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From the "Prisoner's Dilemma" to "Pareto Optimum": Cooperation and Competition in Card Games from a Mathematical Perspective

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Abstract. Card games function as micro-laboratories for studying cooperation and competition using game theory. This paper examines how individual rationality traps players in the mutually harmful "Prisoner's Dilemma," while strategic cooperation can lead to "Pareto Optimal" outcomes. A two-player model (Cooperate/Betray) demonstrates the dilemma: Individual incentives lead to the Nash Equilibrium (Betray/Betray), yielding suboptimal collective payoffs, despite the superior cooperative outcome (Cooperate/Cooperate). Unilateral betrayal destabilizes cooperation, as seen in Bridge (dishonest bidding causing poor contracts) and Poker (alliance betrayal benefiting dominant players). Conversely, Pareto Optimality, where no player improves without harming others, is achieved only through cooperation. In the dilemma matrix, (Cooperate/Cooperate) is Pareto optimal. Bridge partners attain this through honest bidding for optimal contracts; cooperative games like Hanabi require efficient information-sharing to advance collective goals. Card games reveal the tension between individual and collective rationality. The Prisoner's Dilemma explains lose-lose Nash outcomes, while Pareto Optimality defines win-win cooperation, representing the strategic ideal for card game partnerships.

Keywords: Prisoner's Dilemma, Pareto Optimality, Nash Equilibrium, Cooperation, Game Theory

1. Introduction

Card strategy games, such as Bridge, Poker, or Contract Bridge, are not merely contests of skill and luck but also serve as exceptional micro-laboratories for studying human cooperative and competitive behaviors. Beneath seemingly simple rules of card play and bidding lie profound principles of game theory. Players, driven by individual rationality, often find themselves trapped in the mutually detrimental "Prisoner's Dilemma." Yet through strategic design and communication, they can achieve mutual cooperation, striving to reach the collectively superior state of "Pareto Optimality."

Roger B. Myerson's Game Theory: Analysis of Conflict comprehensively examines non-cooperative and cooperative games, covering sequential equilibria, strategic-form game equilibria, and topics like communication, repeated games, bargaining, coalitions, and cooperation. It

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meticulously dissects the Prisoner's Dilemma, establishing a robust theoretical framework [1]. Avinash Dixit and Barry Nalebuff's work made game theory accessible by simplifying complex concepts into practical strategies for real-world decisions. They proposed "Look ahead and reason backward" for dynamic decisions and a three-stage analytical framework—dominant strategies, elimination of dominated strategies, and Nash equilibrium—for simultaneous decisions [2]. Robert Axelrod's research on the Prisoner's Dilemma focused on showed through computer tournaments that cooperation could emerge and persist in repeated Prisoner's Dilemma games among self-interested agents as an Evolutionarily Stable Strategy (ESS) [3]. Drew Fudenberg and Jean Tirole's co-authored "Game Theory" systematized the field with a comprehensive framework covering static and dynamic games with complete and incomplete information [4]. They refined equilibrium concepts, including Pareto-dominant equilibrium, to predict cooperative outcomes in coordination games. Previous research showed that imperfect monitoring disrupts efficient equilibria by inducing strategic uncertainty. Matsushima addressed this by constructing a model for "imperfect private monitoring," proposing the "belief consistency principle," and proving that Pareto efficiency remains achievable under certain conditions [5]. In card games, Chen and Ankenman applied game theory to poker strategy, establishing "expected value maximization" as the ultimate criterion [6]. Additional studies have explored the connection between Nash equilibrium and the Prisoner's Dilemma in poker and other card games. Zagal analyzed poker players' adherence to game-theoretic predictions, while Ramadhan examined team cooperation and Pareto-optimal strategies in Bridge [7,8].

This study explores how individual rationality leads to the "Prisoner's Dilemma" and how strategic cooperation achieves "Pareto Optimal" outcomes in card games. A two-player (Cooperate/Betray) model is constructed to analyze the relationship between individual incentives and collective payoffs, supported by case studies in Bridge and Poker. It provides a theoretical basis for understanding cooperation and competition in card games and advancing related research.

2. The Prisoner's Dilemma: a collectively suboptimal outcome from the mathematical perspective

The Prisoner's Dilemma is the most classic model in game theory. From a mathematical perspective, the Prisoner's Dilemma is a game of individual rationality versus collective behavior; its core lies in individual rational choices leading to collectively non-optimal outcomes.

First, a simple Prisoner's Dilemma scenario is constructed, setting two players in a card game, Player A and Player B, facing two critical choices: cooperation or betrayal. The cooperative strategy (C) involves faithfully conveying card information between partners in Bridge, or temporarily ceasing fire to jointly resist a strong opponent in Poker. The betrayal strategy (D) involves concealing key cards to pursue personal victory in Bridge, or choosing to betray an ally.

Second, a mathematical model is used to map the individual decision-making behaviors and outcome matrix of Players A and B, clearly describing the payoff results under different strategy combinations using mathematical language. Assume Players A and B each have a strategy set {C, D}, and the outcome matrix is illustrated in Table 1, where the corresponding numerical values represent player utility (higher values indicate higher individual utility for the player).

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Table 1. Payoff matrix for the Prisoner's Dilemma

Player A \ B	Cooperate (C)	Betray (D)
Cooperate (C)	(3, 3)	(0, 5)
Betray (D)	(5, 0)	|(1, 1)

Then, the Nash Equilibrium is introduced, defined as a state where, given the strategies of the other participants, no single participant can gain a higher payoff by unilaterally changing their own strategy. Analyzing individual behavioral outcomes using Nash Equilibrium strategy analysis reveal that if Player B chooses Cooperate (C), Player A choosing Betray (D) gets 5, which is greater than choosing Cooperate (C) and getting 3. Under Nash Equilibrium, Player A tends to choose Betray (D). Similarly, if Player B chooses Betray (D), Player A choosing Betray (D) gets 1, which is greater than choosing Cooperate (C) getting 0. Under Nash Equilibrium, Player A tends to choose Betray (D). Similarly, regardless of what Player A chooses, Player B's best choice is also to choose Betray (D).

Therefore, solving for Nash Equilibrium, as shown above, (Betray, Betray) or (D, D) is a Nash Equilibrium point for Players A and B. At this point, the payoffs for Players A and B are both (1, 1). Comparing the collective outcome for Players A and B, the betraying combination (D, D) at the Nash Equilibrium yields the collective total payoff of 1 + 1 = 2. The potential optimal outcome for Players A and B corresponds to the cooperating combination (C, C) with the collective total payoff of 3 + 3 = 6.

Clearly, the potential optimal outcome, the cooperating combination (C, C), yields a superior collective total payoff.

The core essence of the Prisoner's Dilemma is analyzed. Although the cooperating combination (C, C) is optimal for the collective, it is difficult to stabilize at the individual decision-making level. Because, if either Player A or Player B unilaterally chooses Betray (D) when the other chooses Cooperate, they can increase their own payoff from 3 to 5, sacrificing the other's payoff, reducing it from 3 to 0. This enormous temptation for the individual unilaterally makes the cooperating combination (C, C) difficult to maintain. The rational calculation of individuals chasing private gain ultimately locks both Players A and B into the lowest-paying betraying state (D, D). This outcome is the mathematical explanation for the collective tragedy born from individually optimal choices.

3. Examples of the Prisoner's Dilemma in card games

In Bridge, an example of the Prisoner's Dilemma can be seen in bidding misrepresentation. Partner Players A and B should honestly communicate hand strength to find the best contract, which corresponds to the cooperative choice (C). However, if one party, such as Player A, deliberately underreports a strong hand to pursue personal dominance or avoid risk, making the betraying choice (D), it may lure Player B into stopping at a low-level contract. The result of bidding misrepresentation is: Player A might get a safe but low-scoring contract, corresponding to a smaller payoff, while Player B performs poorly due to misinformation, and the total score for Players A and B is far lower than the potential gains of honest bidding reaching a high-level good contract (C, C). Both Players A and B suffer losses, receiving payoffs similar to (1, 1), whereas the cooperating combination could have yielded mutual benefit, similar to (3, 3).

In Poker, an example of the Prisoner's Dilemma can be seen in the collapse of temporary alliances. When multiple players with weaker chip stacks face a single stronger player with a chip

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lead, it is wise for the weaker players to form a temporary alliance, adopting the cooperative combination to jointly suppress the strong player. However, a member of the temporary alliance, like Player A, might suddenly adopt a betrayal strategy, raising and attacking Player B, in order to quickly eliminate another weaker player, Player B, to improve their own ranking and prize money. This will lead to the collapse of the temporary alliance, and ultimately the stronger player with the chip lead will reap the rewards. Player A's betrayal might give them a small short-term reward, such as moving up one rank by eliminating Player B, but they lose the opportunity to gain a higher ranking and prize money by eliminating the strong player through cooperative strategy. In other words, Player A gains a small reward by adopting the betrayal strategy but loses the collectively optimal strategy, ultimately allowing the single chip-leading strong player to win easily.

4. Pareto Optimum from a mathematical perspective is a win-win cooperation outcome

The Prisoner's Dilemma reveals the trap of mutual competition between card players, while the Pareto Optimum points the way towards cooperation. Pareto Optimum or Pareto efficiency refers to a state of strategy or resource allocation, within a given space of feasible strategies or resource allocations, where it is impossible to make any one individual better off without making at least one individual worse off. In short, Pareto Optimum means "cannot make someone better off without harming others".

Mathematical expression of Pareto Optimum:

Assume the set of all feasible strategy combinations is S, and the utility function for participant "i" is $U_i(s)$, $s \in S$. A strategy combination $s \in S$ is Pareto optimal if there does not exist another strategy combination $s' \in S$,

such that $U_i(s') \geq U_i(s)$ for all participants i,

and $U_j(s') > U_j(s)$ for at least one participant j.

In the previously analyzed Prisoner's Dilemma matrix, the Nash Equilibrium point (D, D) is not Pareto optimal for Players A and B. Because (C, C) exists,

such that $U_A(C, C)=3 > U_A(D, D)=1$

and $U_B(C, C)=3 > U_B(D, D)=1$

That is, when Players A and B switch to the cooperating combination (C, C), they can simultaneously improve both players' payoffs. Therefore, in the above Prisoner's Dilemma game matrix, (C, C) is Pareto optimal. Analyzing the following 3 scenarios from a mathematical perspective:

(1) Switching to (D, C), $U_A(D, C)=5 > U_A(C, C)=3$,

but $U_B(D, C)=0 < U_B(C, C)=3$, thus harming Player B's benefit.

(2) Switching to (C, D), $U_B(C, D)=5 > U_B(C, C)=3$,

but $U_A(C, D)=0 < U_A(C, C)=3$, thus harming Player A's benefit.

(3) Switching to (D, D), $U_A(D, D)=1 < U_A(C, C)=3$

and $U_B(D, D)=1 < U_B(C, C)=3$, thus jointly harming the benefits of Players A and B.

The above analysis results show that there does not exist a strategy combination where both Players A and B are better off than under the cooperating combination strategy (C, C). Therefore, the cooperating combination strategy (C, C) for Players A and B is the Pareto optimum point.

5. Examples of Pareto Optimum in card games

In Bridge, an example of Pareto Optimum can be observed in the bidding process. The ultimate goal for partner Players A and B is to find and complete a contract that maximizes their side's score,

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aiming to maximize International Match Points (IMP) or Victory Points (VP). This requires both partner Players A and B, through a precise bidding system, to exchange hand information honestly and fully, including distribution, point count, controls, etc., ultimately arriving at a contract that both Players A and B can accept and that fully exploits the potential of their combined hands. Players A and B's successful bidding, achieving the cooperating combination strategy (C, C), realizes the Pareto optimal score under their combined hand strength. Any concealment or fraudulent betrayal (D) strategy, even if it occasionally brings small rewards to Player A or Player B, will inevitably damage the trust between partner Players A and B and the overall scoring efficiency in the long run.

In cooperative card games, such as "Hanabi," the goal is to collectively complete a high-difficulty task, such as arranging all fireworks in order. Taking "Hanabi" as an example, the game goal is usually to collectively complete a high-difficulty task, such as arranging all fireworks in order. Each player can see other players' hands but cannot see their own hand. Players must give others precise hint information about their own hand cards, meaning each player needs to adopt the cooperating combination strategy. In "Hanabi" game, the optimal strategy requires that the timing and content of information hints must maximize the team's overall information utilization efficiency, making each action advance the collective goal as much as possible. Any ineffective or vague hint (equivalent to inefficient cooperation), or even deliberately providing misleading information (equivalent to adopting a betrayal strategy), wastes team resources, moves away from Pareto optimum, and ultimately prevents achieving the highest score.

6. From Prisoner's Dilemma to Pareto Optimality in "Baohuang"

"Baohuang" (Emperor's Guard), a popular five-player card game in Northern China, features explicit cooperation-competition dynamics.

The game consists of the Emperor's Alliance, which includes the Emperor (known) and two Guards (1 known, 1 hidden), and the Commoners' Alliance, which includes two Commoners (both hidden).

The goals are for the Emperor's Alliance to play all cards quickly and for the Commoners' Alliance to exhaust the Emperor's Alliance.

Strategies include card sequencing, combinations (singles/pairs/straights), and signaling (e.g., specific plays to reveal identity).

Information Asymmetry is introduced by the hidden Guard's identity creates uncertainty. The hidden Guard faces a choice: to Cooperate (C) by revealing their identity to assist the Emperor, which is collectively optimal, or to Defect (D) by hiding their identity to avoid targeting, which is individually rational.

Table 2. Payoff matrix for guards in "Baohuang"

Guard A \ Guard B	Cooperate (C)	Defect (D)
Cooperate (C)	(3, 3)	(1, 4)
Defect (D)	(4, 1)	(2, 2)

Table 2 shows that the highest individual payoffs occur when one Guard defects while the other cooperates, but the highest collective payoff occurs when both Guards cooperate.

Nash Equilibrium (D, D), where both Guards defect, resulting in a collectively suboptimal outcome where the Emperor is suppressed. Pareto Optimality is achieved when both Guards cooperate (C, C) under conditions such as common knowledge, where the Emperor identifies at least

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one Guard and adjusts strategy accordingly, and repeated interaction, where tit-for-tat strategies punish defection.

7. Conclusion

The essence of card games lies in strategic interactions constrained by rules, mathematically abstracted as a binary choice between cooperation and defection. Cooperation entails sharing information and jointly pursuing collective interests, while defection prioritizes concealment and individual goals. This dual structure makes card games a natural laboratory for studying human cooperation.

First, the Prisoner's Dilemma reflects the rupture between individual and collective rationality, exposing card games' core contradiction: individual optimal choices lead to collectively worst outcomes. In two-player card games: Mutual cooperation yields stable returns; Defection while the other cooperates maximizes the defector's gain but inflicts maximum loss on the cooperator; Mutual defection results in a lose-lose deadlock. Mathematically, defection is invariably the individual optimal strategy, forming the sole Nash equilibrium. Yet this equilibrium represents the lowest collective utility, rooted in the "negative externality of strategies": the defector's gain is less than the loss imposed on others, causing net depletion of social welfare. Bidding fraud in Bridge exemplifies this dilemma—misrepresenting card strength may offer short-term tactical advantage but undermines the bidding system's integrity, preventing full utilization of combined hand potential.

Second, Pareto optimality embodies the mathematical ideal of cooperative synergy. It defines the ultimate goal of cooperation in card games: no strategy adjustment can improve one player's outcome without harming another. In card game strategy, the cooperative combination is the unique Pareto optimum, characterized by non-improvable system efficiency, where synergistic value has reached its ceiling. It also features strategic stability, as any deviation from cooperation inevitably damages at least one party's interests. Furthermore, it realizes collective rationality, achieving perfect alignment between individual and collective goals.

In card games, Pareto optimality manifests as partners identifying the optimal contract through precise bidding.

Finally, card games transcend the Prisoner's Dilemma by reconstructing game conditions. Repeated Game Mechanism: When players anticipate ongoing interaction, the "tit-for-tat" strategy (cooperate initially, then mirror the opponent's prior move) makes cooperation a new equilibrium. Defectors face immediate punishment; cooperators receive sustained rewards, satisfying subgame perfect equilibrium conditions.

Signaling Mechanism: In hidden-role games like "Baohuang," the concealed Guard transmits identity via costly signals (e.g., sacrificing key cards). The genuine Guard's signaling cost is far lower than imitators', forming the mathematical basis for a "separating equilibrium". The Emperor dynamically updates beliefs through "Bayesian inference": reassessing identity likelihoods based on signal probabilities, ultimately converging to accurate cognition.

The highest realm of card strategy lies not in defeating opponents but in achieving Pareto optimality through cooperation. In Bridge, this materializes as a Nash bargaining solution—fairly distributing cooperative surplus to synchronously elevate both players' payoffs. When players construct cooperative frameworks with mathematical rationality, card games ascend to proving grounds for collective wisdom. Each precise bid and every synergistic card play pays homage to Pareto optimality—where individual and collective rationality reconcile, and every falling card testifies to the power of collaboration. True victory eternally belongs to those wise enough to bridge the chasm of rationality with cooperation.

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