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# *A Framework for Transportation Network Optimization for Real-World Logistics: Extensions and Applications*

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***Abstract.*** Transportation network optimization is traditionally stated as a classical transportation problem in which supply is allocated to demand at minimum cost. While elegant in an analytical sense, the underlying formulation lacks aspects that are now dominating the real world of logistics: time dependence, binding capacities, multimodal transfers and uncertainty. This paper proposes an integrated framework which retains the interpretability of minimum-cost flow and adds extra dimensions to it along four realism axes. In the baseline, explicit penalty terms for surplus and unmet demand are included to reflect pragmatic trade-offs when exact balance is infeasible. First, spatial routing is linked with intertemporal decisions under inventory carryover and storage costs in a transshipment model. Second, capacity constraints are placed on arcs and facilities so as to obviate a bounded operationally credible feasible set. Third, multimodal routing is added using mode-specific arcs and transfer rules that represent physical compatibilities and service frictions. Fourth, uncertainty in costs, capacities and demands is factored in by separating anticipative decision from adaptive recourse. A dual-price-based diagnostic layer, parametric ranges and scenario testing are used to transform the results of the optimization into management actions such as expanding capacity, mode shifting and buffer-stock. The proposed formulation is a trade-off between tractability and operational fidelity, and provides the building block for scalable solution strategies (exact when possible and heuristic when necessary) that are appropriate for large-scale, practical logistics systems.

***Keywords:*** Minimum-cost flow, Transshipment, Capacity constraints, Multimodal transport, Sensitivity analysis

## 1. Introduction

The smooth flow of goods through a spatial network is one of the core issues in operations research and a key determining factor in supply-chain performance. A classical transportation problem (TP) models this problem as a linear (single-period) assignment of supply to demand that minimizes the total transportation cost. Its structure improves transparency and facilitates the use of specialized

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algorithms and arrives at an intuitive economic interpretation of flows and costs. However, modern-day logistics are far from baseline assumptions. Demand and supply vary over time; arcs and facilities have binding capacity limitations; freight moves over differentiated modes (e.g. road, rail, sea, air), over which transfer frictions exist; and exogenous shocks in costs and availability are introduced. Thus, models that do not go beyond the classical TP framework are at risk of leading to solutions that are internally optimal, but are exogenous infeasible or fragile. In the baseline formulation, explicit penalty terms for surplus and unmet demand are included to capture pragmatic trade-offs whenever exact balance is infeasible.

This paper contributes to a unified modelling framework that preserves the clarity of minimum-cost flow but integrates the features of operations that are responsible for plan failure in practice. The first extension raises the issue of transshipment from being a passive topological possibility to being an explicit decision with inventory carryover and handling costs. Warehouses and cross-docks become active nodes to establish connections between periods as well as routes so that the planner can make a trade-off between transportation and storage and buffer the volatility in time. The second extension has capacity constraints on arcs and storage locations. These provide a regularization of the feasible region and avoid unrealistic corner solutions, which are characterized by overloading single routes or facilities which better represent infrastructure and service commitments.

The third extension is the formalization of multimodality. Heterogeneous costs, speeds, emissions and reliability are represented in mode-specific arcs; transfer rules at intermodal terminals are used to represent compatibility and processing needs. Intermodal transfer centers operate as flow-through facilities that do not hold inventory; storage is restricted to warehousing nodes. This structure makes it possible to compare pure-mode and mixed-mode itineraries in a principled way and to identify where terminal investments would allow the development of cheaper routes. The fourth extension is a two-stage extension that introduces uncertainty through a lens of here-now allocation, where the allocation of foreseeable uncertainty is anticipated and recourse actions are adaptable ex post to the realized state. This separation is robust with regard to changes in demand, congestion, and prices, assuming no perfect foresight.

To convert model outputs to decisions the framework has a sensitivity analysis layer. Dual (shadow) prices give an indication of the marginal value of capacity or storage relaxation and show at what point an increase in amount of capacity will result in a maximum reduction of cost. Parametric and scenario diagnostics determine which region of plan stability is and give directionally correct changes in case thresholds are violated. Together these tools offer a shift between mathematical optimality and managerial action.

The value is double: A consistent network model which is ready to meet the requirements of the real world and a set of diagnostic tools which help to transform the results in a way that they can be used in decision making. This mixture is a highly rigorous, yet viable basis on which to plan big-scale logistics, which can be coupled with accurate solvers on moderate-sized problems and heuristic acceleration of industrial-sized problems.

## 2. Transportation network optimization models

### 2.1. Classic transportation problem

The classic transportation problem is one of the oldest and most widely studied models in operations research. It usually involves allocating goods from multiple supply sources to multiple demand destinations while minimizing the total transportation cost. In short, it is about doing the most efficient thing with the least amount of money or cost. Therefore, in modern society, in addition to

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solving the basic supply and demand balance problem in the model, we also need to introduce capacity constraints, transshipment nodes, and penalty variables for excess or deficiency into the model so that we can make it more realistic while maintaining the linearity of the model [1].

The standard mathematical formulation is:

$$Minimize\ Z = \sum_{(i,j)\in A} c_{ij} + x_{ij} + \sum_{i\in S} q_i r_i + \sum_{j\in D} p_j s_j$$

subject to:

Supply constraints: $\sum_{j:(i,j)\in A} x_{ij+r_i} = s_{i,}\ \forall_i \in S$

Demand constraints: $\sum_{i:(i,j)\in A} x_{ij} + s_j \geqslant d_{j,}\ \forall_j \in D$

Transshipment flow constraints: $\sum_{i:(i,k)\in A} x_{ik} = \sum_{j:(k,j)\in A} x_{kj,},\ \forall_k \in T$

Arc capacity constraints: $0 \leq x_{ij} \leq u_{ij,}\ \forall_{(i,j)} \in A$

Non-negativity constraints: $r_i \geq 0,\ s_j \geq 0$

$S$ : set of supply nodes,

$D$ : set of demand nodes,

$T$ : set of transshipment nodes,

$C_{ij}$ : unit transportation cost along arc $(i,j)$ ,

$S_i$ : available supply at node $i$ ,

$d_j$ : demand requirement at node $j$ ,

$x_{ij}$ : decision variable representing shipment quantity,

$u_{ij}$ : maximum capacity of arc $(i,j)$ ,

$r_i$ : surplus (unused supply) at node $i$ ,

$S_j$ : shortage (unmet demand) at node $j$ ,

$q_{i,}p_j$ : penalty costs for surplus and shortage respectively.

Although this extended model retains some basic features of the classic linear programming (LP) model, it also incorporates additional elements, such as arc capacity and transport balance. While these new elements enable the model to more accurately describe the supply and demand imbalance and transportation situation in the real world, making it more practical in real-world applications [2]. However, it remains applicable only to a single-cycle static environment. This means that it cannot reflect whether supply and demand change over time; that is, while it reflects dynamism, it also ignores the dynamic changes in transportation costs and inventory decisions that occur over time. These limitations lead to the practical application effect. Therefore, to overcome these limitations, we will reconstruct the transportation problem within the framework of network flow theory, for example, by incorporating time dynamics.

## 2.2. Network flow and path problem

Although the extended model we showed in the previous section enhanced the realism of the classic transportation problem, the structure of the classic "transportation problem" is actually very similar to the structure of "network flow" to achieve a unified representation and solution. By abstracting the entire transportation system into a directed network G = (N, A), the nodes correspond to supply points, demand points, or transshipment facilities, and the arcs correspond to transportation routes along with their related costs and capacities [3]. In simple terms, the reason why the transportation problem can be planned as a network flow is that it is itself a directed network from the supply point to the demand point: the constraint is flowing conservation, the capacity limit is capacity, the freight

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cost is the edge cost, and finding the cheapest transportation method is finding the minimum cost flow.

The LP formulation is:

$$min\ z = \sum_{(i,j)\in A} c_{ij}x_{ij} \quad (1)$$

subject to:

Flow balance:

$$\sum_{j:(i,j)\in A} x_{ij} - \sum_{j:(j,i)\in A} x_{ji} = b_i, \forall_i \in N \quad (2)$$

Arc capacity:

$$0 \le x_{ij} \le u_{ij},\ \ \forall (i,\mathrm{j}) \in A \quad (3)$$

Here, $b_i$ is represents the net supply at node i, where positive values indicate supply nodes and negative values indicate demand nodes. Therefore, this formula can be understood as a network flow version.

From a computational perspective, the advantage of linear programming in solving network problems lies in its ability to use specialized algorithms for computation, such as the network simplex method and the continuous shortest path method. These methods are explicitly optimized for network structures, making them faster and more efficient than the general simplex method or interior-point method. However, Dijkstra's algorithm, the Floyd-Warshall algorithm, and others are not optimization models but rather auxiliary tools in linear programming algorithms used to identify low-cost augmenting paths. This is because, whether it involves the most basic transportation problem or a multi-period model involving time and other factors, they can essentially be viewed as the same method: linear programming. Other factors are merely added variables and requirements, which can be understood as a set of "addition and subtraction rules": whether it's freight delivery, inventory management, or traffic scheduling within a network, these calculations are essentially performed using the same linear programming method. Therefore, this commonality ensures that even different formulas have a sure consistency, facilitating theoretical analysis and the use of auxiliary tools to solve a wide variety of problems.

### 2.3. Multi-origin multi-destination

In the models discussed in the previous section, it is evident that these two classic transportation problem models have limitations, specifically that they typically operate in a single-period, static environment. However, as mentioned in the first section, in real-world transportation and supply chain systems, these parameters often change continuously due to factors such as market fluctuations, seasonal changes, or changes in traffic conditions [4]. Therefore, to capture these variables, the classic transportation problem model must be extended to a multi-period linear programming model. The multi-period model retains the linear structure of the classic model while also introducing the explicit dimension of time.

The extended formulation introduces additional decision variables for each period $t \in T$ :

$x_{ij}^t$ : transportation volume along arc $(i, j)$ in period $t$ ,

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$I_i^t$ : inventory carried at node $i$ at the end of period $t$ ,
$s_j^t$ : shortage at demand node $j$ in period $t$ .
The objective function becomes:

$$Minimize\ Z \ = \ \sum_{t \in T} \left( \sum_{(i,j) \in A} c_{ij}^t x_{ij}^t + \sum_{i \in N} h_i^t I_i^t + \sum_{j \in D} p_j^t s_j^t \right) \tag{4}$$

subject to:
Supply balance:

$$\sum_{j:(i,j) \in A} x_{ij}^t + I_i^t \leq s_i^t + I_i^{t-1}, \quad \forall_i \in S, \quad \forall_t \in T \tag{5}$$

Demand satisfaction:

$$\sum_{i:(i,j) \in A} x_{ij}^t + s_j^t \geq d_j^t, \quad \forall_i \in D, \quad \forall_t \in T \tag{6}$$

Flow conservation at transshipment nodes:

$$\sum_{i:(k,i) \in A} x_{ik}^t = \sum_{j:(k,j) \in A} x_{jk}^t \ , \quad \forall_k \in T, \quad \forall_t \in T \tag{7}$$

Arc capacity constraints:

$$0 \leq x_{ij}^t \leq u_{ij} \ , \quad \forall_{(i,j)} \in A, \quad \forall_t \in T \tag{8}$$

Inventory capacity:

$$0 \leq I_i^t \leq K_i \ , \quad \forall_i \in N, \quad \forall_t \in T \tag{9}$$

Non-negativity:

$$x_{ij}^t \geq 0, \ \ I_i^t \geq 0, \ \ s_j^t \geq 0 \tag{10}$$

Here, $\hat{s}_i^t$ and $\hat{d}_i^t$ are forecasted supply and demand, $h_i^t$ is the inventory holding cost, and $p_j^t$ is the penalty for unmet demand.

Although the formula mentioned above increases the problem size, it remains a larger-scale multivariate linear programming model. It can still be solved using precise methods such as the simplex method or interior point algorithm. At the same time, large networks require more advanced computational strategies and more realistic problem constraints.

It is worth noting that introducing time and inventory does not change the fundamental principles of linear programming, as we have repeatedly mentioned; these are merely variable conditions and

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do not change the underlying principles. It allows for a more accurate description of the impact of dynamic changes in the transportation system [4].

However, the above modeling framework is still highly idealistic. Applying the theoretical model to real-world transportation networks still requires considering and incorporating a series of realistic factors. These include capacity limitations at transfer points, routes, and warehouses, as well as the conversion between different modes of transportation, and the uncertainty of supply and demand. These constraints, which extend beyond standard network flow theory, determine whether the theoretical model can be successfully applied in practical settings. This is the core focus of our discussion and analysis in Section 3.

## 3. Model extension and real-world constraints

### 3.1. Transshipment points

Transshipment points are essential for the modern transportation. They help make the entire transportation system efficient and cost-effective. These points gather scattered shipments and combine them into full loads. Additionally, they allow smooth transfer between ships, trains, trucks, and planes. They also provide storage space to balance supply and demand and offer services like sorting and labeling. Together, these functions support large-scale, multi-region, and multi-modal transportation.

To minimize transportation costs and align the model more closely with real-world logistics networks, the original model is extended by incorporating constraints related to transshipment points. Since production and transportation processes are not static but unfold across multiple time periods, inventory can be held at transshipment points from one period to the next, referred in [5]. Therefore, a temporal dimension is introduced based on transshipment points and the original supply and demand constraints are transformed into more general and powerful flow balance constraints.

To achieve the minimum cost using the transportation model, objective function of the model is:

$$Z = \sum_{t \in T} \left( \sum_{i \in N} \sum_{j \in N} c_{ij}^t x_{ij}^t + \sum_{i \in TN} h_i^t I_i^t \right) \tag{11}$$

In the transshipment model, supply points and demand points can both send goods and receive goods, referred in [6]. Constraints of the model are set up:

$$I_i^{t-1} + \sum_j x_{ji}^t + s_i^t = \sum_j x_{ij}^t + I_i^t,\ i \in S \tag{12}$$

$$I_i^{t-1} + \sum_j x_{ji}^t - d_i^t = \sum_j x_{ij}^t + I_i^t,\ i \in D \tag{13}$$

$$I_i^{t-1} + \sum_j x_{ji}^t = \sum_j x_{ij}^t + I_i^t,\ i \in TN \tag{14}$$

$$x_{ij}^t \geq 0 \tag{15}$$

$$I_i^t \geq 0 \tag{16}$$

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Note that $T = 1, 2, \ldots, T$ is time periods set and $N$ is point set. Supply points set, demand points set and transshipment points set are $S$ , $D$ , and $TN$ respectively. These sets belong to point set $N$ . Moreover, the amount of goods produced at node i in period t is $s_i^t$ , the amount of goods used at node i in period t is $d_i^t$ , unit transportation cost from node i to j in period t is $c_{ij}^t$ , unit inventory holding cost at node i in period t is $h_i^t$ , initial inventory level at node i at the beginning of period t=1 is $I_i^0$ . Furthermore, quantity shipped from node i to node j in period t is $x_{ij}^t$ , inventory level at node i at the end of period t is $I_i^t$ .

To better align the model with real-world operations and enhance its practical accuracy, loading and unloading costs must be incorporated into the unit transportation cost, referred in [7]. Loading cost, $l_i$ , refers to the expense incurred when goods are loaded onto a vehicle at an origin point i; unloading cost, $u_j$, represents the cost associated with removing goods from a vehicle at a destination j. The formula of unit cost, $c_{ij\text{-transport}}^t$ , is:

$$c_{ij\text{-transport}}^t = c_{ij}^t + l_i + u_j \tag{17}$$

This extension changes the parameter and the constraints stay the same. The objective function is:

$$Z = \sum_{t \in T} \left( \sum_{i \in N} \sum_{j \in N} c_{ij\text{-transport}}^t x_{ij}^t + \sum_{i \in TN} h_i^t I_i^t \right) \tag{18}$$

While these added extensions make the model more complex and larger, they shape a new and more realistic feasible solution space. This optimal solution may have a higher total cost in absolute terms compared to the basic model, but it represents the true global optimum solution that can actually be achieved in the real world. Therefore, this added complexity is necessary and valuable.

### 3.2. Capacity constraints

Introducing capacity constraints into the transportation cost minimization model is a critical step to make the model more realistic. Capacity constraints are defined as upper limits on the processing capabilities of various elements within the network, such as restrictions on the maximum transportation capacity of routes and maximum loading capacity, referred in [8]. Specifically, these constraints improve the model by influencing the structure of the feasible set.

First, capacity constraints can directly exclude infeasible solutions to avoid impractical solutions. In a model without capacity constraints, the feasible set is very large because any transportation route and any quantity allocation are permitted as long as total supply meets total demand. This may lead to an unrealistic optimal solution. For example, all goods for the entire period are transferred using the single cheapest route. This approach completely ignores the actual capacity of the route in the real world. The introduction of capacity constraints ensures that all the solutions will comply with all network capacity restrictions, makes them applicable in the reality.

Furthermore, from the perspective of the mathematical properties of the feasible set, capacity constraints change the fundamental shape of the solution space. Without capacity constraints, the feasible set is unbounded in the direction of the decision variables $x_{ij}^t$ , and its geometric shape is a polyhedral cone extending infinitely outward from a vertex. After adding capacity constraints, these

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upper-bound restrictions cut the originally unbounded solution space into a bounded convex polyhedron, known as a polytope. This transformation guarantees the existence of an optimal solution and changes the location of the optimal solution. This accurately reflects the real-world decision-making logic.

According to the capacity constraints, Arc capacity from node i to j in period t, $u_{ij}^{t}$ , is added to restrict the maximum transportation capacity of routes and the constraint is:

$$0 \leq x_{ij}^{t} \leq u_{ij}^{t} \quad \forall i, j \in \mathscr{N}, \forall t \in \mathscr{T} \tag{19}$$

Moreover, storage capacity at node i, $V_i$ , is added to restrict the maximum inventory holding capacity of warehouses and the constraint is:

$$0 \leq I_{i}^{t} \leq V_{i} \quad \forall i \in \mathscr{T}\mathscr{N}, \forall t \in \mathscr{T} \tag{20}$$

In conclusion, capacity constraints are the core element that connect theory and practice in transportation optimization models. By restricting path capacity and node capacity, they ensure that solutions align with real-world limits and prevent the idealized results that are impractical.

### 3.3. Multi-modal transport

In transportation cost optimization model, using different transport methods and allowing changes between them can greatly improve efficiency and save money. In real situations, a single type of transport may not balance cost, speed, and coverage well. Optimum solution can be improved by combining truck, train, ship, or plane depending on their strengths, referred in [9]. For example, trains are good for moving large amounts of goods over long distances at low cost, while trucks are more flexible and better for local delivery. Even though changing transport mode may add some handling and management costs, it can greatly reduce the total cost of transportation and make the whole network more reliable. Therefore, mode changes in the model make the results more practical and useful for real-world logistics.

To include the mode transformation in the transportation model, another set, P, which is the points set where modes transformed is added and $P \subseteq N$ . The unit transportation cost with different modes, $c_{ij\text{-modes}}^{t}$ , is the transportation cost from node i to j using mode m in period t. The total cost per unit, $c_{ijm}^{t}$ is the sum of unit transportation cost with different modes, loading cost at original node i and unloading cost at destination j. The formulation is:

$$c_{ijm}^{t} = c_{ij\text{-modes}}^{t} + l_i + u_j \tag{21}$$

Since transportation center that transfer different modes cannot hold inventory, another constraint should be added:

$$\sum_i \sum_m x^t_{(ipm)} = \sum_j \sum_m x^t_{(pjm)} \tag{22}$$

Note that $x^t_{(ipm)}$ is the quantity shipped in from node i to transportation center p in period t, $x^t_{(pjm)}$ is the quantity shipped out from transportation center p to node i in period t

The final multi-modal transportation model is, referred in [10]:

$$Min.Z = \sum_{t \in T} \left( \sum_{i \in N} \sum_{j \in N} \sum_{m \in M} c^t_{ijm} x^t_{ijm} + \sum_{i \in TN} h^t_i I^t_i \right) \tag{23}$$

$$I^{t-1}_i + \sum_j x^t_{ji} + s^t_i = \sum_j x^t_{ij} + I^t_i,\ i \in S \tag{24}$$

$$I^{t-1}_i + \sum_j x^t_{ji} - d^t_i = \sum_j x^t_{ij} + I^t_i,\ i \in D \tag{25}$$

$$I^{t-1}_i + \sum_j x^t_{ji} = \sum_j x^t_{ij} + I^t_i,\ i \in TN \tag{26}$$

$$0 \le x^t_{ijm} \le u^t_{ijm}, \quad \forall i, j \in \mathscr{N}, m \in \mathscr{M}, t \in \mathscr{T} \tag{27}$$

$$0 \le I^t_i \le V_i \quad \forall i \in \mathscr{TN}, \forall t \in \mathscr{T} \tag{28}$$

$$\sum_i \sum_m x^t_{(ipm)} = \sum_j \sum_m x^t_{(pjm)} \tag{29}$$

In conclusion, the multi-modal transportation model significantly improves decision-making by adding different unit cost and capacity limits for different transport methods. This allows the model to accurately compare and choose the best combination of transfer options.

### 3.4. Uncertainty

In the transportation model, all parameters are assumed to be certain. However, unexpected changes should be considered as well, such as fluctuations in customer demand and delays in the loading process. If plans are made only based on fixed values, the solutions will be fragile. Even a small disruption can cause the entire plan to fail, leading to high costs such as penalties for late deliveries, wasted resources, and loss of customers. Therefore, considering uncertainty is important to build strong and efficient transportation strategies.

One solution to deal with uncertainty is stochastic programming. This program involves two-stage decisions, referred in [11]. The first stage decisions are made before the uncertainty is resolved. Second-stage decisions are made after the uncertainty is resolved, which act as a corrective measure. The overall objective is to minimize the total cost of the first-stage decisions plus the expected cost of the second-stage recourse actions.

The objective function is, referred in [12]:

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$$\min. \sum_{S\in\mathscr{S}} w_S^{\mathrm{I}} x_S + \sum_{A\subseteq U} \sum_{S\in\mathscr{S}} p_A w_S^{\mathrm{II}} r_{A,S} \tag{30}$$

Note that $w_S^{\mathrm{I}}$ is the cost to select $S$ in the first stage, and $x_S$ is the decision variable representing the fractional amount of set $S$ selected upfront. Moreover, $\sum_{A\subseteq U} \sum_{S\in\mathscr{S}} p_A w_S^{\mathrm{II}} r_{A,S}$ is the expected second stage cost, which is the sum of over all possible scenarios $A$. Note that $p_A$ is the probability that scenario $A$ occurs. $w_S^{\mathrm{II}}$ is the scenario-dependent cost to select set $S$ after the scenario is known. $r\_\{A,S\}$ is the decision variable for how much of set $S$ to select in the second stage if scenario $A$ occurs.

The constraints of the stochastic programming are:

$$\sum_{S:e\in S} x_S + \sum_{S:e\in S} r_{A,S} \geq 1 \quad \forall A \subseteq U, \forall e \in A \tag{31}$$

$$x_S, r_{A,S} \geq 0 \quad \forall S \in \mathscr{S}, \forall A \subseteq U \tag{32}$$

In conclusion, stochastic programming offers a theoretical foundation that makes full use of historical data to obtain optimal solutions in the expected sense. However, it suffers from high computational complexity. As the number of scenarios increases, the model scales explosively. Moreover, this method relies heavily on accurate probability distributions of uncertain parameters, which can be difficult to obtain in practice.

## 4. Sensitivity analysis in transportation models

### 4.1. Shadow price

As a key indicator reflecting the marginal value of constraint conditions in linear programming, shadow price is mainly used in transportation models to quantify the effect of resource scarcity on the optimal solution of transportation schemes. It indicates the change in the optimal total cost if a single unit of supply or demand constraint is relaxed. Specifically, shadow price represents the marginal value of an additional unit of a resource or the cost of not meeting one more unit of demand. In the process of calculating shadow prices, we need to define a dual price $u_i$ for each supply constraint and a dual price $v_j$ for each demand constraint. To compute these $u_i$ and $v_j$, the cost coefficient must be equal to $(u_i + v_j)$ for each basic cell. Then, one can solve the system of equations for all remaining $u_i$ and $v_j$ values by setting any one $u_i = 0$ or $v_j = 0$. Every nonbasic cell is associated with the net evaluation index, $c_{ij} - (u_i + v_j)$, which reflects the incremental change in the overall cost that will occur when one unit of flow is allocated to the corresponding cell. For an optimal solution, we use the term shadow price for $(u_i + v_j)$ values and present them as a shadow price matrix. Evidently, for each nonbasic cell, its shadow prices are less than the corresponding cost coefficients [13].

In addressing transportation problems, shadow price improves the precision of cost-benefit trade-offs, a decision-making process where decision-makers weigh the potential cost of an action against its expected benefits to determine whether the choice is worthwhile. Additionally, by understanding

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the marginal cost of changing constraints, companies can make more informed decisions about where to allocate resources to lower transportation costs. For instance, if the shadow price of transportation capacity on a main highway is high, it indicates that the current route has become a bottleneck in the transportation network. At this point, investing funds to expand the road or add freight lines will produce greater cost-saving benefits than the investment cost does. On the contrary, if the shadow price is relatively low, it means that the route has redundant transportation capacity. Therefore, no additional investment is needed, and resources can be redirected to other constrained links with higher shadow prices [14].

### 4.2. Practical application of sensitivity analysis in transportation problems

The core parameters of transportation systems (such as transportation cost, time, and supply-demand volume) are susceptible to fluctuations due to external factors. For example, the rise of fuel price can lead to an increase in the unit cost of road transportation; extreme weather can extend transportation period; and urgent orders may increase the demand for materials. Sensitivity analysis is then an essential tool to address such fluctuations, and its practical applications can be divided into three main scenarios.

#### 4.2.1. Risk prediction

Sensitivity analysis can eliminate the uncertainty about transportation networks by modeling operational risks and allocation decisions. First, analysts use historical data (such as freight volumes, route costs, or delivery time records) and foreseeable assumptions to build a base case. It is crucial for the company to identify and acknowledge the most anticipated result of every decision. Typically, this refers to the most conservative outcome, in which no catastrophic or amazing events occur. The next step is to determine the input and output variables for the transportation system that are important to the company, such as transportation costs and vehicle capacity. Then, to test all the variables, analysts use spreadsheets to build financial models and use “what-if” analysis to determine the impact of each variable in every outcome. By figuring out how changes in these variables affect the system’s outputs, analysts can determine the significance of each variable to the transportation model. To assess whether investments in transportation optimizations will bring desired returns, analysts adopt the formula NPV = (Cash Flow / (1 + Required Return)^t) – Initial Investment. Here, “cash flow” refers to annual cost savings from optimized routes or increased delivery revenue, and “initial investment” covers expenses like road expansion or fleet purchases. If the result of the NPV calculation is positive, the transportation investment will yield the desired returns; otherwise, it will not. As a result, when companies analyze a broad range of variables, they can better visualize future outcomes for their decisions. This simplifies and optimizes decision-making related to capital budgeting and transportation strategies [15].

#### 4.2.2. Scheme optimization and adjustment

Sensitivity analysis provides data basis for the dynamic optimization of transportation schemes. When parameters deviate from expected values, it can be quickly adjusted without rerunning the entire linear programming model. Its core value lies in identifying the adjustment threshold of key parameters, that is, the maximum fluctuation range that the current optimal scheme (such as route selection, vehicle allocation, supply and demand matching, etc.) remains valid, and the optimal adjustment direction once the threshold is exceeded [16].

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#### 4.2.3. Resource allocation

Sensitivity analysis optimizes the efficiency of limited resources by linking resource input to marginal improvements in transportation system. It calculates the marginal benefit coefficient of each resource — the degree to which increasing one unit of resource input improves key performance indicators (such as cost reduction, delivery time shortening, demand satisfaction rate, etc.). Based on the analysis, decision-makers can prioritize resources [17]. For example, in a transportation network, investing in an additional freight vehicle can increase distribution efficiency by 8% and reduce transportation costs by 5%, with a high marginal benefit coefficient. Adding one storage point can only increase the demand satisfaction rate by 3%. Accordingly, policymakers prioritize resources for additional freight vehicles. For storage resource investment with low marginal benefit coefficient, it can be suspended or reduced, and the saved resources can be invested in other links with higher marginal benefits.

## 5. Methods and computational tools

### 5.1. Applied mathematical modeling

This study builds upon the multi-period transportation-inventory model presented in the previous article as its starting point. Stepping away from the classic single-period perspective, we introduce time as a key variable into the model, which makes decision-making three-dimensional: not only must we consider "how to transport," but we must also plan "when to store." In reality, supply chain demand, inventory accumulation, and transportation capacity are all dynamic characteristics that change over time. All of these characteristics are reflected in the model.

Although this model introduces a time dimension, its core remains linear in nature. The objective function and constraints maintain the characteristics of linear functions. Therefore, we can continue to use methods for solving linear programming problems.

In this approach, we ensure both the transparency and operability of the model, while also addressing several potential issues that may arise during subsequent algorithm verification.

### 5.2. Constraint formulation

Designing constraints is a key step in mapping real-world operational logic into mathematical form. For example, imagine we are building a digital twin for a supply chain. Constraints are the "laws" that this world must adhere to. The first law is flowing conservation—each logistics node is like a precisely measured miniature hub, with the exact amount of cargo entering and leaving. However, the real world does not have infinite resources, so the second law assigns a clear "capacity label" to each transport line and warehouse. Inventory constraints are like a magician, allowing goods to traverse time periods while also recording the costs of each stockout. Within the complex dance of multimodal transport, we have designed additional "transfer rules" to prevent certain transit stations from becoming hotels where goods can be stranded for extended periods. In situations with uncertainty, scenario decomposition can be used to separate "ex-ante" decisions from "ex-post" adjustments, thereby capturing the system's response to external fluctuations. These constraints are not just theoretical formulas; they represent the model's mapping of real-world operating procedures.

### 5.3. Sensitivity analysis

In any context, constraints are the context of the problem being solved.

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For example, suppose we are building a digital twin model of a supply chain.

Constraints are the "laws" of this world. The first law is the conservation of flow—each logistics node acts as a precisely measured miniature hub, with the quantity of goods entering and leaving accurately. However, the world's resources are not infinite, so the second law is capacity, which means assigning a clear "capacity tag" to each transport route and warehouse.

This "capacity tag," or capacity constraint, acts like a magician, allowing goods to traverse time periods while recording the cost of each shortage. In such complex multimodal transport, we design additional "transfer rules" to prevent goods from being stranded for extended periods at specific transit points.

In situations of uncertainty, scenario decomposition can be used to distinguish between "pre-emptive" decisions and "post-emotive" adjustments, thereby capturing the system's response to external changes.

These constraints are not merely theoretical formulas; they represent a mirror image of the real-world operational processes within the model.

## 5.4. Implementation workflow

To effectively transform the theoretical model into an operational tool, we designed an implementation process. Due to the inherent multi-period dynamics and uncertainties of the model, this study employs multiple methods to enhance its rationality and practicality in real-world applications.

Specifically, this can be summarized in the following four steps:

1. Data Preparation and Standardization: Comprehensive collection of key information, including supply data, cost parameters, and capacity constraints, followed by unified dimensional normalization to establish a data foundation for subsequent model development.
2. Model Construction and Solution: Based on the theoretical framework, objective functions and constraints are set in the solver, and appropriate computational parameters are configured to ensure the stability of the solution process and the reliability of the results.
3. Result Extraction and Visualization: Based on the numerical solution, graphical tables are used to visualize the transportation route flow, node inventory levels, and utilization rates in the logistics network. This clearly demonstrates potential bottlenecks and resource redundancy in the system, providing an intuitive basis for analysis.
4. Sensitivity Analysis: By systematically adjusting key parameters, examining the changes in model output under different scenarios, and assessing its robustness, we ensure the relevance and reliability of the research results.

This method retains academic rigor while being integrated into the company's actual logistics planning process, thereby forming a complete closed loop from model building to solution calculation, ultimately informing business decisions.

## 6. Conclusion

This study approaches the problem of transportation in the context of minimum-cost flow and extends it with feasibility and resilience-determining features in modern logistics. By linking spatial allocation with intertemporal buffering, the framework incorporates inventory and handling costs in a transshipment model, showing how unequal access to storage absorbs variability and reshapes effective unit costs. As the linking of the facility with the process introduces capacity constraints on arcs and facilities, an unconstrained allocation problem is converted into a resource-balanced plan

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that meets physical constraints on and commitments to service. A heterogeneous reality of freight movement (cost, speed, reliability) is ultimately condensed into one coherent network by means of various multimodal routing and transfer rules. Finally, uncertainty is handled through a two-stage structure that decouples anticipative commitments from adaptive recourse, producing solutions that remain robust to shocks without sacrificing efficiency.

The diagnostic layer which is linked to the optimization is also of equal importance. Shadow prices identify the highest-leverage expansions across arcs and facilities, parametric analysis delineates the stability ranges of recommended plans, and scenario analysis prescribes directionally correct adjustments to the mix of capacity, modes, and inventory when conditions vary. These tools may be used to map the results of mathematical models into actionable suggestions about the best combination of capacity, modes, and inventory to be used. Considering the fact that the diagnostics is local and interpretable, it becomes possible to support the prompt corrections of courses in case of the fuel prices, service times or demand profiles change.

Methodologically, the framework remains consistent with precise linear optimization and heuristics can be scaled up to nonlinearity or dimensionality in case of more dimensions. This balance can be interpreted, expeditiously make decisions in an environment, and be modular in its execution. The organizations can use the baseline model and add multimodal and capacity layers before introducing uncertainty and sensitivity diagnostics when the data is mature and a budget is free to run calculations. Practically, the methodology is more of a planning science: it sheds light on tradeoffs, exposes bottlenecks, bridges targets and levers including terminal capacity, fleet mix and inventory buffers. It hence bridges tactical dispatch and strategic design and can be able to make consistent decisions at any given horizon.

Several limitations point to the directions for deepening the contribution. The present abstraction does not endogenize demand, congestion formation and learning effects in lead times, as well as considering compliance, contracts and emissions as exogenous constraints and not strategic choices. Decomposition schemes for multi-period stochastic instances could be added to the framework to allow finer-scale temporal details. Distributionally robust variants could relax assumptions on the uncertainty, without losing tractability, and tighter integration with forecasting systems would allow for alignment of the planning process with signals on an up-to-date basis. Incorporating service-level guarantees and environmental objectives as constraints would be a further step in making the policy relevant and accountable.

In sum, the paper is a serious and operationally credible template for network design and planning. By combining realistic constraints with a clear economic interpretation of flows - and combining optimization with sensitivity-based diagnostics - it provides principled benchmark of research and a worthwhile framework for practitioners in managing complex, uncertainty-laden, capacity-limited, multi-modal systems.

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