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Numerical Simulation of Electric Potential Distribution in Capacitor Structure Using the Finite Difference Method

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Abstract. This research used numerical simulation based on Finite Difference Method to investigate the arrangement of electrode in parallel capacitor. A python-based solver has used NumPy to do high efficiency operations. Using SciPy to build sparse matrix and solve the linear system, by using Matplotlib to visualize codes. The Laplacian operator is discretized on a two-dimensional grid and apply boundary conditions to simulate Neumann and Dirichlet these two situations. This solver can calculate electric potential field from charge source and indicate corresponding electric field component. The results are presented by contour maps, vector field visualizations and three-dimensional surface representations, representing the variations of potential and electric field in the domain. The modularization of codes is helpful to change the grid resolution, the boundary conditions and domain size, in order to expand into different geometry shapes. This work indicates that numerical simulation provided an effective way to analyze complex electrostatic configuration. This method also can further applicated to optimize capacitors' designs and other relevant electronic devices.

Keywords: Finite Difference Method, Parallel capacitor, Numerical simulation, Electric potential and field, Boundary conditions

1. Introduction

The structure of capacitors are basic components in electronic engineering, playing an important role in energy storage, signal filtering, voltage adjustments. Its efficiency is closely related to geometrical structure, material properties and the way that electrodes are arranged. These factors all determined the electrical distribution inside the device. Therefore, precise modeling and analyzing electrical distribution are essential for optimize capacitance, lower the energy loss to ensure devices' reliability. In the process of modeling, the Poisson equation provided some mathematical basics to describe the relationship between the change in electrical potential as the space changes and the charge distributions[1]. By solving the equation in proper boundary conditions, engineers can predict parameter, such as medium thickness, material components and electrode shape. Even though in simple geometry structures can get analytical solution, but actual design in capacitance are more complex and have irregular boundary conditions. This need advance numerical simulation method to get a more precise result[2].

In lots of different methods to solve Poisson equation, we choose Finite Difference Method, because it is very suitable for our research about the 2D capacitance model. Finite Difference Method

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convert differential equation into a algebraic equation system, through change the continuous derivatives into segregate finite difference method approximation[3]. This method can handle regular geometry structure very directly, and can integrate common boundary conditions into segregation equation, such as Dirichlet condition and Neumann condition. Besides that, coefficient matrix that get from Finite Difference Method have sparse character, in order to lower the calculation cost. Based on these characters, Finite Difference Method provide a high efficiency and practical choice[4].

Even though some research has used analysis or numerical simulation to solve Poisson equation, and extend it into devices design[5][6], there are still some deficiencies. From one perspective, the perspective of numerical simulation in traditional structured capacitance' electric potential distribution, there are very few public references, such as numerical integration is still needed in complex context for MOS capacitor structures. In these kinds of circumstances, analytical method can provide simple result, but when handling complex structure or irregular boundaries, numerical simulation are more adaptable and precise.

This research aims to analyze under different geometry conditions, the changes of internal electric potential in a parallel plate capacitor. This paper will focus on the distribution of equipotential lines as the research object. By analyzing it's shape and change movement, to discuss the relationship between geometry structure and electric field distribution. Therefore, we choose the numerical simulation method, on the 2D grid to solve Poisson equation to get parallel capacitor's different electric distribution in different boundary conditions. By simulating same electric potential's field line, we can verify the accuracy of numerical method, and based on this, the distribution of equipotential lines and electric field lines under different plate shapes are visualized and analyzed. Using this visualized process as analytical method, it will help to reveal the influence on electric field line distribution by the changes of geometry.

2. Theoretical Background

2.1. Basics of Electrostatics and Potential Theory

Electrostatics describes the relationship between electric fields and stationary charge distributions. In the context of this study, it provides the physical principles to express Poisson's equation, the equation of electric potential due to the presence of charge. For a continuous charge density $\rho(r)$, $V(r)$satisfies the equation

$$\nabla^2 V = -\frac{\rho(r)}{\varepsilon_0} \tag{1}$$

where ε_0 is the permittivity of free space. In charge free regions, this function can simplify to Laplace's equation,

$$\nabla^2 V = 0 \tag{2}$$

The electric field E can be expressed by:

$$E = -\nabla V \tag{3}$$

This function can support the different situations of field distributions when the electrical potential is known. The uniqueness theorem stated that, under certain area which electrical potential V satisfies the Poisson equation or Laplace equation, and in the boundary of this region, the electric potential is given, or the component of electric field is given, then the potential distribution can only have one solution in this specific region. This concept is important for the modeling process, because it ensures the calculated solution matches the actual physics of the system. Potential theory worked

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as a mathematical basis to analyze these equations. In electrostatics, the harmonic functions have some properties, such as the mean value property and the maximum principle, that help to shape the development of calculation methods and the way results that are understand. For example, in the capacitance problems, the region between conductors can satisfy Laplace's equation, but regions that have free charge needs to solve the full Poisson's equation. In real life applications such as the capacitor modeling, analytical solutions can only work for simple geometries. However, in the circumstances of complex shapes, particularly when boundaries are irregular or charges are irregularly distributed, numerical methods are needed to solve the solution.

2.2. Derivation of Laplace's and Poisson's Equations

The electrostatic behavior of a system is fundamentally governed by Poisson's and Laplace's equations, which relate spatial variations in electric potential to charge distributions. These equations are direct consequences of Gauss's law—one of the four Maxwell's equations—which, in differential form, asserts a relationship between the electric field E and the charge density $\rho(r)$:

$$\nabla \cdot E = \frac{\rho(r)}{\varepsilon_0}, \tag{4}$$

where ϵ_0 denotes the permittivity of free space. Since the electric field E can be written as the negative gradient of a scalar potential $\phi(r)$,i.e.,

$$E = -\nabla\phi\left(r\right), \tag{5}$$

we substitute into Gauss's law to obtain:

$$\nabla \cdot -\nabla\phi(r) = \frac{\rho(r)}{\varepsilon_0}, \tag{6}$$

This simplifies to the classic form of Poisson's equation:

$$\nabla^2\phi\left(r\right) = -\frac{\rho\left(r\right)}{\epsilon_0} \tag{7}$$

Poisson's equation thus describes how the electric potential responds to the presence of electric charges. In regions absent of free charges, i.e., where $\rho\left(r\right) = 0$, the equation reduces to Laplace's equation:

$$\nabla^2\phi\left(r\right) = 0 \tag{8}$$

This special case applies, for example, within ideal conductors under electrostatic conditions or throughout charge-free dielectric media. In the context of a parallel-plate capacitor, a uniform charge density is typically assumed within the inter-electrode gap. Under this condition, Poisson's equation takes a simplified form with a constant source term. Boundary conditions—whether Dirichlet (fixed voltage on plates) or Neumann (specified electric field)—are applied to close the system. Beyond the plates, where $\rho\left(r\right) = 0$, Laplace's equation prevails. This mathematical framework is essential for predicting the spatial variation of both potential and electric field within capacitive systems, serving as the theoretical basis for the finite difference approach adopted in this study.

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2.3. Boundary Conditions and Uniqueness Theorem

As the Poisson's equation provide us with core differential relationship to solve electric potential of the field, the equation alone cannot uniquely determine the solution to a certain physical problem. The equation describes local behavior, but the global solution depends critically on domain boundary conditions. This section discusses key boundary condition types and introduces the Uniqueness Theorem, which ensures solution existence and uniqueness under appropriate boundary conditions and provides the foundation for numerical methods. The boundary conditions are conventionally categorized into three fundamental types: Dirichlet conditions, which gives a certain value at a certain boundary (eg:$u_{x=a} = b$) Neumann conditions, which provides a derivative value of a specific direction and boundary (eg: $\frac{\partial u}{\partial x}_{x=a} = b$) Robin (mixed) conditions, which prescribe a linear combination of the field variable and its normal derivative on the boundary (eg: $\frac{\partial u}{\partial r}_{r=a} + u_{r=a} = b$)

Taking a simple parallel plate capacitor as an example to show how these types of boundary conditions be used in our numerical simulation, which consists of two conducting plates separated by an insulting region. Under this certain physical circumstance, the type of boundary conditions that applies to both upper and bottom plates is naturally the Dirichlet conditions, describing the voltage applied to the capacitor, and the Neumann conditions are applied to the left and right edge because they physically means the tangential component of the electric field strength is zero. If we take the edge effect into consideration, the mixed condition will be applied to all boundary the parallel plate capacitor has. The specification of these boundary condition types is not arbitrary—it crucially determines the practicality of the boundary value problem, as formalized by the uniqueness theorem, which is described as following: The potential in a region is uniquely fixed if (a) the charge density is given everywhere; (b) the value of V or its normal derivative is specified on all boundaries. Where V is the voltage in this region. The uniqueness theorem provides the mathematical foundation for well-posed electrostatic field solutions, guaranteeing that Poisson's (or Laplace's) equation admits a unique physical solution under consistent boundary constraints. This theoretical assurance is fundamental to numerical stability, ensuring that discretized approximations converge to the correct potential distribution. In the following section, we will leverage this principle to construct a discrete framework for solving the boundary value problem via the finite difference method.

3. Methodology

3.1. Finite Difference Method for Poisson's equation

Our project mainly focuses on the potential distribution of parallel capacitor in two dimensions. The regular geometry structure and small scale makes FDM the superior numerical method for our project.

FDM is a numerical method, it discretizes the continuous space into finite grid and iterates depending on boundary condition until the result met the tolerance. For Poisson's equation, the continuous form is:

$$\begin{cases} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -f(x,y) = -\frac{\rho_{i,j}}{\epsilon_0}, (x,y) \in \Omega \\ u(x,y) = \varphi(x,y), (x,y) \in \Gamma \end{cases} \tag{9}$$

where Ω is the space and Γ is the boundary. To make it appliable for FDM, Poisson's Equation must be written as discretized form. If the grid is m$\times$ n in size, then the equation would be:

$$\begin{cases} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)\Big|_{(x_i,y_i)} = -f\left(x_i, y_i\right) = -\frac{\rho_{i,j}}{\epsilon_0}, \left(x_i, y_i\right) \in \Omega \\ u\left(x_s, y_t\right) = \varphi\left(x_s, y_t\right), \left(x_s, y_t\right) \in \Gamma \end{cases}, \tag{10}$$

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where 1≪ i≪ m-1, 1≪ j≪ n-1, s and t are line and column indices of boundary dots. Keeping simplifying by replace derivative with difference and ignore the higher order term, we can get:

$$\begin{cases} \frac{u_{i+1,j}+u_{i-1,j}-2u_{i,j}}{\Delta x^2} + \frac{u_{i,j+1}+u_{i,j-1}-2u_{i,j}}{\Delta y^2} = -f\left(x_i, y_i\right) = -\frac{\rho_{i,j}}{\epsilon_0} \\ u\left(x_s, y_t\right) = \varphi\left(x_s, y_t\right), \left(x_s, y_t\right) \in \varGamma \end{cases} \tag{11}$$

Each calculation of above equations would include 5 points. All equations would form a matrix, but the matrix can't be written as a simple form of Ax=b, it can only be expressed as:

$$\begin{aligned} &-\frac{1}{\Delta y^2}\begin{pmatrix} 1 & \dots & 0 \\ & 1 \ddots & \vdots \\ \vdots & & 1 \\ & 0 \dots & 1 \end{pmatrix}\begin{pmatrix} u_{1,j-1} \\ u_{2,j-1} \\ \vdots \\ u_{m-2,j-1} \\ u_{m-1,j-1} \end{pmatrix} - \frac{1}{\Delta y^2}\begin{pmatrix} 1 & & & \\ & 1 & 0 & \\ & & \ddots & \vdots \\ & & \dots & 1 \end{pmatrix}\begin{pmatrix} u_{1,j+1} \\ u_{2,j+1} \\ \vdots \\ u_{m-2,j+1} \\ u_{m-1,j+1} \end{pmatrix} \\ &-\begin{pmatrix} 2\left(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}\right) & -\frac{1}{\Delta x^2} & & & \\ -\frac{1}{\Delta x^2} & 2\left(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}\right) \dots & & 0 & \\ \vdots & & \ddots & & \\ 0 & & \dots 2\left(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}\right) & -\frac{1}{\Delta x^2} & \\ & & -\frac{1}{\Delta x^2} & 2\left(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}\right) & \end{pmatrix}\begin{pmatrix} u_{1,j} \\ u_{2,j} \\ \vdots \\ u_{m-2,j} \\ u_{m-1,j} \end{pmatrix} \\ &= \begin{pmatrix} f\left(x_1, y_j\right) + \frac{1}{\Delta x} u_{0,j} \\ f\left(x_2, y_j\right) \\ \vdots \\ f\left(x_{m-2}, y_j\right) \\ f\left(x_{m-1}, y_j\right) + \frac{1}{\Delta x} u_{m,j} \end{pmatrix} \end{aligned} \tag{12}$$

by substituting the variables the matrix equations can be simplified to a great extent, take:

$$\begin{aligned} &U_j = \left(u_{1,j}, u_{2,j}, \dots, u_{m-1,j}\right)^T, 0 \ll j \ll n, \\ &2\left(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}\right) = \alpha, \\ &\frac{1}{\Delta x^2} = \beta \\ &\frac{1}{\Delta y^2} = \gamma \end{aligned} \tag{13}$$

which can be expressed as:

$$A\left(U_{i-1}+U_{i+1}\right) + BU_i = F_j, 1 \ll j \ll n-1, \tag{14}$$

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then the matrix equation can be simplified as:

$$\begin{pmatrix} -\gamma & & \dots & 0 \\ & -\gamma & & \\ \vdots & & \ddots & \vdots \\ 0 & & \dots & -\gamma \\ & & & -\gamma \end{pmatrix} \begin{pmatrix} u_{1,j-1} \\ u_{2,j-1} \\ \vdots \\ u_{m-2,j-1} \\ u_{m-1,j-1} \end{pmatrix} + \begin{pmatrix} \alpha & -\beta & \dots & & 0 \\ -\beta & \alpha & & & \\ & \vdots & & \ddots & \\ & & & \alpha & \\ 0 & & \cdots & \alpha & -\beta \\ & & & -\beta & \alpha \end{pmatrix} \begin{pmatrix} u_{1,j} \\ u_{2,j} \\ \vdots \\ u_{m-2,j} \\ u_{m-1,j} \end{pmatrix}$$
$$+ \begin{pmatrix} -\gamma & & & \\ & -\gamma \cdots & & 0 \\ & \vdots & \ddots & \\ 0 & & \cdots -\gamma & \\ & & & -\gamma \end{pmatrix} \begin{pmatrix} u_{1,j+1} \\ u_{2,j+1} \\ \vdots \\ u_{m-2,j+1} \\ u_{m-1,j+1} \end{pmatrix} = - \begin{pmatrix} f(x_1, y_j) + \frac{1}{\Delta x} u_{0,j} \\ f(x_2, y_j) \\ \vdots \\ f(x_{m-2}, y_j) \\ f(m-1, y_j) + \frac{1}{\Delta x} u_{m,j} \end{pmatrix}, 1 \ll j \ll n-1 \tag{15}$$

where:

$$A = -\gamma I, I \text{ is an identity matrix of } n-1 \text{ order}$$
$$B = \begin{pmatrix} \alpha & -\beta & & \\ -\beta & \alpha & \cdots & 0 \\ & \vdots & \ddots & \\ & & \vdots & \\ & & & -\beta \\ & & \alpha & \end{pmatrix}$$
$$F_j = \begin{pmatrix} f(x_1, y_j) + \frac{1}{\Delta x} u_{0,j} \\ f(x_2, y_j) \\ \vdots \\ f(x_{m-2}, y_j) \\ f(m-1, y_j) + \frac{1}{\Delta x} u_{m,j} \end{pmatrix} \tag{16}$$

eventually, the matrix can be simplified as:

$$\begin{pmatrix} B & A & & \\ A & B \cdots & & \\ \vdots & & \ddots & \\ & & \vdots & \\ 0 & & \cdots & B \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \\ \vdots \\ U_{n-2} \\ U_{n-1} \end{pmatrix} = \begin{pmatrix} F_1 - AU_1 \\ F_2 \\ \vdots \\ F_{n-2} \\ F_{n-1} - AU_n \end{pmatrix} \tag{17}$$

The characteristics of above equation are as follows: the coefficient matrix is symmetric, positive definite, and the vast majority of them are zero elements. Each row has at most five non-zero elements, making it a sparse matrix. For solving linear equations of low order, the direct method is very effective. However, for linear equations of high order and sparse coefficient matrices, the direct method would cause unnecessary waste since a large number of zero elements need to be stored. To reduce the operation times and save memory, iteration is a better approach. Gauss Seidel method refined from Jacobi method by making use of the newly calculated components for each iteration, which contributes to its less iteration times and higher accuracy. It was applied in our project because

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of its high accuracy and efficiency. The deduction are as followed: for any linear equations:

$$\begin{array}{c} a_{11}x_1 - a_{12}x_2 - \cdots - a_{1n}x_n = b_1 \\ a_{21}x_1 - a_{22}x_2 - \cdots - a_{2n}x_n = b_2 \\ \cdots \\ a_{n1}x_1 - a_{n2}x_2 - \cdots - a_{nn}x_n = b_n \end{array} \tag{18}$$

if the matrix is not single and $a_{i,i} \neq 0$, $i = 1, 2, \ldots, n$, then it can be rewrite as:

$$\begin{array}{l} x_1 = \frac{1}{a_{11}} \left(b_1 - a_{12}x_2 - \cdots - a_{1n}x_n \right) \\ x_2 = \frac{1}{a_{22}} \left(b_2 - a_{21}x_1 - \cdots - a_{1n}x_n \right) \\ x_n = \frac{1}{a_{nn}} \left(b_n - a_{n1}x_1 - \cdots - a_{nn-1}x_{n-1} \right) \end{array} \tag{19}$$

the Gauss Seidel iteration can be expressed as:

$$\begin{array}{l} x_1^{(k+1)} = \frac{1}{a_{11}} \left(b_1 - a_{12}x_2^{(k)} - \cdots - a_{1n}x_n^{(k)} \right) \\ x_2^{(k+1)} = \frac{1}{a_{22}} \left(b_2 - a_{21}x_1^{(k+1)} - \cdots - a_{1n}x_n^{(k)} \right) \\ x_n^{(k+1)} = \frac{1}{a_{nn}} \left(b_n - a_{n1}x_1^{(k+1)} - \cdots - a_{nn-1}x_{n-1}^{(k+1)} \right) \end{array} \tag{20}$$

in a simpler way, it can be expressed as:

$$x_i^{(k+1)} = \frac{1}{a_{ii}} \left(b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij}x_j^{(k)} \right), i = 1, 2, \ldots, n \tag{21}$$

substitute all coefficients in (9) by matrix in (6), the Gauss Seidel iteration of (6) is:

$$\begin{array}{c} u_{i,j}^{(k+1)} = \frac{1}{\alpha} \left[f\left(x_i, y_i\right) + \beta u_{i-1,j}^{(k+1)} + \gamma u_{i,j-1}^{(k+1)} + \beta u_{i+1,j}^{(k+1)} + \gamma u_{i,j+1}^{(k+1)} \right] \\ u_{i,j}^{(k+1)} = \frac{1}{2\left(\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2}\right)} \left[f\left(x_i, y_i\right) + \frac{1}{(\Delta x)^2} u_{i-1,j}^{(k+1)} + \frac{1}{(\Delta y)^2} u_{i,j-1}^{(k+1)} + \frac{1}{(\Delta x)^2} u_{i+1,j}^{(k+1)} + \frac{1}{(\Delta y)^2} u_{i,j+1}^{(k+1)} \right] \end{array} \tag{22}$$

in our project, we specified it to:

$$u_{i,j}^{(k+1)} = \frac{1}{4} \left[u_{i-1,j}^{(k+1)} + u_{i,j-1}^{(k+1)} + u_{i+1,j}^{(k+1)} + u_{i,j+1}^{(k+1)} \right] \tag{23}$$

3.2. Model Geometry, Grid Design and Boundary Implement

Three dimension capacitor is simplified in to two dimension in order to reduce the computational complexity and improve efficiency while still capturing the essential physics of the problem. This simplification is premised on the core assumption that the electrode plate extends infinitely in z direction, resulting in translational symmetry and, consequently, a potential distribution ϕ that is independent of the z-coordinate.

Second, we take rectangular grid to solve this problem in our code as the plates in two dimension is rectangular and lower the difficulty of calculation. Specifically, we set both of the width and height of interval to be 20, the plate length to be 20, and the plate gap to be 10, then we used the linspace and meshgrid function in NumPy library to create the grid whose size is 40×40.

The third step is to apply our boundary conditions. In order to achieve this goal, we need to get the indices of dots on the plates. The upper plate y will be 0 + half of the plate gap, which will 5 in

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total, and the lower position is minus 5. After this, we programming to get the line indices. We used where and isclose function in NumPy to pick out elements in Y whose value lies between 4.5 and 5.5, because the values of matrix are not integral. Y is a matrix so the output of code would be two indices arrays of line and column, which is [24,24. . . 24],[1,2,. . . ,39] in this case. The lower plate indices run the same way. Then we used a simple comparison to pick out dots in x axis. Now we have both ine and column indices and can apply the potential to these dots. We set the upper plate potential to be 1 and the lower to be -1.

At last, we choose iteration way to find solutions to the capacitor problem. For all the dots in the grid except those on the plates, we apply the FDM to iterate. And we chose the Gauss-Seidel method because it is more efficient in solving such problem.

3.3. Programming Language, Libraries and Code Structure

In this research, the numerical simulation solver is achieved by using Python, applying Python because its grammar is simple, has strong community support and abundant scientific calculation system. Our three main libraries are: NumPy, used for array operations and construction of grid, SciPy is used to construct sparse matrix and solve linear system, Matplotlib is used to visualize results. These tools make sure the high-efficient calculations and represents data clearly.

Within the help of Finite Difference Method in the working process, in order to improve readability and reusability. Firstly, solver defines spatial discretization and generate calculation grid, then building two dimensional Laplace by sparse format. Boundary conditions (No matter the Neumann or Dirichlet) were applicated in updated matrix. The linear system that generated by discretized Poisson equation will use sparse linear algebra to solve electric potential field. Through the calculation in electric potential and use contour maps to visualize the result. This method allowed to change the boundary conditions, grid resolution ratio and domain size.

The full Python implementation and detailed function definitions are provided in the Appendix.

4. Result

4.1. Simulation Results

We conducted 2 groups of numerical simulations. For the first group the ratio of plate length and the plate gap is taken as variable, and the rotated angles is taken as variable for the second group. The ratio of plate length and plate gap is a vital quantum to measure whether the capacitor can be viewed as idea capacitor. It can be shown clearly how this approximation becomes increasingly reasonable as b range from 0.5 to 5.

We define an electric field to be a uniform field if the change of the field density is within 10%. According to the result, the proportion of uniform field between plates are 18%, 46%, 82% and 98% for (a), (b), (c) and (d) respectively, indicating that the capacitor can be seen as ideal when b is larger then 5. In another aspect, the potential line per unit length in the uniform field area for (a), (b), (c) and (d) are 1.1, 1.3, 2.5, 6.2 respectively, indicating the increase of the electric field density as b grows larger.

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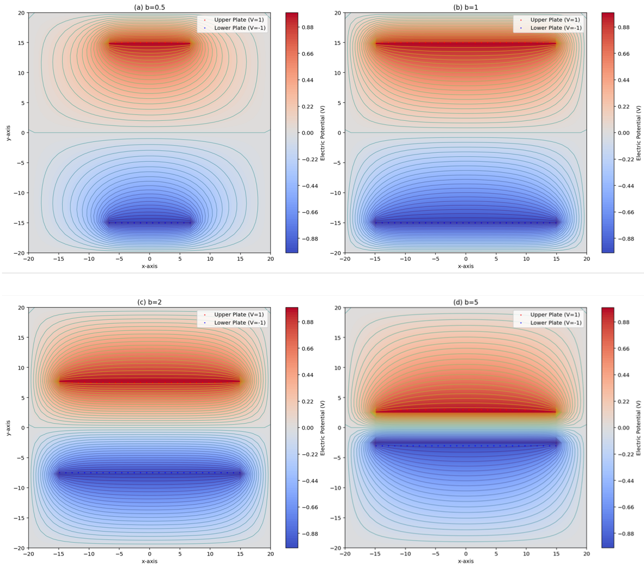

Figure 1: Equipotential lines of Parallel Capacitor

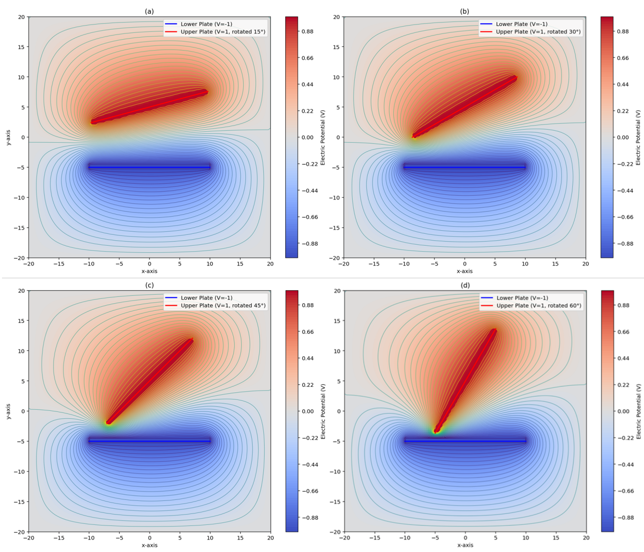

Figure 2: Equipotential Lines of Tilted Capacitor

The tilted capacitor plays a great role in circuit matching. We set the angle at 15, 30, 45, 60 (Figure 2 (a)(b)(c)(d)) because these angles are mostly used in the circuit and can express a clear change.

It was found that when the inclination angles were 15, 30 and 60 degrees (Figure2 (a)(b)(d)), obvious edge effects occurred near the upper plate. As the angle increasing, the edge effects were more concentrated at the lower end of the upper plate. The electric field is no longer uniform, and the difference between the plates gradually increases. An abnormal phenomenon occurred when the inclination angle was 45 degrees (Figure2 (b)). The equipotential lines around the upper and lower plates were very smooth, and no significant edge effect was observed. Further explanation will be shown in the next section.

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4.2. Analysis of Results and Interpretation

Having presented the key simulation results, we now proceed to interpret these findings within the established framework of electrostatic theory. First, our simulation results are highly consistent with the basic theory of electrostatics. According to Gauss's law, in the infinite parallel plate approximation, the electric field between plates is uniform. And it can be seen in Figure 1 that in the central area of the plates, the equipotential lines are uniformly distributed, and the electric field lines are parallel, perfectly matching this ideal model and verifying the accuracy of our simulation.

However, this uniform potential distribution becomes distorted near the edges in Figure 1, which is a perfect reflection of the fringing effect. The fringing effect arises because of a fundamental constraint of electrostatics: The electric field is always normal to the surface of a conductor to achieve the continuity of the electric field under electrostatic conditions[7]. Therefore, at the edge of the plate, in order to meet both the requirements of the boundary condition and the electric field starting from the positive plate and ending at the negative plate, the electric field lines can no longer remain parallel and must bend outward, resulting in local concentration of electric field lines in the edge region, manifested as an increase in equipotential line density, which is in accordance with our detailed results in Figure 3(a).

Having established the baseline performance of the ideal parallel-plate configuration, we now extend our analysis to a more complex and practically relevant structure: the tilted parallel-plate capacitor. As clearly presented in Figure 2, the tilted structure leads to geometric asymmetry, which completely changes the distribution of the electric field lines. First, according to the formula $E = \frac{V}{d}$, where V is the voltage applied to the capacitor and d is the plate spacing varying with horizontal position, our simulation clearly shows that on the left side with the smallest spacing, the electric field lines are most dense (the electric field is strongest); on the right side with the largest spacing, the electric field lines are sparser (the electric field is weakest), intuitively verifying that basic relationship between electric field strength and local plate spacing that is inversely proportional. Second, the tilted structure also causes a localized concentration of the fringing effect compared to that of a parallel plate capacitor, which surrounds the entire plate in Figure 3(b). In the inclined plate, the lower end of the upper plate are all the area with the smallest spacing and the edge. The combination of these two factors causes the electric field strength at this point to reach its extreme value, making it the most dangerous breakdown point. As the angle increases, the minimum spacing between plates decreases, and this effect becomes more significant (Figure 3(c), (d)). This is completely consistent with our theoretical expectations: the electric field tends to concentrate at the tips with large curvature or small spacing.

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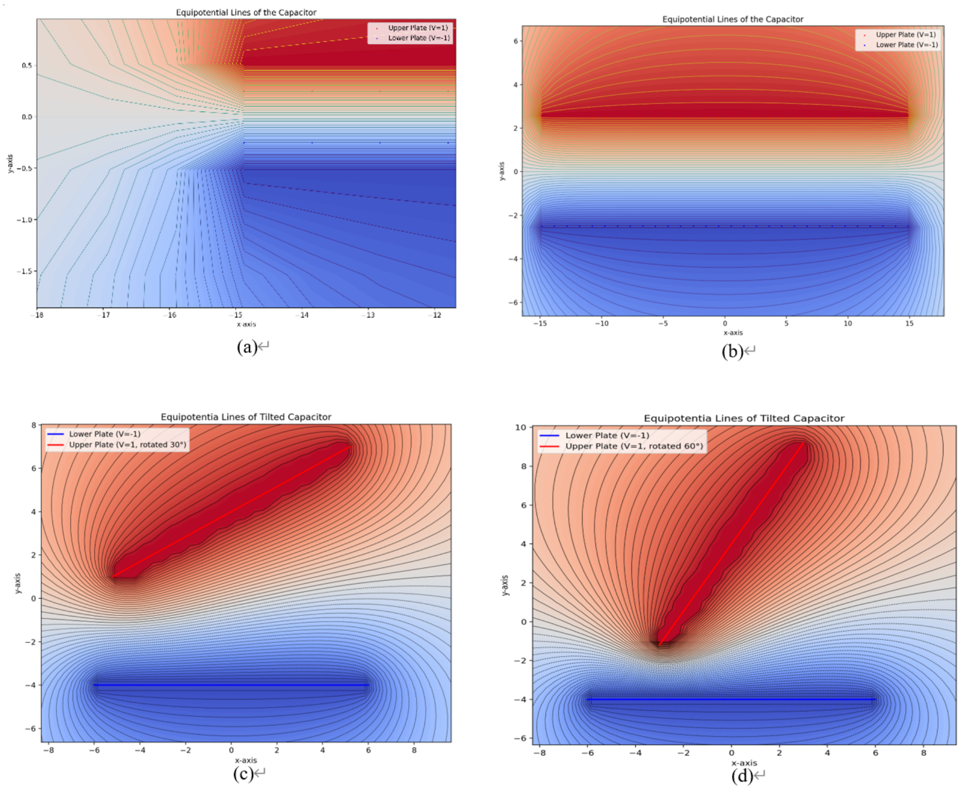

Figure 3: Detailed Simulation Results

Last is about an abnormal phenomenon when the angle of inclination of upper plate equals to 45 degrees. Compared with other inclined simulation results, its equipotential lines near the edge are abnormally smooth(figure4(a), (b)). Our preliminary analysis suggests that this may be due to some compensatory effect of geometric symmetry, at a specific angle (such as 45°), the asymmetry introduced by the inclination of the upper plate may unexpectedly cancel or balance with the distribution pattern of the edge field in the two-dimensional simulation. This points out the limitations of our model and provides direction for an interesting future research topic: systematically studying the relationship between tilt angle and electric field uniformity to find possible optimal engineering configurations.

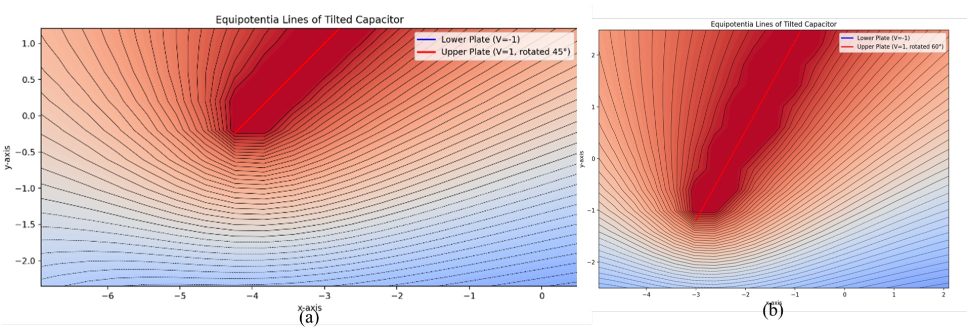

Figure 4: Detailed Comparison

4.3. Limitations and Sources of Error

In numerical simulation performed in this work, certain limits and sources of error possibly inherent must be considered. One is that due to numerical discretization. We employed Finite Difference Method (FDM) to generate approximations of solutions to Poisson's equation. Though this approach is quite simple to use and computationally economical, it is very resolution sensitive. With a rough grid, desirable characteristics of electric potential distribution will be lost. With a very highly refined grid, however, the computation will be too expensive due to a loss of efficiency. A balance between being computationally feasible and accurate is thus difficult to reach.

Secondly, code and calculation will be prone to numerical and round-off errors. With floating-point operations available, numerical uncertainties will be accumulated within iteration schemes. This problem will be made worse if a large number of iterations is required in order to achieve convergence since small imperfections will be propagated iteratively and produce large deviations from theoretical findings. Furthermore, some code parameters—the convergence criterion or maximum number iterations—are subjectively chosen. These choices can influence results quality and produce further uncertainties.

Another limitation applies to idealization of the physical model. We computationally simulated only electric potential distribution within a two-dimensional grid. In practice, electric fields reside within a three-dimensional world. It will always dampen some effects associated with space in treating a three-dimensional phenomenon within a two-dimensional model. However, it renders the problem less challenging to analyze and visualize owing to utilization of a 2D approach. This discrepancy occurs especially in situations where irregular boundaries or edge effects exist such that a two-dimensional model does not treat adequately richness associated with physical system complexity.

Finally, visualization and interpretation of data have limitations too. Although plots of distributions of electric potentials and equipotential lines provide intuitive insight, resolution and readability remain tied to plotting schemes utilized within programs. Sometimes plots formulated will be too coarse to be reproducibly interpreted or will obscure small changes within a data set.

5. Conclusions

In this paper, we intuitively reveal the complexity and diversity of the electric field distribution in capacitors through numerical simulations. And our simulation results show that: first, the distribution of electric field lines is determined by both geometric structure and conductor boundary condition; second, the loss of field uniformity is primarily dictated by the fringe effect, which becomes pronounced and localized in tilted structures, forming points of high risk. Also, we find some interesting results which can be our future research direction. The interesting results happened when the angle of inclination of the upper plate equals to 45°, the equipotential lines become abnormally smooth near the edge compared with that under other situations. It may provide a better distribution of electric field lines in 2D, which can be enlightening for the future design of capacitors.

Overall, this study not only deepen the understanding of electrostatic field theory, but also provide important theoretical basis and engineering inspiration for the optimization design of capacitors.

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