) can be calculated:

$$T \approx \frac{2R_s}{\frac{2\pi a}{P}} \approx \frac{PR_s}{\pi a} \quad (5)$$

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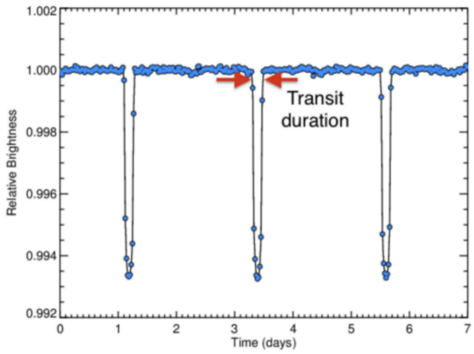

Figure 3. Time between the complete interval points of the transit event [8]

After knowing the transit orbital period of the planet, using Kepler's Third Law, which describes the relationship between the transit duration of the planet and the distance between the star and its planet, the value of the semi-major axis can be obtained.

The statement of the law: The square of the orbital period of a planet (T) is proportional to the cube of the semi-major axis (a) of the orbit. (G : the gravitational constant, with the value of $6.67 \times 10^{-11} \mathrm{m}^3 \mathrm{kg}^{-1} \mathrm{s}^{-2}$

$$T^2 \propto a^3 \tag{6}$$

$$a^3 = \frac{G M_s T^2}{4\pi^2} \tag{7}$$

2.2. Transit probability

The transit method of detecting exoplanets needs to measure the dimming of a star's light. However, whether one can always see transit signs is something that needs to be constantly considered. This probability strongly depends on the inclination of the planet's orbit. The approximate equal sign used when calculating the transit duration above was due to the variation in inclination. The inclination (i) is of an orbit is the angle between the orbital plane of the planet and the reference plane.

In the below Fig. 4, shows the model image of different angle of inclination.

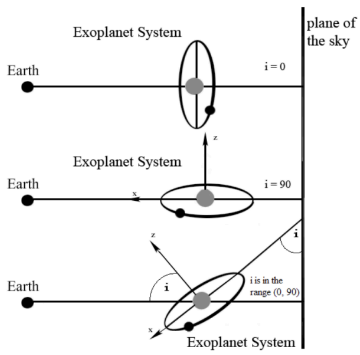

Figure 4. Different angle of inclination between the observer and orbital plane [9]

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When $i = 0^\circ$, means the observer is facing the orbital plane, so in this moment, no transit is visible. When $i = 90^\circ$, means the observer can see the edge of the orbital plane, which is the perfect status, where the full transit sign can be observed.

These two degrees of inclination are ideal and easy to calculate, but most of the time, the angle of i is hard to tell, like the third image in the figure. So the inference of the minimum orbital inclination angle is necessary and useful.

The transit sign can be seen only if the change of planetary altitude deviation is less or equal to the radius of the star:

$$a \cos i \leq R_s \tag{8}$$

$$\cos i_{min} = \frac{R_s}{a} \tag{9}$$

$$i_{min} = \cos^{-1}\left(\frac{R_s}{a}\right) \tag{10}$$

More in-depth analysis of transit probabilities:

if $\frac{R_s}{a}$ decreases as a increases, means $\cos i_{min}$ decreases, which leads to the increase in i_{min}

$$\lim_{a \to \infty} i_{min} = 90^\circ \tag{11}$$

So simply, when the distance between the star and planet increases, the transit probability will decrease. $P_{transit}$ if $\frac{R_s}{a}$ increases as R_s decreases, means $\cos i_{min}$ increases, which leads to the decrease in i_{min} . So when the radius of the star increases, the transit probability will increase. $P_{transit} \uparrow$.

2.3. Limitations

Although using the transit method--transit spectroscopy and some photometry techniques can provide the size of the planet, transit duration, and the value of semi-major axis, still, not all parameters of the planet can be obtained by this method. Meanwhile, the transit probability is uncertain due to the change in the angle of inclination. The transit signal, which can be easily observed, requires nearly edge-on orbits.

So merging manifold techniques allows the most overarching exoplanet delineation [6]. The radial velocity technique is another very common and useful way to help discover more information about exoplanets, which will be explained in the content below.

3. Radial velocity method

3.1. Doppler effect

When the star and planet orbit around each other, a portion of their motion is within our line of sight (toward or away from the observer). This kind of motion can be measured by using the Doppler effect and the spectrum of the star. Since the star is moving back and forth around the system's center of mass as it orbits, its motion is influenced by the gravitational pull of the orbiting planet, causing the lines in its spectrum to shift back and forth in response. When the star is moving away

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from the observer, the lines in its spectrum show a small-scale redshift; when it is moving toward the observer, they show a small-scale blueshift [7]. As shown in Fig. 5.

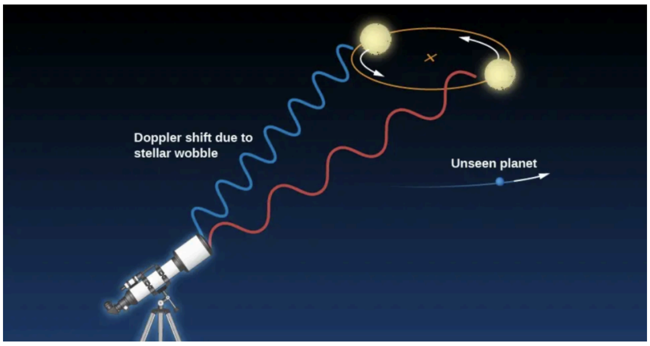

Figure 5. The picture displays the blueshift and redshift of the Doppler effect [6]

Take the blueshift as an example, the wavelength seen by the observer is $\lambda^{'} = \lambda - d$, d is the distance of the planet's movement. The time it takes the planet to move is the time between emitting two crests, which is the period P of the wave. If the planet moves with the speed v , then d = vP .

Therefore,

$$\lambda^{'} = \lambda - vP \tag{12}$$

Also, $\lambda = cP$, c is the speed of the wave.

So

$$\lambda^{'} = \lambda - \frac{v}{c}\lambda \tag{13}$$

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda^{'}-\lambda}{\lambda} = -\frac{v}{c} \tag{14}$$

If $v < 0$, $\Delta\lambda > 0$ the planet is moving away the observe, which is a redshift. $v > 0$, $\Delta\lambda < 0$ the planet is moving toward the observer, which is a blueshift.

3.2. Binary systems

There are pairs of stars, with the two components moving in bound orbits about their common center of mass, which we call binary systems of stars, or just binary stars [10].

The paper assumed that the planet orbit around the star because it is received the force exerted by the star: $\underset{F_{star\to planet}}{\longrightarrow}$, but the star is also acted on by the force $\underset{F_{planet\to star}}{\longrightarrow} = -\underset{F_{star\to planet}}{\longrightarrow}$, due to Newton' Third law. So both the planet and the star orbit around their center of mass, which the object with total mass M moves as if all is mass were focused at point C. As shown in Fig. 6.

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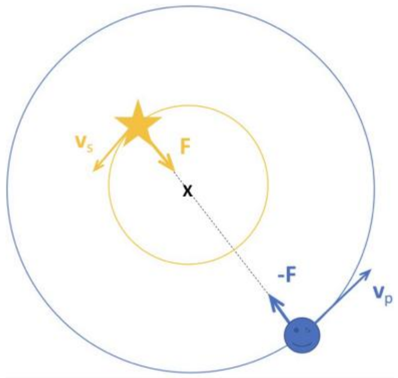

Figure 6. Planets and stars exert forces on each other to move in a circular motion around the center of mass. (photo/picture credit: original)

3.3. Formula derivation

According the Newton's Second law: $M_s \vec{a_s} = \vec{F}$, $M_p \vec{a_p} = -\vec{F}$. Also $\mathrm{F} = \frac{\mathrm{GM_s M_p}}{\mathrm{a^2}}$ (a: distance between star and planet), the gravitation force.

So

$$a_s = \frac{GM_p}{a^2} \tag{15}$$

same force on both the star and the planet, but $a_s \ll a_p$.

Then, through the uniform circular motion,

$$a_s = \frac{v_s^2}{r_s} \tag{16}$$

$$v_s^2 = \frac{GM_p r_s}{a^2} \tag{17}$$

Based on center of mass $M_s r_s = M_p r_p$, the radius of star can be got $r_s = \frac{M_p}{M_s} r_p$

Therefore,

$$v_s^2 = \frac{GM_p r_s}{a^2} = \frac{GM_p^2 r_p}{M_s a^2} \tag{18}$$

Because the planet is greatly less massive than the star, for example, HD 209458 b, $\frac{M_p}{M_s} \approx \frac{1}{1667}$. $M_p \ll M_s$, means $r_s \ll r_p$, so $a = \mathrm{r_s} + \mathrm{r_p} \approx \mathrm{r_p}$.

Then r_p can be replaced by the above equation a :

$$v_s^2 = \frac{GM_p^2}{M_s a} \tag{19}$$

v_s can also be demonstrated in terms of the orbital period T, since $v_p = \frac{2\pi r_p}{T} \approx \frac{2\pi a}{T}$ and $v_p^2 = \frac{GM_s r_p}{a^2} \approx \frac{GM_s}{a}$.

Therefore,

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$$a = \left(\frac{\mathrm{GM_sT^2}}{(2\pi)^2} \right)^{\frac{1}{3}} \tag{20}$$

$$v_s^2 = \frac{GM_p^2}{M_s a} = \frac{GM_p^2}{M_s} \left(\frac{(2\pi)^2}{GM_sT^2} \right)^{\frac{1}{3}} = \left(\frac{2\pi G}{T} \right)^{\frac{2}{3}} \frac{M_p^2}{M_S^{\frac{4}{3}}} \tag{21}$$

Or

$$v_s = \left(\frac{2\pi G}{T} \right)^{\frac{1}{3}} \frac{M_p}{M_s^{\frac{2}{3}}} \tag{22}$$

Using the angle of inclination equitation to relate the $v_r \; and \; v_s$,

$$v_r = v_s \bullet \sin(\omega t) \bullet \sin i \tag{23}$$

$$v_r = -\left(\frac{2\pi G}{T} \right)^{\frac{1}{3}} \frac{M_p sini}{M_s^{\frac{2}{3}}} \bullet sin(\omega t) = -K sin(\omega t) \tag{24}$$

K represents the semi-amplitude of the radial velocity curve. It is the quantity that can be directly observed through telescopes, the peak of the sine curve of the radial velocity of a star varying with time.

The mass of the planet M_p can be calculated if v_s, M_s and T are known. Brief steps to get the Mass of star:

Wien's Law

$$\lambda_{max} T_s = 0.0029 \text{ m k} \tag{25}$$

Since λ_{max} can be measured, the temperature of the star can be easily got.

Stefan's Law

$$\mathrm{P} = \delta \mathrm{A} T_s^4 \tag{26}$$

(Stefan-Boltzmann constant: $\sigma = 5.67 \times 10^{-8} \mathrm{Wm^{-2}K^{-4}}$)

Since the luminosity and T_s are known, the surface of the star, also the radius and mass of the star, can be calculated.

3.4. Limitation

Radial velocity can provide many necessary parameters that are needed to study exoplanets, but this method still has its own flaws. Radial velocity only can easily detect massive planets on close orbit, because $v_r = -k \sin(\omega t)$, $\mathrm{k} = \left(\frac{2\pi \mathrm{G}}{\mathrm{T}} \right)^{\frac{1}{3}} \frac{\mathrm{M_p \sin i}}{\mathrm{M_s^{\frac{2}{3}}}}$ to get a obvious and larger signal, small T, and large M_p are required. Considering Kepler's Third law $a^3 = \frac{GM_sT^2}{4\pi^2}$, small a leads by small T. Besides, the orbits of binary stars and planets, especially their eccentricity and inclination, contain information about the angular momentum of these systems. In multiple-star systems, the magnitudes of angular momentum vectors among each star, disk, and planets, as well as their (in)consistencies reveal the complex dynamical processes that guide their formation and evolution [11]. Therefore, using more methods might help to relieve this complex status.

4. Combining two methods makes the planetary parameters more

M_p can be obtained through the radial velocity method. If adding the information provided by the transit method, which is the value of T(period) and a(semi-major axis).

Then, K(velocity amplitude) can be measure on the curve, leads to the value of $M_p \sin i$ due to

$$K = \left(\frac{2\pi G}{T}\right)^{\frac{1}{3}} \frac{M_p \sin i}{M_s^{\frac{2}{3}}} \tag{27}$$

Also, the Transit method gives the value of R_p radius of planet. Therefore, the density of the planet can be obtained.

Therefore, the brief comparison between the transit method and radial velocity is obvious and easy to tell.

The transit method measures planet size, and Radial velocity measures planet mass. Combining both yields planet density, constraining composition. Meanwhile, Transit needs edge-on orbits, and Radial velocity operates for an extensive range of inclinations. However, Radial velocity is more sensitive to massive planets [6].

5. Conclusion

This paper provides the principle, basic formula, and information of two common and useful methods - Transit and Radial velocity to detect exoplanets. The transit method, the observation of the dimming - part of the brightness of the star is blocked by the planet, can give some obvious parameters through the direct photometry image and the U-shape light curve, but the change in the angle of inclination always needs to be considered, which is closely related to the transit probability. Besides, these several parameters do not satisfy a profound study; more elements are necessary. So, using the advantages of the other method, Radial velocity can help to step into further research. Radial velocity, according to the Doppler effect principle and circular motion of a binary system, involves lots of formula derivation, which leads to more parameters of the planet. The mass and density of the exoplanets can finally be obtained due to the combination of these two methods. Although radial velocity has its own flaw, which is only sensitive to the massive planets, Transit is good on this side since spectroscopy is general and easy to access all the target planets. Overall, each methods have its merits and limitations. Therefore, learning to combine the methods to achieve the research goal is an essential skill. The paper tries to explain the principle, display the advantages and disadvantages, and provide a fundamental background of these two methods, and hopes to guide readers to develop a deeper interest and reflection on the study of exoplanets. With the advanced technology and ingenious methods, more and more data of target exoplanets and even more complex multi-planets systems can be easily studied, greatly increasing the possibility of finding the Earth-like exoplanets in the habitable zone. Then the next step is the previous detecting and study of the chemical components on the surface of the exoplanets, such as exploring whether there is any liquid present, whether there is an atmosphere similar to that of the Earth, and whether there are any signs of life.

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