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Gravitational Slingshot Research

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Abstract. The gravitational slingshot effect, also known as gravity assist or gravity assist. As we know, it is a technique that uses the gravitational field of a celestial body to increase the speed and trajectory of a spacecraft in outer space. In this paper, the basic rules of how gravitational slingshot works are discussed, and the calculation formula of slingshot velocity is according to the conservation of momentum and energy. We also look at a realistic situation. When there is an angle, and came up with a second formula for speed. We used Python programming to simulate trajectories of different qualities and speeds and made a table to compare the differences in different situations. Gravity slingshot focuses on making the orbits of missions. The article also looks at successful cases that have used the technology, such as Voyager and spacecraft, to illustrate its practical applications and benefits. The goal is to enhance the understanding of how gravity slingshots can be used to achieve efficient and cost-effective space travel.

Keywords: gravity assist, Voyager, Galileo, gravity slingshots, orbits of missions

1. Introduction

A gravitational slingshot uses the gravity of a planet or other celestial body to let a spacecraft travel with minimal fuel consumption. In the 1960s, Michael Minovitch, a student at MIT, as with being one of the key figures in discovering and using the concept. Minovitch then developed the theoretical basis for gravity-assisted trajectories in his 1961 undergraduate thesis. Minovich's ideas were further developed and applied by NASA and other space apartments. The discovery allows the spacecraft to travel farther to outer planets [1,2].

Research into gravitational slingshots is necessary for some reasons. It can achieve high speeds and does not carry too much onboard fuel to extend flight to distant destinations. This is especially valuable in exploring outer planets such as Jupiter and Saturn, where conventional propulsion is not practical. Previous methods relied on air propellants. In fact, it had limitations, including the fact that certain celestial bodies could not be reached in a limited amount of time and would increase overall costs. These shortcomings show the need for continued research into gravitational slingshots.

The aim of this paper is to reduce the disadvantages of traditional methods. By combining advanced orbital plans and mature techniques, spacecraft can make more efficient uses of gravity assistance. This cannot only optimize the orbit to maximize gravity but also ensure accuracy around the celestial body. By not relying on airborne propellants, gravity slingshots greatly reduce costs, expand the range of space missions, and continue the exploration of outer planets such as Jupiter,

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Saturn, Uranus, and Neptune. It has also played an important role in visiting comets, providing valuable scientific data. Many discoveries in planetary science, astrophysics, and cosmology have been made using gravitational slingshots.

2. Theoretical background

In physics, there are two rules we have to follow, which are conservation of energy and conservation of momentum, these are the rules in order to allow gravitational slingshot effect to happened as well.

Conservation of energy and momentum is energy, and momentum in a closed system stays constant the whole time, and there are no other interventions [3].

So, the basic reason for using the gravitational slingshot effect is to gain extra momentum and save money. As we are in the solar system, we need to deal with the gravity of the sun if we are travelling away from it, and we need to have enough momentum to fight against the sun. The way we get this momentum now is by using the fuel we bring on the spacecraft, but the amount of fuel the spacecraft can bring is limited. Therefore, we have to use the gravitational slingshot effect to gain extra momentum in order to get to a further distance.

Using kinetic energy (KE) equation and equation for conservation of momentum:

$$1.\ MU - mv = M\left(U - \Delta \mathrm{U}\right) + \mathrm{m}v_1$$

$$2.\ \frac{1}{2}mv^2 + \frac{1}{2}M\mathrm{U}^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}M\left(U - \Delta U\right)^2$$

Where M is the mass of the large object, m is the mass of the small object, U stands for the initial velocity of the large object, is the velocity change of the large object after experiencing the slingshot effect, v stands for initial velocity of small object, and v1 stands for final velocity of the small object after the slingshot effect.

We can derive the equation for the ideal situation of the gravitational slingshot effect, which is in 180.

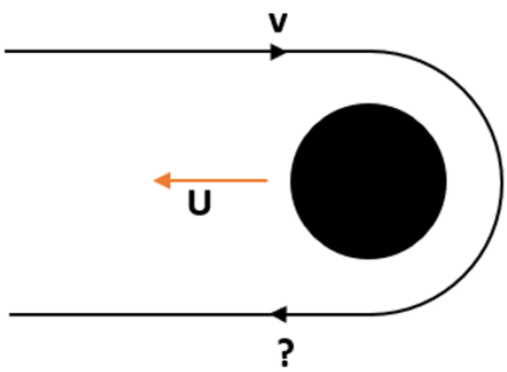

Figure 1. The velocity of larger objects and small objects

Figure 1 shows the initial velocity of a small object and the final velocity of it after the slingshot effect.

First, we are able to get the expression for :

DOI: 10.54254/2753-8818/2025.31297

$$MU - mv = M\left(U - \Delta U\right) + mv_1$$

$$MU - MU + M\,\Delta\,U = \mathrm{m(v\ +\ v1)}$$

$$M\,\Delta\,U = \mathrm{m(v\ +\ }v_1)$$

$$\Delta U = \frac{\mathrm{m(v\ +\ }v_1)}{M}$$

Then we substitute into the KE equation:

$$m\mathrm{v}^2 + M\mathrm{U}^2 = mv_1^2 + M\left(U - \Delta U\right)^2$$

$$M\left(\mathrm{v}^2 - v_1^2\right) = M\left(U - \Delta U\right) - M\mathrm{U}^2$$

$$m\left(\mathrm{v}^2 - v_1^2\right) = -2MU\,\Delta\,\mathrm{U} + M\,\Delta\,\mathrm{U}^2$$

In this situation, U >>, there is nearly no effect to the large object when the effect happens, which we are able to neglect ^2 as it is extremely small. Then we substitute the expression for:

$$m\left(\mathrm{v}^2 - v_1^2\right) \approx -2mU\left(\mathrm{v} + v_1\right)$$

$$\mathrm{v}^2 - v_1^2 \approx -2U\left(\mathrm{v} + v_1\right)$$

$$\left(\mathrm{v} + v_1\right)\left(\mathrm{v} - v_1\right) \approx -2U\left(\mathrm{v} + v_1\right)$$

$$\mathrm{v} - v_1 \approx -2U$$

$$v_1 \approx v + 2U$$

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As the situation becomes more realistic, there will be angles when the slingshot effect happens. Based on the equation at the ideal situation and using the Pythagoras theorem, we can find out an expression for v_1:

$$v_x = vcos\theta$$

$$v_y = vsin\theta$$

Where θ is the angle at which the small object comes in.

$$v_1^2 = (\mathrm{vcos\theta} + 2\mathrm{U})^2 + v_y^2$$

$$v_1^2 = (\mathrm{vcos\theta} + 2\mathrm{U})^2 + (\mathrm{vsin\theta})^2$$

$$v_1^2 = \mathrm{v}^2\mathrm{cos}^2\theta + 4\mathrm{Uvcos\theta} + 4\mathrm{U}^2 + \mathrm{v}^2\mathrm{sin}^2\theta$$

$$v_1^2 = \mathrm{v}^2\mathrm{cos}^2\theta + \mathrm{v}^2(1 - \mathrm{cos}^2\theta) + 4\mathrm{Uvcos\theta} + 4\mathrm{U}^2$$

$$v_1^2 = \mathrm{v}^2\mathrm{cos}^2\theta - \mathrm{v}^2\mathrm{cos}^2\theta + \mathrm{v}^2 + 4\mathrm{Uvcos\theta} + 4\mathrm{U}^2$$

$$v_1 = \left(\mathrm{v}^2 + 4\mathrm{Uvcos\theta} + 4\mathrm{U}^2\right)^{1/2}$$

We can find the final angle as well:
Let the final angle be α ,

$$\mathrm{tan\alpha} = \frac{vy}{vx + 2U}$$

$$\mathrm{tan\alpha} = \frac{vsin\theta}{vcos\theta + 2U}$$

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3. Result

During this project, we use python to code a simulation of gravitational slingshot effect in 2D plane, which only has the x-axis and y-axis. In the code, the mass of the large object, the velocity of the large object and the velocity of the small object are all controllable. We compare the difference of the trajectory as we are changing the mass and velocity of the large object (set the velocity of small object constant).

Then we get a set of pictures like this:

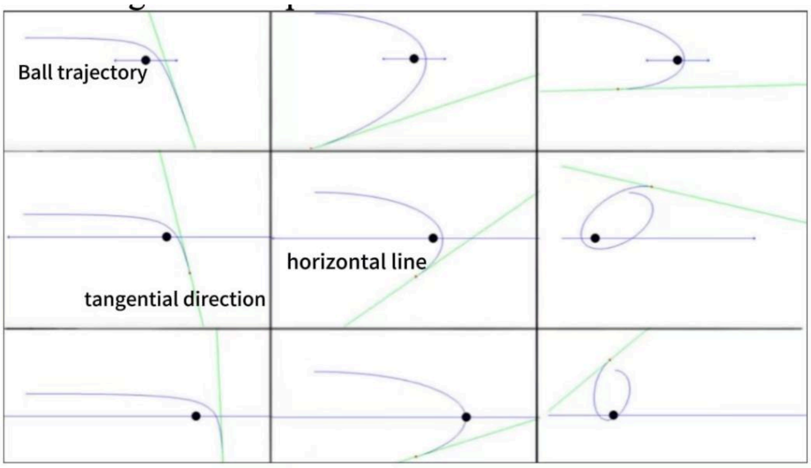

Figure 2. The movement of small object in python

As shown in Figure 2, in this set, the vertical rolls have the same mass, and the horizontal rolls have the same velocity. Horizontal is the level line, then curved is the trajectory of the small ball, and the last one is the direction of the tangent.

4. Conclusion

Based on the above-mentioned facts, the substantial energy economies gravitational slingshot affords can markedly diminish the financial burden of spacecraft propulsion and it is advisable to make preparations for future explorations. In the present study, the sophisticated orbit algorithm, along with precise navigational techniques, are employed to elucidate and derive the principles underlying the gravitational slingshot effect and the correlation between the accelerated influence and the planetary mass, velocity, and orientation angle. This paper believes that the innovation is not only capable of enhancing the efficiency of space travel for humanity but also merits further contemplation and emphasis in the realm of space exploration. The wisdom crystallization of the gravitational slingshot will provide significant support for future exploration and development in the space domain [4]. Whether it pertains to its inherent role or the use of harnessing natural forces, it merits our thorough investigation and profound analysis. However, gravitational slingshot mechanisms also possess certain constraints, an inherent aspect of their operation. For example, the gravitational slingshot may not work when it is too far away from the celestial body, or the gravitational slingshot may be affected by other external factors that lead to inaccurate calculations [5]. This paper hopes that in future scientific development, more new theories and experiments can be found to fill the defects and optimize the scheme.

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