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Differential Equation Models of Resource-Limited Population Growth

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Abstract. Human population and urban systems are typical resource-limited dynamic systems, in which growth is constrained by land, food, energy, and ecological capacity. Classical logistic differential equations describe these processes with an S-shaped trajectory, yet most applications still treat the environmental carrying capacity as a fixed constant. This paper uses recent theoretical and empirical studies to revisit resource-limited population growth from the perspective of differential equations. Drawing on the logistic, θ-logistic, and resource feedback models, a unified framework is first summarized, in which both population size and carrying capacity can evolve. Several representative case studies are then analyzed. These include logistic fitting of Wuhan's population, resource-limited growth models that embed explicit resource dynamics, and logistic-type models of urbanization and ecological environmental quality in Chinese cities. The discussion highlights new advances after 2020, especially models that link carrying capacity to food availability and other socio-ecological variables. The results show that logistic-type differential equations remain powerful tools for capturing long-term trends, but their explanatory power is greatly enhanced when feedback from resource production, technological progress, and policy is incorporated. The paper concludes that future research should move from single-equation curve fitting toward coupled socio-ecological systems, providing a stronger quantitative basis for sustainable population management and urban planning.

Keywords: Differential Equations, Population Growth, Logistic Model, Urbanization, Resource Limitation.

1. Introduction

Against the backdrop of continuous population growth and rapid urbanization, human society is confronted with increasingly severe pressures on resources and the environment. How to achieve sustainable development under resource constraints has become a central concern in demography, urban planning, and environmental science. Since Verhulst proposed the logistic growth equation, ordinary differential equation models have been widely adopted to describe the S-shaped growth trajectory of populations under environmental limits [1]. However, traditional models often treat environmental carrying capacity $K(t)$ as a constant, overlooking the dynamic influences of resource availability, technological progress, and institutional change.

Recent progress has shifted towards variable load-bearing capacity models. This model has been developed based on factors such as food production, energy consumption, and policy intervention [2,3]. In China, logistics-oriented models have been widely applied in regional population prediction, urbanization analysis, and environmental assessment, and are often combined with spatial econometric methods to capture regional heterogeneity and spillover effects [4]. These developments highlight the need for a more comprehensive modeling framework that combines population dynamics with resource feedback and spatial interactions.

This paper reviews the theoretical and empirical progress of differential equation models for resource-limited population growth in recent years, with a focus on their application in the urban system of China. By integrating logistic, θ -logistic, and resource feedback models, this study demonstrates how incorporating variable carrying capacity and spatial coupling can enhance the model's performance in predicting population trends, urbanization paths, and environmental outcomes. This study aims to illustrate how these improved models can provide a stronger quantitative basis for sustainable population management and urban planning under resource constraints.

2. Theoretical foundation

2.1. Classical logistic model and self-limiting growth

The study of differential equations for population and urban growth can be traced back to the Logistic model proposed by Verhulst. The model describes the population dynamic process under environmental constraints by the following formula:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) \tag{1}$$

Among them $N(t)$ represents the population size at time t, r represents the intrinsic growth rate, K represents the maximum population size that the environment can support (environmental carrying capacity). This equation can generate a typical S-shaped curve, effectively depicting the self-limiting behavior that gradually becomes saturated from rapid growth in the real population system, and has been verified in both ecological and human population data [1].

2.2. Variable carrying capacity and the θ-logistic extension

With the deepening of research, scholars have begun to view environmental carrying capacity as a dynamic quantity that changes over time or with resources, and have proposed the variable carrying capacity model. The work of Safuan et al. And Zulkarnaen & Rodrigo expressed population and carrying capacity as a coupled ordinary differential equation system respectively:

$$\frac{dN}{dt} = aN\left(1 - \frac{N}{K}\right) \tag{2}$$

$$\frac{dK}{dt} = b\left(K - K_0\right) \tag{3}$$

In this coupled system, N denotes the population size at time t, while K represents the environmental carrying capacity at time t, that is, the maximum population level that available resources and technology can sustain. The constant K_0 is a reference or baseline carrying capacity, describing the long-run level toward which $K(t)$ tends under given socio-ecological conditions.

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The parameter a is the intrinsic per capita growth rate of the population, governing how fast $N(t)$ increases when the population is far below the carrying capacity, whereas b is the adjustment rate of the carrying capacity, determining the speed at which $K(t)$ moves toward its baseline value K_0 in response to changes in resources, technology or policy. The evolution equation of $K(t)$ transforms the influence of factors such as food supply, technological progress, and policy adjustment on the upper limit of population into a feedback mechanism within the dynamic system, making the model closer to the real human-land system [3].

On this basis, the θ Logistic model further introduces shape parameters θ, obtaining a more general form:

$$\frac{dN}{dt} = rN\left(1-\left(\frac{N}{K}\right)^{\theta}\right) \tag{4}$$

when $\theta=1$, it degenerates into the classical Logistic equation; when $\theta>1$, the population shows a more acute "braking effect" near the carrying capacity, which can be used to fit the growth trajectory under different ecological pressures [5]. This model can also calibrate empirical data through the change of the inflection point position $N_{inflection}$, thereby improving the fitting accuracy.

2.3. Localized population forecasting models in China

Chinese scholars have localized and extended these models in the field of regional population prediction. For instance, Ren Yunping and Yang Jianya improved the classical logistic equation by introducing “human creativity” as a compensation term for the environment. Using United States census data from 1790 to 1995, they showed that the modified model achieves a very high goodness of fit to the historical population series and can explain why the estimated long-term population ceiling keeps rising as technological progress and human innovation enhance the effective carrying capacity of the environment [6].

Building on this line of research, Gao Mei and coauthors analyzed the population data of Xi’an from 1985 to 2015 and used both the Malthus and logistic models to predict the city’s population over the following 20 years. By comparing simulated values with the historical data, they found that both models generate relatively small prediction errors, but the Malthus model performs better in short-term forecasting, whereas the logistic model provides higher accuracy for medium- and long-term horizons and thus yields more reliable estimates of the future population scale [6].

Taken together, these studies indicate that logistic-type models based on differential equations have been effectively adapted to the Chinese context and have become fundamental tools for analyzing population dynamics, assessing model accuracy under different time horizons, and supporting discussions of resource constraints and the urbanization process.

3. Application

3.1. Logistic fitting of Wuhan’s population

To demonstrate the specific application of differential equations in population and urban research, this section selects several representative cases for analysis.

Firstly, the population prediction of Wuhan City is a typical example of the regional application of the Logistic model. Yanguang Chen established a Logistic equation:

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$$P_t = \frac{9390532}{1+0.15435e^{-0.10517t}} \tag{5}$$

The determination coefficient obtained through fitting $R^2 = 0.997$, indicating that this model has a high explanatory power and predictive ability for long-term population evolution. Yanguang Chen obtained a linear relationship by performing a logarithmic transformation on the Logistic equation, which allowed the parameters to be estimated via least squares regression

$$\ln \frac{y}{y-1} = \ln a - bt \tag{6}$$

The least squares method was used to estimate the Logistic model parameters, showing that under resource and policy constraints, Wuhan's population follows an S-shaped saturation trend, approaching a carrying capacity of about 9.39 million. Yanguang Chen interpreted this as reflecting the transition from rapid growth to gradual stabilization. The model demonstrated high reliability, with a symmetry index of 0.023, confirming its consistency with observed trends from 1985 to 2002 [7].

3.2. Resource constrained growth and feedback mechanism models

Nowadays, research increasingly integrates resource feedback mechanisms into population dynamic models, significantly enhancing the performance of the models in data fitting and policy evaluation. For instance, Zhang proposed a resource feedback model that directly combines population growth with the food production index $\mathrm{F}(\mathrm{t})$ [8]:

$$\frac{dN}{dt} = rN(1 - \frac{N}{K(F)}), K(F) = K_0 + \alpha ln(1 + \frac{F}{F_c}) \tag{7}$$

Calibrated with provincial panel data from China (2000–2020), the model achieved a long-term goodness-of-fit R^2 of 0.992, representing an improvement of approximately 5 $\%$ over traditional fixed-Klogistic models. Moreover, the model successfully captured the synchronous slowdown in population growth and food production growth between 2015 and 2020, indicating that resource feedback effectively explains the "soft landing" of population growth.

Another study by Liu and Wang incorporated energy consumption into the carrying capacity function, constructing a coupled population-energy differential equation system [9]:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K(E)}\right) \tag{8}$$

$$\frac{dE}{dt} = \beta E - \gamma N \tag{9}$$

Using panel data from 30 Chinese cities, numerical simulations showed that the model achieved a mean absolute percentage error (MAPE) of only 3.7 $\%$ in predicting the 2030 population peak, significantly lower than the 7.2 $\%$ error of the traditional logistic model. The study further revealed that a 10 $\%$ increase in energy efficiency could raise sustainable population carrying capacity by 4–6 $\%$, providing a quantitative basis for low-carbon transition policies.

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3.3. Logistic modeling of urbanization levels and its extensions

Recent studies have introduced time-varying parameters and asymmetric S-shaped curves into urbanization modeling, enhancing the model's ability to capture differentiated urbanization pathways. Based on panel data from 287 prefecture-level cities in China (2000–2020), Chen proposed an extended logistic model with a time-varying maximum urbanization rate $U_{max}(t)$[10]:

$$\frac{dU}{dt} = b(t)U\left(1 - \frac{U}{U_{max}(t)}\right) \quad (10)$$

$$U_{max}(t) = U_0 + kt \quad (11)$$

The model achieved an average R^2 of 0.986 for cities in eastern coastal regions and over 0.975 for central and western cities. Compared with the fixed U_{max} model, its prediction error (RMSE) was reduced by approximately 30 % . The model also identified that the timing of urbanization inflection points was significantly correlated with infrastructure investment intensity and household registration policy openness.

Furthermore, Wang integrated spatial panel data into a spatial autoregressive version of the logistic urbanization model, incorporating neighboring cities' urbanization levels as explanatory variables [11]:

$$U_i(t) = \frac{U_{max,i}}{1+a_i e^{-b_i t}} + \rho \sum_{j \neq i} w_{ij} U_j(t) \quad (12)$$

In an empirical application to the Yangtze River Delta urban agglomeration, the model not only improved fitting accuracy (adjusted R^2 increased to 0.994) but also quantified spatial spillover effects: a 1 % increase in the urbanization rate of neighboring cities boosted the local urbanization growth rate by 0.15–0.22 percentage points.

3.4 Spatially coupled models of population density and eco-environmental quality

In recent years, an increasing number of studies have integrated spatial dynamics into differential equation models to better capture the heterogeneity and interactivity of population and environmental systems. Based on previous work, Zhong proposed a spatial explicit differential equation model that combines population density and ecological environment quality (EEQ) in a dynamic feedback framework [4]:

$$\frac{dP_i}{dt} = rP_i\left(1 - \frac{P_i}{K_i(E_i)}\right) \quad (13)$$

$$\frac{dE_i}{dt} = -\alpha P_i + \beta \sum_j w_{ij} E_j \quad (14)$$

Using panel data from 2000 Chinese counties between 2000 and 2020, the model was calibrated through a hybrid approach combining geographically weighted regression and system dynamics. The results demonstrated a strong explanatory effect. This model accounted for 71.3 % of the variation in county-level EEQ, which was approximately 18 % higher than that of traditional linear spatial models. Population density has a significant negative impact on the local environmental quality. For every 1 % increase in population density, the EEQ decreases by 0.05 to 0.09 percentage points. Furthermore, the model also revealed a significant spatial spillover effect, that is, the environmental deterioration of one county led to a 0.02 percentage point reduction in the average EEQ of neighboring counties. This spatial coupling enables the identification of concentrated areas with high population pressure and low environmental quality, such as the North China Plain and the Yangtze River Delta urban agglomeration, providing insights into spatial positioning for cross-regional governance.

Further extending this line of inquiry, Li et al. developed a tripartite coupled differential equation system that incorporates both resource consumption and pollution emissions into the environmental carrying capacity function. Applied to the Beijing–Tianjin–Hebei region, the model demonstrated enhanced predictive capability, achieving an R^2 of 0.89 in simulating PM2.5 concentration changes. Notably, the model succeeded in identifying environmental quality inflection points 3–5 years earlier than previous approaches, underscoring its utility as a policy early-warning tool for sustainable urban and regional planning [12].

4. Discussion

4.1. Limitations of classical logistic models

The assumption of a fixed carrying capacity limits the realism of classical logistic models in dynamic socio-ecological systems. The traditional logistic model assumes a constant environmental carrying capacity K which fails to capture the long-term effects of resource fluctuations, technological innovation, and policy shifts. As demonstrated in Section 3, models with a fixed K tend to underestimate population ceilings and miss inflection points in urban growth trajectories, particularly in rapidly developing regions such as China.

4.2. Advances in variable carrying capacity frameworks

Two-equation systems with dynamic carrying capacity significantly improve model flexibility and explanatory power. Rodrigo and Zulkarnaen propose a coupled system in which both population size $N(t)$ and carrying capacity $K(t)$ are governed by differential equations [3]. Their framework accommodates a range of growth patterns, including θ-logistic and Gompertz curves, and allows for the explicit incorporation of resource feedback, as illustrated in the food- and energy-linked models in Section 3.2.

4.3. Integrating resource availability into carrying capacity

Modeling carrying capacity as a function of food production helps enhance the understanding of the mechanism of population sustainability. Zulkarnaen and Rodrigo demonstrated that carrying capacity can be expressed as a self-limiting function of food availability, thereby achieving a more precise simulation of population transition [3]. Incorporating this approach into food and energy feedback has enhanced the long-term model fit and policy relevance.

4.4. Toward coupled socio-ecological and spatial models

Future models should integrate resource dynamics, spatial interaction, and random factors, so as to better guide sustainable planning. The cases in the article show that urbanization and environmental quality models benefit from spatial coupling and temporal variation parameters. Extending these methods to include the unified growth law, fractional differential equations, and spatial econometric

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methods will further enhance the model's ability to cope with nonlinearity, uncertainty, and regional spillover in urban systems.

5. Conclusion

This paper mainly examines the differential equation model of population growth with limited resources, emphasizing the latest progress in variable carrying capacity, resource feedback and spatial coupling. The theoretical review highlights the evolution from the classical logistics model to a more flexible framework, in which carrying capacity is regarded as a dynamic function of resource indicators such as food and energy. Empirical applications in Chinese cities, such as the logistical fitting of the population in Wuhan, resource-constrained growth models, and spatially defined environmental population systems, indicate that these enhanced models achieve higher accuracy, stronger explanatory power, and greater policy relevance compared to traditional methods.

The discussion underscores the importance of moving beyond single-equation curve fitting toward integrated socio-ecological systems. Key directions for future research include: (1) explicitly modeling carrying capacity as a function of time-varying resource and technology indicators; (2) incorporating spatial econometric and stochastic elements to handle heterogeneity and uncertainty; and (3) applying these advanced frameworks to high-resolution spatiotemporal data to support sustainable population and urban governance in China and globally. Ultimately, differential equation models remain indispensable tools for understanding and managing growth under resource constraints, provided they continue to evolve to reflect the dynamic and interconnected nature of real-world systems.

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