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Optimal Strategy of a Simplified Yahtzee Game

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Abstract. Yahtzee is a popular dice game that combines simple rules with complex strategic decisions. While the optimal strategy for the standard Yahtzee game has been solved computationally, those results rely on fixed rules and specific parameters. In this study, we investigate a simplified version of the game that replaces dice with coins, allowing for analytical treatment while preserving the core probabilistic features of the original game. Using probability analysis and game theory, we derive the optimal strategies in three different cases, each characterized by distinct scoring systems and free parameters. Our analytical solutions reveal general principles underlying decision-making in sequential stochastic games. Specifically, we find that optimal play requires balancing the goal of maximizing immediate scores against maintaining the potential for higher rewards in later rounds. In addition, an effective strategy must manage risk by preserving diverse coin outcomes to keep multiple scoring categories open. These results provide a clear mathematical understanding of strategy formation and offer conceptual insights applicable to more complex games such as Yahtzee.

Keywords: Yahtzee, optimal strategy, game theory, probability

1. Introduction

The popular Yahtzee game, which was invented by Milton Bradley, consists of playing a classic dice game, where each player makes a game with five dice, in which all players toss.[1] There are 13 rounds and three tosses per round. At the final round, the players choose a score to score [2].Since its introduction in 1956, Yahtzee has received around 50 million copies sold yearly [3].Mathematical interpretations of Yahtzee and its variants have attracted much attention and frequently are the source of exercises for mathematics and computer science students [4]. Previous work [5] calculated the optimal approach to maximize the score under well-known rules and derived around 255 expected average score from all choices. This is also a computational approach that only applies to specific rule sets and does not generalize.It is also short-sighted to provide more mathematical knowledge that are needed for adaptation of the game to other situations (e.g. for children, casual players, or in a tournament) - e.g., Triple Yahtzee, Challenge Yahtzee, Jackpot Yahtzee [6].More analysis and generalization mathematical solutions that could guide both game designers and players are needed.Many of the most frequent game designs, for instance, rely on counting every turn of a particular(uniformly) digit. We study a simplified model using coins instead of dice; this unbalanced model incorporates variable parameters like number of coins, tosses/rounds, and scoring rules. We calculate this optimal strategy analytically and expressed the satisfactions expected score as a function of these parameters

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in a rigorous mathematical manner that can be used to apply mathematical analysis to game design and strategic play.

2. Model

2.1. Game rules and parameters

Consider a game with N fair coins, each having distinct heads (H) and tails (T) sides. The object of this game is to toss the coins for the highest score in the given categories. The score pad includes only 2 categories, and the game is played for 2 rounds. After each round, the coins show a pattern, the player will make a decision to choose one category that has not been chosen yet. The score is based on the category description and the current coin pattern. The score of the first category is the number of heads in the coin pattern multiplied by S_1; the score of the second category is the number of tails in the coin pattern multiplied by S_2. The goal is to win the highest total score of both categories.

Within each round, the player is allowed to toss the coins up to T times. For the first toss, the player needs to toss all the N coins. After the first toss, the player can set any coins as 'keepers', and continue the round with the non-keeper coins. For other tosses, the player needs to toss the non-keeper coins, and the player can re-set keepers and non-keepers after the toss. At any toss, if the player set all coins as keepers, the round stops and the player scores a category.

Table 1: Score Categories

Category	Score
1	The number of heads $\times S_1$
2	The number of tails $\times S_2$

For example, consider the game with $N = 5$, $T = 3$, $S_1 = 1$, $S_2 = 2$. In the first toss of the first round, a player tosses the 5 coins. The distribution of the coins is 3 heads and 2 tails (HHHTT). Then, the player sets the 2 tails (TT) as keepers and 3 heads as non-keepers. In the second toss of the first round, the player tosses the three non-keepers and gets 2 tails and 1 head (HTT). So, the pattern is HTTTT. Then the player sets 1 head and 1 tail as keepers, and the other 3 coins as non-keepers. In the third toss of the first round, the player tosses the three non-keepers and gets 3 heads. So the pattern is HHHHT. The player chooses category 1 to score, getting $4 \times S_1 = 4$ points, and the first round ends. In the first toss of the second round, the player tosses the 5 coins. The pattern is TTTTT. Then the player decides to keep all 5 coins and end the second round. Now the player can only choose category 2 to score, getting $5 \times S_2 = 10$ points. Therefore, the total score of the player is $4 \times S_1 + 5 \times S_2 = 14$.

2.2. Methods

We have our study of the simplified Yahtzee game in stochastic dynamic programming. Our task is to formulate a strategy which maximizes the total expected score in both rounds, by maximizing two different types of decisions at each moment, i.e. deciding which scoring group should win, or which coins to keep or reroll after each toss. The effects of these decisions are two-fold: they immediately limit the number of categories available at future rounds, and the distribution of later coin-toss results follows a binomial distribution. We exploit the so-called backward induction principle of dynamic programming by first examining the last possible decision – last toss of the second round – and taking

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the expected value of being in any possible game state at the time. We then recursively move back in the decision tree. In each previous step, we select the decision that maximizes all of the immediate reward and the expected values of all future outcomes. And play then. This backward reasoning formalized by the Bellman equation allows us to transfer the results of last-round constraints and opportunities from a first-down decision. In this way the model shows how the optimal approach for early tosses and categories is made essentially depend on the scoring parameters and the number and number of tosses, so that every local decision contributes to the global goal of maximising the total expectation score.

Since there are two kinds of decisions, we can focus on only one kind first. The xpected score is determined S_1, S_2, which makes the problem non-symmetric. To be simple, we start from the symmetric case. So we will first investigate Case 1: $T = 1, S_1 = S_2$. Case 2: $T > 1, S_1 = S_2$. Case 3, $T > 1, S_1 > S_2$.

3. Results

3.1. Case 1: $T = 1$, $S1 = S2 = 1$

We compute the expected score for each toss separately and then sum them to obtain the total expected score. The coins are fair, so the probability of heads or tails on any coin is 0.5, and all coins are independent.

The optimal strategy is as follows.

After the second toss: The player has no choice; they must select the face opposite to their first choice. The score is then calculated on the basis of this fixed choice.

After the first toss: The player should always choose the face (heads or tails) that appears more frequently in the outcome. This maximizes the score for the first toss, as it selects the majority face since the expected score in the second toss is independent to the score of second toss.

For all situations similar to this with different numbers of coins, the expected points for the second round will be the expected number of tails, which equals $N/2$ where m is the number of coins. We can then calculate the expected points for the first round. Since the player will choose the face that appears more frequently in the outcome to maximize the score, the expected points when the player get n tails are:

$$P(n) = \max(n, N-n) \times \frac{\binom{N}{n}}{2^N}. \tag{1}$$

The expected points for this round will be

$$\sum_{k=0}^{N} P(k) \tag{2}$$

Thus, the expected points for the whole game will be

$$\sum_{k=0}^{N} P(k) + \frac{N}{2}. \tag{3}$$

For example, when $N = 5$, we can calculate the expected points using the formula:

$$\sum_{k=0}^{5} P(k) + \frac{N}{2} = 3.4375 + 2.5 = 5.9375 \tag{4}$$

Thus, the expected points for the whole game will be 5.9375. We notice that the expected points is higher than the number of coins.

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3.2. Case 2: $T > 1$, $S_1 = S_2 = 1$

For this case, there are 2 rounds and $2T$ decision times. Similarly, we can consider the player who chooses the head in the first round. Then, the optimal strategy for the second round is to keep the tails for each toss. For each coin, the expected number of tails is

$$(1 - (\frac{1}{2})^T) \times N \tag{5}$$

Then, the optimal strategy for the first toss in the first round is to keep the side with the most amount. The optimal strategy for the first round t-th tosses will be keeping the side you choose at first round first tosses. Similarly, the expected points when the player gets n tails are:

$$P(n) = (\max(n, N - n) + (N - \max(n, N - n)) * (1 - (\frac{1}{2})^{T-1})) \times \frac{\binom{N}{n}}{2^N}. \tag{6}$$

We can also simplify this equation into

$$P(n) = (N - (\frac{1}{2})^{T-1} * Min(N, N - n)) \times \frac{\binom{N}{n}}{2^N}. \tag{7}$$

The expected points for this round will be

$$\sum_{k=0}^{N} P(k) \tag{8}$$

Thus, the expected points for the whole game will be

$$\sum_{k=0}^{N} P(k) + (1 - (\frac{1}{2})^T) \times N \tag{9}$$

For example, when $N = 5$ and $T = 2$, there are 2 rounds and 4 decision times. Using the given formula, the expected point for the second round is

$$(1 - (\frac{1}{2})^2) \times 5 = 3.75 \tag{10}$$

And the first round, the formula give the expected points

$$\sum_{k=0}^{5} P(k) = 4.21875 \tag{11}$$

Therefore, the expected points for 2 rounds will be 7.96875.

3.3. Case 3: $S_1 > S_2$

Now there is a situation where the heads and tails are different. According to the symmetric feature of the heads and tails, we assume $S_1 > S_2$ and $S_2 = 1$. As before, for the second round, the optimal strategy is to keep most of the remaining choices (heads or tails). The expected number of faces to be scored will be

$$(1 - (\frac{1}{2})^T)N \tag{12}$$

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3.3.1. Last step in first round

For the last step in the first round, we should make decision for categories. Suppose there are h heads and $N - h$ tails in the end. If we choose to keep heads, the expected point is

$$hS_1 + (1 - (\frac{1}{2})^T)NS_2. \tag{13}$$

If we choose to keep tails, the expected point is

$$(N - h)S_2 + (1 - (1/2)^T)NS_1. \tag{14}$$

For the optimal strategy, we choose the first category when

$$hS_1 + (1 - (1/2)^T)NS_2 > (N - h)S_2 + (1 - (1/2)^T)NS_1. \tag{15}$$

By simplifying this equation, we get

$$\frac{h}{N} > \frac{(1 - (1/2)^T)(S_1 - S_2) + S_2}{S_1 + S_2} := h^*. \tag{16}$$

For any $T >> 1$, we have

$$h^* \approx \frac{S_1}{S_1 + S_2}. \tag{17}$$

This formula means that we choose the heads to score only if the proportion of heads is larger than the points of heads compared to tails. For example, when $S_1 = 2, S_2 = 1$, when $h/N < 2/3$, we choose tails to score and keep the opportunity of getting higher score in the second round.

3.3.2. Second last step in first round

Now consider the step before the last step in the first round of the game given the optimal strategy of later steps. Suppose that there are h' heads and $N - h'$ tails at this step. We can choose to keep k_1 heads and k_2 tails, such that $0 \le k_1 \le h', 0 \le k_2 \le N - h'$, and re-roll the remaining $N - k_1 - k_2$ coins.

The optimal strategy is based on the probability distribution on the number of heads h in the next step. If we choose $k_1 \ge h_*$, we will always choose the heads to score in the next step because $P(h > h^*) = 1$. In this case, the expected number of heads in the next step is

$$E_0 = k_1 + \frac{N - k_1 - k_2}{2} \tag{18}$$

On the other hand, if we choose $k_1 < h^*$, the probability of $h > h_*$ is given by

$$P(h > h^*) = \sum_{m=h_*-k_1}^{N-k_1-k_2} \binom{N - k_1 - k_2}{m} \times \frac{1}{2^{N-k_1-k_2}}. \tag{19}$$

In this case, under the condition $h > h^*$, the expected number of heads will be

$$E_1 = \sum_{m=h_*-k_1}^{N-k_1-k_2} \binom{N - k_1 - k_2}{m} \times \frac{k_1 + m}{2^{N-k_1-k_2}} \times \frac{1}{P}, \tag{20}$$

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while under the condition $h \le h^*$, the expected number of heads will be

$$E_2 = \sum_{m=0}^{h_*-k_1-1} \binom{N-k_1-k_2}{m} \times \frac{k_1+m}{2^{N-k_1-k_2}} \times \frac{1}{1-P}. \tag{21}$$

To summarize, the expectation of the final score when we choose $k_1 \ge h^*$ is

$$E(k_1,k_2) = E_0(k_1,k_2)S_1 + (1-\frac{1}{2^T})NS_2. \tag{22}$$

When we choose $k_1 < h^*$,

$$E(k_1,k_2) = P\times\left(E_1(k_1,k_2)S1 + (1-\frac{1}{2^T})NS_2\right)+(1-P)\times\left((N-E_2(k_1,k_2))S_2 + (1-\frac{1}{2^T})NS_1\right). \tag{23}$$

We can computationally show the expectations. The figure below shows the example for $N = 10$, $T = 3$, $S_2 = 2$, $S_1 = 1$. If there are $h' = 6$ heads in the second last step of the first round, we look at the region $[0,6] \times [0,4]$ and find out that we should keep $k_1 = 6$ heads and $k_2 = 0$ tails for the game.

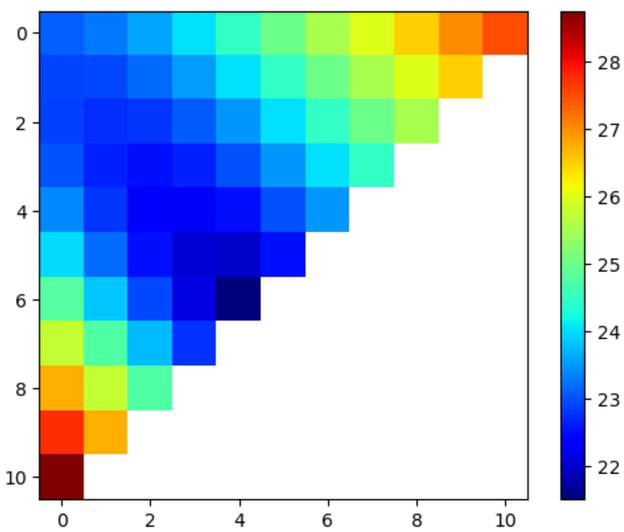

Figure 1: Expected score under different decision. The y-axis represent k_1, and the x-axis represent k_2. The color is the expected score $E(k_1,k_2)$.

4. Conclusion

Based on the probability analysis and strategic derivation of the simplified Yahtzee game using coins, two key conclusions emerge regarding optimal decision-making. First, when choosing which scoring category to assign at the end of a round, players must carefully balance maximizing the immediate score against preserving high-value categories for future rounds. This trade-off becomes particularly critical when the point values per head and tail differ . In such cases, the optimal strategy involves selecting the higher-scoring category only if the current count of that face exceeds a specific threshold h^*; otherwise, it is more advantageous to reserve it, sacrificing short-term gain for greater long-term expected score.

Second, concerning keeper/non-keeper decisions after each toss, the optimal approach favors maintaining flexibility and avoiding over-commitment to a single outcome. Rather than locking all coins to one face early on, players should distribute risk by retaining some coins of the less frequent face, thereby preserving the ability to adapt to subsequent tosses and mitigate the impact of unfavorable random outcomes. This strategy of risk diversification, combined with forward-looking category selection, underscores a fundamental principle: in multi-round stochastic games, the globally optimal strategy often requires sacrificing local optima to enhance overall expected performance. These insights not only apply to this simplified model but also offer valuable guidance for game design and strategic play in broader contexts of sequential decision-making under uncertainty.

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