

Physical Mechanisms and Impacts of Different Noises in Gravitational Wave Detection

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Abstract. Gravitational wave detection is modernly an important detection method of celestial phenomena like black hole merges or neutron star merges. Since the change in the arm length of the interferometer caused by gravitational waves is extremely small, the requirement of detection accuracy is significantly high. However, current detection accuracy is limited by multiple noises. Among them, the influences of quantum noise, thermal noise, and environmental noise are the most intense. Quantum, thermal, environmental noises limit high, medium, and low frequency range detecting accuracy respectively. This paper aims to explain the basic mechanisms of the noises through mathematical deduction. For each part, the paper also introduces common noise suppression strategies, include using compressed light and frequency-dependent compression (quantum noise), using low thermal influence suspension and coating materials (thermal noise), and setting multi-stage filtration system when solving seismic noise (environmental noise). With the strategies introduced, the paper also put forward ideas for future development.

Keywords: Gravitational wave, noise mechanism, noise suppression measures

1. Introduction

Theory of general relativity predicts the existence of gravitational wave (GW). After a hundred years of continuous research, this solution, which only existed in theory, has been confirmed. The discovery of gravitational waves marked a significant progression of physics and astronomy research, and also created a new research strategy for extreme celestial bodies in the universe. In 2015, gravitational wave was first detected by the Laser Interferometer Gravitation Wave Observatory (LIGO) program. The source of observed gravitational wave is from the merger of black holes. From then on, detection of gravitational waves became an important modern branch of astronomy [1]. Follow up studies, like LIGO, Virgo, and KAGRA projects, have discovered more gravitational wave sources, like neutron star combinations. These studies provide new research perspectives for intense galactic activities in the universe. However, since gravitational wave detection focuses on extremely small size changes on the detection arm (much less than the atomic diameter), it has very high requirements on detection accuracy and equipment sensitivity. Hence, noises in detection will affect results significantly. And not only the accuracy of data, noises may also cover some gravitational wave signals, which lowered the sensitivity of the detector [2].

Major noises of gravitational wave detection are quantum noises, thermal noises, and environmental noises. They all cause significant inaccuracy in detection. Quantum noises are caused by the statistical fluctuations of photons within the laser interferometer. They bring noises at high frequency range (hundreds Hz) [3]. Thermal noises are caused by the thermal effect of detection instruments (optical components, suspension systems, and sensors). Tiny thermal expansions in instruments' sizes, especially laser mirror, bring medium-frequency range noises (hundreds to thousands Hz) [4]. Environmental noises are caused by natural and human aided interference include wind, vehicle, earthquake, etc. These influences cause detectors occur displacements and vibrations, which bring noises at low frequency range (less than 10 Hz) [5]. These noises will also interact with each other to bring further impacts to detection.

As the research progresses, the requirements for detection accuracy keep increasing. Hence, these noises become one of the biggest obstacles to development. This article aims to explain the mechanisms of various types of noises (quantum noise, thermal noise, environmental noise) and their effects on gravitational wave detection at different frequency ranges. The author's research on the underlying logic of different noises is expected to contribute to the development and proposal of targeted improvement measures.

2. Theoretical background of gravitational waves

Gravitational wave is one of the major predictions from the general relativity. The theory states that accelerating masses will cause disturbance of spacetime curvature. These disturbances transverse in space under light speed, and will remain their characteristic morphology during long-distance transmission [6].

Mathematically, generation and diffusion of gravitational waves can be described by the the weak field approximation of general relativity. When the spacetime metric is assumed to be flat, and the perturbation is extremely small, equation of their superimposed form is like:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, |h_{\mu\nu}| \ll 1 \quad (1)$$

In equation (1), $\eta_{\mu\nu}$ is the Minkowski spacetime metric, and $h_{\mu\nu}$ is the disturbance. When connecting it into the Einstein field equation and retaining the approximations about the perturbation terms, the linearized field equation can be obtained [6]:

$$\bar{h}_{\mu\nu} = -\frac{16\pi}{c^4} T_{\mu\nu} \quad (2)$$

In equation (2), is D' Alembert operator, $\bar{h}_{\mu\nu}$ is the trace inversion tensor. Under vacuum condition ($T_{\mu\nu}=0$), the equation simplifies into:

$$\bar{h}_{\mu\nu} = 0$$

This result implies that gravitational disturbances (gravitational waves) travel at wave form under light speed. Further simplify with transverse - traceless gauge (TT gauge), disturbances can be described with only two components [7]:

$$h_{\mu\nu}^{TT} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (3)$$

Matrix (3) implies gravitational wave has two independent polarization modes: “+” polarization and “ $\times$ ” polarization. The polarization modes brings periodic stretching and compression in testing particles’ relative positions (against time). This theoretical deduction brings possible experiment strategy. In practical experiments, resultant gravitational wave effect is recorded by strain:

$$h(t) = \frac{\Delta L(t)}{L} \quad (4)$$

In equation (4), L is the interferometer arm length, and $\Delta L(t)$ is the disturbance of arm length against time.

3. Analyzing noises

3.1. Quantum noises

Quantum noises are originated from the quantization characteristics of electromagnetic fields. The principle of uncertainty sets a limitation of accuracy when detecting length change under such small orders of magnitudes [8]. Quantum noises are mainly classified into two categories. These noises cause inaccuracy in high and low frequency respectively.:

Shot noise: Measurement uncertainty caused by fluctuation of photons.

Radiation pressure noise: interferometer mirror surface displacement caused by photons’ momentum.

3.1.1. Analyzing shot noise

Light is considered as quantized flow of photons. When reaching the photoelectric detector, photons’ statistical distribution follows the Poisson distribution. When power of detection light is “ P ”, and photon energy is “ $E_\gamma = h\nu$ ”, the number of photons arriving per second is shown in equation (5) [9].

$$N = \frac{P}{h\nu} \quad (5)$$

From variance of the Poisson distribution, $\Delta N = \sqrt{N}$, relative fluctuation is deduced in equation (6) and (7). Inaccuracy is presented in phase difference of signals.

$$\frac{\Delta N}{N} = \frac{1}{\sqrt{N}} = \sqrt{\frac{h\nu}{P}} \quad (6)$$

$$\Delta\phi_{shot} \propto \sqrt{\frac{h\nu}{P}} \quad (7)$$

From the equation, characteristic of shot noise can be deduced: inaccuracy brings by shot noise is inversely proportional to power of incident light. To study the actual impact of shot noise, data from practical experiments will be substituted for comparison here. Deduction data (shown in table 1) is from Advanced LIGO project [10]:

Table 1. Data used for calculation [10]

Optical maser wavelength (λ)	1064 nm
Optical maser frequency (ν)	2.82×10^{14} Hz
Photon energy ($h\nu$)	1.87×10^{-19} J
Interference arm length (L)	4 km
Mass of mirror (m)	40 kg
Observing frequency (f)	100 Hz
Angular frequency (ω)	$2\pi f$

According to equation (7), inaccuracy in displacement form will be:

$$\Delta\theta_{shot} \sim \sqrt{\frac{h\nu}{P}} \sim \sqrt{\frac{1.87 \times 10^{-19}}{7.0 \times 10^5}} \sim 5.2 \times 10^{-13} \quad \Delta x_{shot} \sim \frac{\lambda}{4\pi} \Delta\Phi_{shot} \approx 4.4 \times 10^{-20} m$$

In terms of order of magnitude, the result is consistent with LIGO's sensitivity at high frequencies (10×10^{-20}) [10].

3.1.2. Analyzing radiation pressure noise

Different from shot noise, radiation pressure noise is caused by the interaction between photons and mirrors. When photons reach the mirror and be reflected, their momentum will change as what equation (8) describes [9]:

$$\Delta p = \frac{2h\nu}{c} \quad (8)$$

Assuming within time Δt , ΔN amount of photons randomly fluctuating and incident, then the uncertainty of the force acting on the mirror is as shown in equation (9).

$$\Delta F = \frac{\Delta N \cdot \Delta p}{\Delta t} \quad (9)$$

This force causes fluctuations of mirror's position. To convert it into the displacement noise spectral density is approximately:

$$S_x^{rad}(\omega) \propto \frac{P}{m\omega^2} \frac{\hbar P}{mL^2\omega^2 c\lambda} \quad (10)$$

From the equation, characteristic of radiation pressure noise can be deduced: inaccuracy brings by radiation pressure noise is proportional to power of incident light. However, Data from Table 1 will be used again for comparison [10]:

Table 2. Summary of calculation results for radiation pressure noise

Frequency (f)/Hz	Result (m^2/Hz)
10	9.802×10^{-44}
50	3.921×10^{-45}
100	9.802×10^{-46}

However, the formula is in scale function form, many steps and constant terms were abandoned during the derivation to minimize calculation. Therefore, the results shown can only be used for a reference in terms of magnitude but not accurate engineering calculation. The accurate form of formula is shown in formula (11), detailed derivation steps are shown in reference [11]. Considering the computational load, this article does not present it.

$$S_x^{rad}(\omega) = \frac{4\hbar\omega_0 P}{c^2 m^2 \omega^4} \left(\omega_0 = \frac{2\pi c}{\lambda} \right) \quad (11)$$

3.1.3. Introducing Standard Quantum Limit (SQL)

From the equations, shot noise inversely proportional to the square root of power and radiation pressure noise straightly proportional to power. Increasing power will reduce shot noise but increase radiated pressure noise, and vice versa. When the two noises are equal in magnitude, the resultant noise becomes minimized. So, theoretically, quantum noise can never be eliminated, and this minimized noise is called the Standard Quantum Limit (SQL) [12].

It is now theoretically impossible to eliminate the SQL, but there are strategies break through this limitation. One famous method is to squeeze the injected light. To squeeze one quadrant of the light (from phase quadrant X_2 or amplitude quadrant X_1 , orthogonal quadrant of these two forms light field), its variance shrinks by e^{-2r} and the orthogonal quadrant is "anti-squeezed" by e^{+2r} . Therefore, if phase quadrant is squeezed, the shot noise at high frequency can be lowered. However, this method will increase the radiation pressure noise at low frequency [13]. One way to further develop this strategy is through using frequency-dependent compression. This method uses filter cavity to realize squeezing phase quadrant in the high-frequency band and squeezing amplitude quadrant at low-frequency band. Currently, this strategy is tested on large-scale interferometer and planned to use in LIGO A+ program.

3.2. Thermal noises

3.2.1. Introduction

Overall, thermal noises are caused by the thermal equilibrium fluctuations of detectors' mechanical system and inner environment. The fundamental physics principle for this mechanism follows Fluctuation - Dissipation Theorem (FDT). The spectral density of thermal fluctuation force of FDT is shown in equation (12) (γ is the damping coefficient). It states that the thermal noise power spectrum of a system is directly related to its energy dissipation [14].

$$S_F(\omega) = 4k_B T m \gamma \quad (12)$$

In the interferometer of gravitational waves, common thermal noise source include:

- Suspension thermal noise. It is caused by the mechanical dissipation of the suspension wire.
- Substrate & coating thermal noise. It is dominated by the loss of optical materials themselves
- Gas thermal noise. It is caused by the collision between residual gas particles and measuring instruments, but it has only very less effect under high vacuum environment.

3.2.2. Formula derivation

For a simple harmonic oscillator with mass m , damping coefficient γ , and resonant frequency of ω , its motion function follows formula (13).

$$m\ddot{x}(t) + m\gamma\dot{x}(t) + m\omega_0^2x(t) = F_{th}(t) \quad (13)$$

Combine equations (13) and (12) will get the displacement noise spectrum. At high frequency range where resonance is eliminated, the function can be further simplified into equation (14) [14]

$$S_x(\omega) = \frac{S_F(\omega)}{|-m\omega^2 + im\gamma\omega + m_0^2|^2} = \frac{4k_B T m \gamma}{|-m\omega^2 + im\gamma\omega + m_0^2|^2} \approx \frac{4k_B T \phi(\omega)}{m\omega^5} \quad (14)$$

$\phi(\omega)$ is the loss angle of the material.

Formula (14) states that thermal noise is proportional to material dissipation, and reversely proportional to frequency to the power of five.

3.2.3. Suspension thermal noise

The suspension system is hanged by fiber suspension to insulate vibration from the ground. However, these fibers have internal friction, so to introduce suspension thermal noise is necessary. Its spectrum density is shown in equation (15)

$$S_x^{susp}(f) \propto \frac{4k_B T}{m(2\pi f)^5} \phi_{susp} \quad (15)$$

ϕ_{susp} is the equivalent loss angle of the suspension filament

Formula (15) indicates that at low-frequency range (10Hz - 50Hz), suspension thermal noise is major source of noises. But the magnitude of suspension thermal noise decreases at an exponential rate when frequency increases. To minimize the noise, common strategy is to use low-loss materials (minimize the “ ϕ_{susp} ”) or lower operation temperature (minimize “T”). Both strategies are employed in KAGRA program [15].

3.2.4. Substrate & coating thermal noise

In practical experiments, material noise of the mirror surface is significantly important. Commonly, the material noise is calculated by “energy dissipation method”. For the thermal noise of coating, its spectral density is shown in equation (16) [15].

$$S_x^{coating}(f) = \frac{2k_B T}{\pi^2 f} \bullet \frac{1-\sigma^2}{\pi^{\frac{3}{2}} Y \omega_0} \phi_{eff} \quad (16)$$

“Y” is the Young modulus, “ σ ” is the Poisson's ratio, “ ω_0 ” is the radius of the light spot, and “ ϕ_{eff} ” is the equivalent loss Angle.

The equation indicates that increasing spot radius can reduce thermal noise, but high-loss coating materials increases noise.

3.3. Environmental noises

3.3.1. Introduction

Environmental noises are from external influences, which are not quantum or thermal noise sources. They affect the interferometer arm length through ground, air, or electromagnetic path coupling. These noises are not from interferometer itself, but from earth's environment or external conditions. Environmental noises limits the sensitivity of the interferometer at low frequencies (1Hz-50Hz) in a great extent [16]. Common environmental noise sources include seismic noise and Newtonian gravity gradient noise

3.3.2. Seismic noise

Seismic noise is sourced from vibration of earth's the earth's surface, include natural earthquake, wind, human activities, etc. The suspension system of interferometer can be seen as a mechanical filter. Its transfer function is shown in equation (17). And the displacement spectral density of seismic noise is shown in equation (18) [17]. It is proportional to the square of the transfer function.

$$T(f) = \left(\frac{f_0}{f}\right)^n \quad (17)$$

f_0 is the resonant frequency of the system and n is the filtering order.

$$S_x^{seis}(f) = s_x^{ground}(f) \bullet |T(f)|^2 \quad (18)$$

s_x^{ground} is the displacement spectrum of surface vibration.

Seismic noise significantly obvious at low-frequency range (1Hz-10Hz). However, through suspension for multiple stages, this noise can decrease exponentially. This is now LIGO's strategy for how to minimize the seismic noise.

3.3.3. Newtonian gravity gradient noise

Newtonian gravity gradient noise is not caused by mechanical interactions, but the gravitational field fluctuation sourced by environmental density change. Since it is the change of the gravitational field, it will straightly join the detected signals, so it cannot be eliminated through device re-design. When ground density disturbance is $\delta\rho(r,t)$, the gravity disturbance at position r_0 is shown in equation (19) [18, 19].

$$\delta g(r_0, t) = G \int \frac{\delta\rho(r,t)(r-r_0)}{|r-r_0|^3} d^3r \quad (19)$$

Commonly, the strategy to minimize gravity gradient noise is through environmental sensor array to detect noise in advance and make compensation on the data.

4. Conclusion

This article systematically discussed gravitational waves and its three major noises - quantum noises, thermal noises, and environmental noises. The writer introduced their physical mechanisms, mathematical expressions, and deduced their effect on sensitivity through equations and practical

measurements. And with the analysis of the noises' characteristics, the article also discussed the common strategies to minimize the noises.

At high frequency range (hundreds Hz), quantum shot noise brings significant effect. It decides the sensitivity of interferometers in a great extent. Through injection of compressed light together with frequency-dependent compression, the noise can be lowered. However, the effect of quantum radiation pressure noise at low-frequency range is still severe, which shows the SQL. At medium frequency range (10Hz-100Hz), thermal noise is the main bottleneck of the interferometer's sensitivity. Mirror material is the crucial variable. The article deduced the equation to describe the thermal noise and analyzed the relationship between noise and material. This points out the necessity of material development. At low frequency range (1Hz-10Hz), environmental noise takes the major role, especially seismic noise and Newtonian gravity gradient noise. The article proves that multi-stage filtration system reduces seismic noise exponentially. For Newtonian gravity noise, since it is the fluctuation of gravitational field, it cannot be prevented by mechanical system upgrade.

Quantum, thermal, environmental noises limit the sensitivity of detection respectively at different frequency range. To achieve a higher detecting accuracy, developments on quantum technology, material exploration, gravity noise pre-feedback technology is necessary.

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