

Introduction to the Line-of-Sight Approach to CMB and Refine It to Incorporate Effects from Reionization

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Abstract. Cosmic Microwave Background (CMB) is the relic microwave radiation emitted at last scattering surface and fills all space in our universe. The Friedmann–Lemaître–Robertson–Walker metric predicts a homogeneous and isotropy CMB temperature distribution, but in fact people observe some anisotropic temperature fluctuation of CMB across the universe. This paper will provide history of exploring CMB and discuss anisotropic property of CMB across the sky. A possible explanation of this anisotropy behaviour of free CMB photons, the Line-of-Sight (LoS) approach of Boltzmann equation, including Sachs Wolfe effect, gravitational Doppler effect and Integrated Sachs Wolfe effect, and mathematical derivation of LoS from Boltzmann equation will also be included. Then this paper will provide a possible way to modify the LoS to include effects from the Reionization and include error propagation analysis. Finally this paper will discuss the importance in studying the anisotropy of CMB, the LoS and why we need to refine it.

Keywords: Line-of-Sight approach, Reionization, CMB anisotropies

1. Introduction

During 20th century, there were a variety of discussions in cosmology. In discussing the Big Bang theory, cosmic microwave background was first predicted in 1948 by Ralph Alpher and Robert Herman, where they were able to estimate temperature of CMB to be 5K. In 1965, two American radio astronomers, Arno Penzias and Robert Wilson, accidentally discovered the CMB while working at Bell Labs in New Jersey. They were investigating the noise in their radio receiver and found a persistent, uniform background signal, which seemed to come from all directions in the sky, which is known to be Cosmic Microwave Background. In order to explore CMB, NASA sent Cosmic Background Explorer (COBE) in 1989 and sent Wilkinson Microwave Anisotropy Probe (WMAP) in 2001, which corroborate the anisotropic property of CMB.

One of the most famous theories, the Big Bang theory, illustrates the expanding of the universe from an initial state of high density and temperature. It predicts that when the universe was undergoing adiabatic expansion, the temperature would decrease to a point eventually where electron-baryon plasma was no longer able to trap photons, and the escaped photons are CMB photons which made the universe transparent, and that period is called recombination era. Nowadays, average temperature of CMB is $2.72548 \pm 0.00057\text{K}$. Since CMB is said to be the relic radiation from early universe, studying its behaviour could provide people a more precise way in

understanding early universe. It carries information and conditions of the universe during its earliest moment, which are important evidence for the Big Bang theory. Through studying anisotropy of CMB photons, people could explore the evolution of the universe including change in density of matter, radiation and dark energy. At the same time, anisotropy of CMB contains wealthy information of inhomogeneous distribution of matter in early universe.

The reionization epoch is the period where first celestial body forms, around 150 million to 1 billion years after the Big Bang. Prior to this epoch, our universe is full of neutral hydrogen, where CMB photons have a long mean free path. However, at Reionization, formation of stars ionize surrounding hydrogen [1], releasing tons of protons and electrons. If the ionization is uniform as presumed by LoS, then the optical depth is a constant. If the ionization is patchy, then we need to take account for this patchiness and the optical depth would be a 2D field rather than constant [2-7].

In 1872, Ludwig Boltzmann devised Boltzmann equation which is used to describe thermal dynamic system not in a state of equilibrium, and this equation is the key in explaining anisotropy of CMB. While assuming CMB photons would not experience any collision when traveling through the universe, the particle distribution function of CMB photon is conserved during the path. The corresponding result from Boltzmann equation is the line-of-sight solution [8], which explains anisotropy of free streaming CMB photons from three distinct terms: Sachs Wolfe term, Doppler term and Integrate Sachs Wolfe term, as this paper will discuss later in this paper. Although the LoS approach has become a standard approach for computing CMB temperature anisotropies, it often ignores the effects of Reionization epoch on the evolution of optical depth. As the precision of measurement increases as time goes by, understanding how does the Reionization affect the optical depth and hence the source function becomes increasingly important. When CMB photons travel through large scale structures, they would have a great chance to interact with free particles through SZ effects [9]. That is the reason why people need to modify the LoS to incorporate effects from patchy Reionization. In section 2, this paper show the derivation of the standard LoS from Boltzmann equation. In section 3, this paper discuss the terms in standard LoS and incorporate effects from Reionization on optical depth and source function via a tanh model, including error propagation analysis.

2. Derivation of LoS from Boltzmann equation

In describing distributions of CMB photons, kinetic theorem can be used. A CMB photon can be described by one particle distribution function $f(x^\mu, p^\mu)$, which is a function depends on position and four-momentum. From Boltzmann equation, we have this relation [10]:

$$\frac{df}{d\eta} = C[f, f_e] \quad (1)$$

Where the η is the conformal time. We will expand both LHS and RHS of (1) to find the LoS approach now. The first thing is to expand the LHS to partial derivatives:

$$\frac{df}{d\eta} = \frac{\partial f}{\partial \eta} + \frac{\partial f}{\partial x^i} \frac{dx^i}{d\eta} + \frac{\partial f}{\partial \ln(\epsilon)} \frac{d\ln(\epsilon)}{d\eta} + \frac{\partial f}{\partial p^i} \frac{dp^i}{d\eta} = C[f] \quad (2)$$

Where $\epsilon = p \cdot a$ is the comoving energy, as this paper will discuss later. Here, $\frac{dx^i}{d\eta}$ equals to p^i for zeroth order. The $\frac{\partial f}{\partial p^i} \frac{dp^i}{d\eta}$ term can be dropped since it is in second order as photons direction would not change with time for unperturbed universe, and the discussions here are restricted up to first

order. In order to simplify (2), conformal Newtonian gauge metric is chosen for a linear perturbed FLRW metric, the metric is shown below:

$$ds^2 = a(\eta)^2 \left[-\left(1 + 2\Psi\right) d\eta^2 + \left(1 - 2\Phi\right) \delta_{ij} dx^i dx^j \right] \quad (3)$$

Ψ and Φ here are Newtonian potentials. After calculating Christoffel symbols using this metric, defining 4-momentum to be $P^\mu = \frac{dx^\mu}{ds}$ and set $ds^2=0$,

$$g_{\mu\nu} P^\mu P^\nu = -a^2 \left(1 + 2\Psi\right) (P^0)^2 + p^2 = 0 \quad (4)$$

Where

$$p^2 = a^2 \left(1 - 2\Phi \delta_{ij} P^i P^j\right) \quad (5)$$

Equations (4) shows that temporal component of 4-momentum can be expressed as $P^0 = p/a * (1 - \Psi) = \varepsilon / a^2 * (1 - \Psi)$, where $\varepsilon = p * a$ is the comoving energy shown in (2). These equations show that spatial component is $p/a * (1 + \Phi) p^i$. In order to find time dependence of ε and p^i , solving the geodesic equation with convective differentiation up to first order is helpful:

$$\frac{d \ln(\varepsilon)}{d\eta} = -\frac{d\Psi}{d\eta} + (\Phi' + \Psi') \quad (6)$$

Consider $\ln(\varepsilon)$ instead of ε because gravitational red shift occurs uniform for all wavelength of light, which means it is achromatic, as shown in (6) (derivative of $\ln(\varepsilon)$ over time is independent of ε , thus independent of frequency).

For zeroth order solution(i.e. when photon is traveling through a homogeneous and isotropic universe), using Bose-Einstein distribution to describe distribution function f .

$$f\left(\varepsilon\right) = \frac{1}{e^{\frac{\varepsilon}{a * T(\eta)}} - 1} \quad (7)$$

Where $T(\eta) = \frac{T_0}{a(\eta)}$ is the solution to zeroth order Boltzmann equation. When solving up to first order, collision term now takes part and distribution function now could have position, energy and direction dependence. Temperature perturbation factor Θ should be included now:

$$T(x^i, p^i, \eta) = T(\eta) (1 + \Theta(x^i, p^i, \eta)) \quad (8)$$

Substitute (8) back into Bose-Einstein distribution function (7) and apply binomial approximation:

$$f\left(\varepsilon, x^i, p^i, \eta\right) = \frac{1}{e^{\frac{\varepsilon}{a * T(1+\Theta)}} - 1} \cong \frac{1}{e^{\frac{\varepsilon * (1-\Theta)}{aT}} - 1} \cong f\left(\varepsilon\right) \left(1 - \Theta\left(x^i, p^i, \eta\right) \frac{\partial \ln(f)}{\partial \ln(\varepsilon)}\right)$$

When dealing with RHS of (2), the velocities of electrons are important since photons were colliding with moving baryons rather than stationary ones at last scattering surface. Here, defining scattering rate:

$$\Gamma = a\sigma_T n_e \quad (9)$$

Where σ_T is Thomson scattering rate $\approx 6.65 \times 10^{-29} \text{m}^2$ and n_e is number density of free electrons. The collision can be expressed as $e^-(q_{in}) + \gamma(p_{in}) \rightarrow e^-(q) + \gamma(p)$

In finding the RHS in (2) in electron's rest frame S' , integrating over all directions and momenta and get:

$$C' \left[f'(\epsilon', p') \right] = \frac{df'}{d\tau'} = -\Gamma f'(\epsilon', p') + \Gamma \int dp'_{in} \frac{f'(\epsilon', p'_{in})}{4\pi} \quad (10)$$

Where Γ is defined in (9). This is the scattering rate with respect to proper time in electron's rest frame. Then transform into background frame S (electrons moving at v_e) using Lorenz Boost, where $\epsilon' = \gamma\epsilon(1 - \mathbf{p} \cdot \mathbf{v}_e)$ and $\epsilon = \gamma\epsilon'(1 + \mathbf{p}' \cdot \mathbf{v}_e)$.

Since Thomson scattering would not change energy of photon, we have $\epsilon' = \epsilon'_{in}$

Using the invariance of the phase space density and expanding around the black-body distribution function:

$$f'(\epsilon'_{in}, p'_{in}) = f(\epsilon_{in}, p_{in}) = f(\epsilon) - \frac{df}{d\ln(\epsilon)} \left[(\mathbf{p} - \mathbf{p}'_{in}) \cdot \mathbf{v}_e + \Theta(\mathbf{p}_{in}) \right] \quad (11)$$

And

$$f'(\epsilon', p') = f(\epsilon, p) = f(\epsilon) - \frac{df}{d\ln(\epsilon)} \Theta(\mathbf{p}) \quad (12)$$

Substitute (11) and (12) back into (10) and simplify, the RHS of Boltzmann equation turns out to be:

$$C = \Gamma \frac{df}{d\ln(\epsilon)} \left(\Theta - \Theta_0 - \mathbf{p} \cdot \mathbf{v}_e \right) \quad (13)$$

Where $\Theta_0 = \int p'_{in} \frac{\Theta(p_{in})}{4\pi}$ is the monopole of the temperature distribution

Combine (26) and the LHS, this equation is obtained:

$$\frac{d\Theta}{d\eta} = \frac{d\Psi}{d\eta} + (\Phi' + \Psi') - \Gamma \left[\Theta - \Theta_0 - \mathbf{p} \cdot \mathbf{v}_e \right] \quad (14)$$

Solving this equation straightforwardly is hard, but constraining conditions to photons that they would not experience any collision after emitted from last scattering surface would simplify the situation, and the corresponding solution is the line-of-sight solution which will be discussed later. To help finding the line of sight solution, introduce optical depth here:

$$\tau = \int_{\eta}^{\eta_0} d\eta' \Gamma(\eta') \quad (15)$$

The optical depth here is the inverse of mean free path of photons, and η_0 is the time today. The physical interpretation of $e^{-\tau}$ is the probability of photons not being scattered from η to η_0 . Next introduce the visibility function:

$$g = \frac{\partial}{\partial \eta} (e^{-\tau}) = \tau' e^{-\tau} = \Gamma e^{-\tau} \quad (16)$$

The visibility function corresponds to the probability of photons being scattered at η , where g is 0 everywhere except a peak at recombination epoch. Now define S , the source function:

$$S(\eta, \mathbf{x}(\eta), \mathbf{n}) = e^{-\tau} [\Phi' + \Psi'] + g [\Theta_0 + \Psi - \mathbf{p} \cdot \mathbf{v}_e] \quad (17)$$

After multiplying both sides of (14) by $e^{-\tau}$ and rearrange, the equation now becomes neat:

$$\frac{d}{d\eta} (e^{-\tau} (\Theta + \Psi)) = S \quad (18)$$

So far, this paper has rearranged Boltzmann equation up to a simple equation (17), where solution is the line of sight solution. Suppose a photon is moving in $\mathbf{p} = -\mathbf{n}$ and arriving at $\mathbf{x}_0$ at $\eta = 0$, the trajectory is then $\mathbf{x}(\eta) = \mathbf{x}_0 + (\eta_0 - \eta) \mathbf{n}$. Using the fact that $\tau(\eta_0) = 0$ for present and $\tau(0) = \infty$ for early past and present day potential Ψ leads to monopole that people are not able to observe thus Ψ can be dropped, the solution reduces to

$$\Theta(\eta_0, \mathbf{x}_0, \mathbf{n}) = \int_0^{\eta_0} d\eta' S\left(\eta', \mathbf{x}_0 + \left(\eta_0 - \eta'\right) \mathbf{n}, \mathbf{n}\right) \quad (19)$$

This solution can be further simplified if assuming photons will not being scattered after recombination (i.e. $e^{-\tau} = 1$ after recombination and $e^{-\tau} = 0$ before recombination) and visibility function can be simplified to $\delta(\eta - \eta_{\text{recombination}})$ where δ here is the direc-delta function. The solution (18) further simplifies to

$$\Theta(\eta_0, \mathbf{x}_0, \mathbf{n}) = (\Theta_0 + \Psi)_{\text{at recombination}} - (\mathbf{n} \cdot \mathbf{v}_e)_{\text{at recombination}} + \int_{\eta_{\text{recombination}}}^{\eta_0} d\eta' (\Phi' + \Psi') \quad (20)$$

Which shows that there are three independent contributions to anisotropy of free streaming CMB photons. This paper will discuss these three contributions in section 3.

3. Discussions

3.1. Sachs Wolfe effect

(20) shows there are three independent solutions in the line-of-sight solutions. The first $\Theta_0 + \Psi$ component corresponds to the Sachs Wolfe effect [11,12]. Θ_0 by definition refers to the temperature of CMB photons on last scattering surface and Ψ by definition refers to the change in gravitational potential between that point on last scattering surface and now. It states the relationship between CMB temperature and difference in gravitational potential energy. This relationship can be modeled as $\frac{\Delta T}{T} = -\frac{1}{3} \Psi$ where Ψ here refers to gravitational potential [13]. It shows that when

CMB photons are propagating through the universe, it might experience a gravitational red shift due to climb out of a gravitational potential well ($\Psi > 0$, $\Delta T < 0$) or experience a gravitational blue shift due to climb into a gravitational potential well ($\Psi < 0$, $\Delta T > 0$). That means over-densities in photon distribution on large scale appear as cold spot across the CMB temperature graph. Adopting graph from [14]:

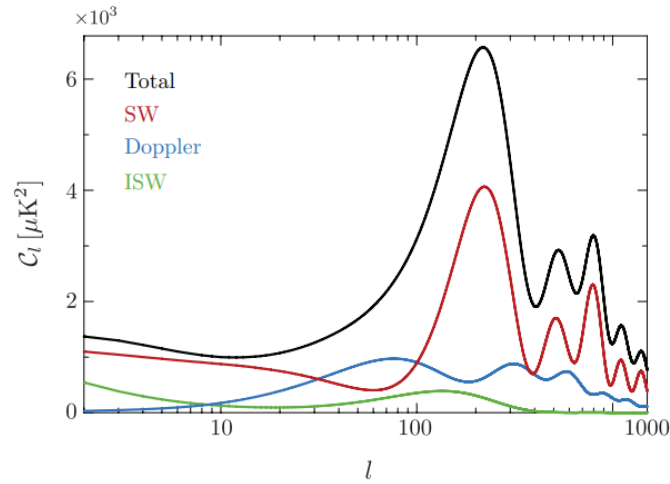


Figure 1. from Figure 6 of [14]. Contributions on CMB temperature anisotropies from SW, Doppler and ISW

This shows that most of the anisotropy on large angular scale is dominated by Sachs Wolfe effect, secondly dominated by Doppler Effect and least dominated by integrated Sachs Wolfe effect. Also, on small angular scale, SW is the strongest contribution as shown in the graph.

3.2. Doppler contributions

The middle term on the right-hand side of (20) shows the Doppler contribution. Since before the recombination era, the universe is full of electron-baryon-photon plasma. Due to quantum effect which causes small perturbations in pressure, the plasma is moving around. That means when photons were undergoing Thomson scattering, they scattered with moving electrons rather than stationary ones, which is the Doppler contribution here. As shown in figure 1, Doppler contribution goes to zero at small multipoles (i.e large scale) because it has straight relationship between electrons' velocity ($\mathbf{n} \cdot \mathbf{v}_e$) and electron's velocity is negligible on large scale. On all other scales, it's contribution has roughly the same order as SW effect.

3.3. Integrated Sachs Wolfe effect

The third term on the right-hand side of (20) is the contribution from ISW [15-17]. It shows the relationship between time dependence of gravitational potential well and CMB photons' temperature fluctuation. From [18], he explores the ISW effect on temperature fluctuation at low multipole:

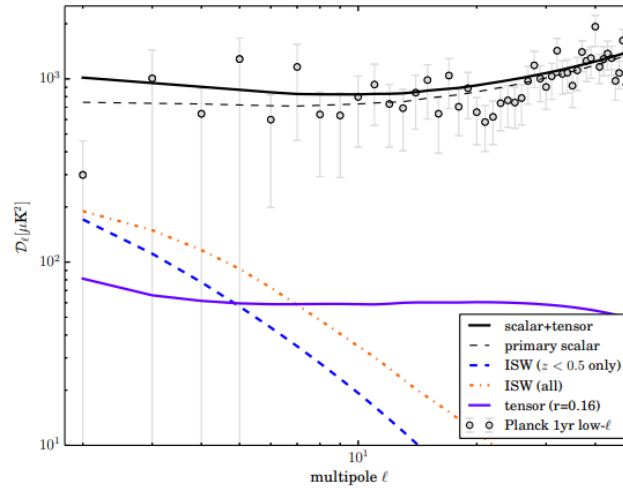


Figure 2. from Figure 1 of [18]. Early ISW and late ISW effect at low multipole

Figure 2 shows that there are several different time scales for ISW effect. In fact, since ISW refers to the evolution of gravitational potential well with respect to time, it could happen when the universe is not dominated by matter since expansion of large-scale structure is almost negligible in matter domination era. That causes two different ISW effect: the early ISW and later ISW.

The early ISW occurs just after the recombination era ($z \sim 3000$), which arises due to the shift from radiation dominated universe to matter dominated universe. Since it occurs when the universe is relatively small, it corresponds to relatively small angular scale ($l \sim 100-200$ on figure 1), where it helps in increasing amplitude of first acoustic oscillation peak.

The late ISW occurs when the universe is shifting from matter dominated to dark energy dominated ($z < 1$). It arises since dark energy would let the universe undergoing accelerated expansion, thus gravitational potential well around large scale structures would have a strong time dependence. It corresponds to low multipole moments ($l < 10$ on figure 1 and blue dotted line in figure 2), where it has roughly same order as SW effect on large angular scale. The late ISW could be one of the pieces of evidence for the existence of dark energy.

3.4. Refine the LoS to incorporate effects from reionization

Now if the Reionization epoch is patchy, the optical depth in (15) can no longer be defined as a constant. Construct a 2D field in describing the optical depth, where

$$\tau(\mathbf{n}, z) = \sigma_T n_{p,0} \int_0^z \frac{dz' (1+z')^2}{H(z')} x_e(\mathbf{n}, z') \quad (21)$$

This equation comes from [19], where $n_{p,0}$ is the present-day value for number density of protons, $H(z')$ is the Hubble parameter and $x_e(\mathbf{n}, z')$ is the ionization fraction in direction $\mathbf{n}$ at redshift z' . After computing the 2D field of $\tau(\mathbf{n}, z)$, some variables in previous part have to be modified to account for patchy Reionization. A simpler model can also be used. To estimate x_e , a simple tanh model [20] can be used, where x_e can be expressed as:

$$x_e = \frac{1}{2} \left(1 + \tanh \left(\frac{y(z_{re}) - y(z)}{\Delta y} \right) \right) \quad (22)$$

Where $y(z) = (1+z)^{2/3}$, $\Delta y = 3/2(1+z_{\text{re}})^{1/2} \Delta z$ with $\Delta z = 0.5$ [21]. When substitute x_e back into the equation, a relation between τ and z_{re} is obtained, where z_{re} is the central redshift for the Reionization epoch.

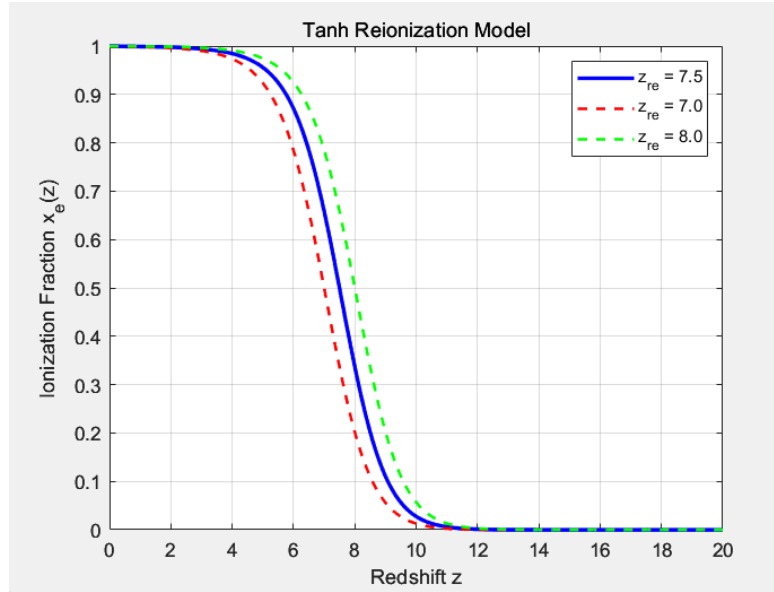


Figure 3. Ionization fraction as a function of redshift with different central redshift value of reioinzation. Without reionization, the ionization fraction would stay as a straight line near zero all the way up to today

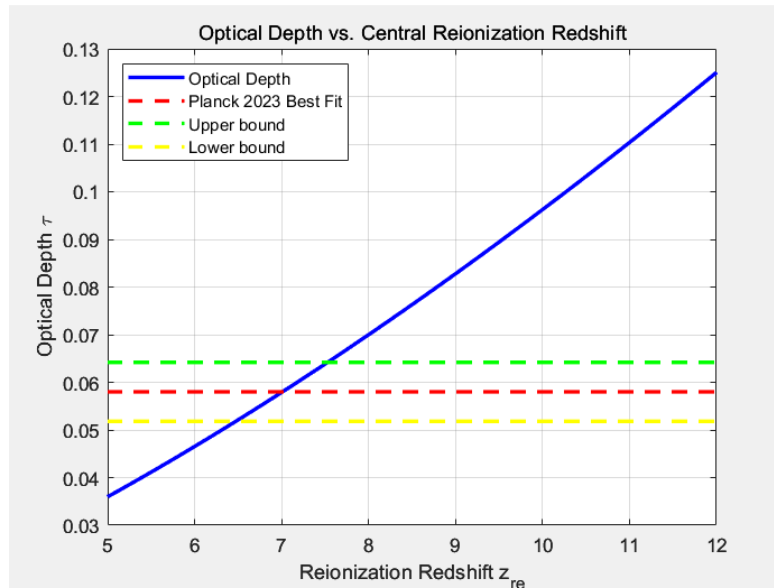


Figure 4. The optical depth as a function of central reionization redshift as a consequence of tanh model. The red line shows the value from 2023 Planck data [22], the region between green and yellow line is the Planck 1σ band

In order to connect back to the LoS model, the visibility function also needs to be modified. In the standard cosmological model, the visibility function exhibits a prominent peak at $z \sim 1100$, corresponding to the epoch of recombination. However, the onset of reionization at later times

introduces an additional population of free electrons into the intergalactic medium, thereby modifying the optical depth and the visibility function. Specifically, reionization induces a secondary, broad bump in the visibility function at redshifts $z \sim 6-10$, reflecting the probability of CMB photons being rescattered by newly liberated free electrons. Although this low-redshift contribution is subdominant in magnitude compared to the recombination peak, it has significant observational implications, particularly for large-scale CMB polarization.

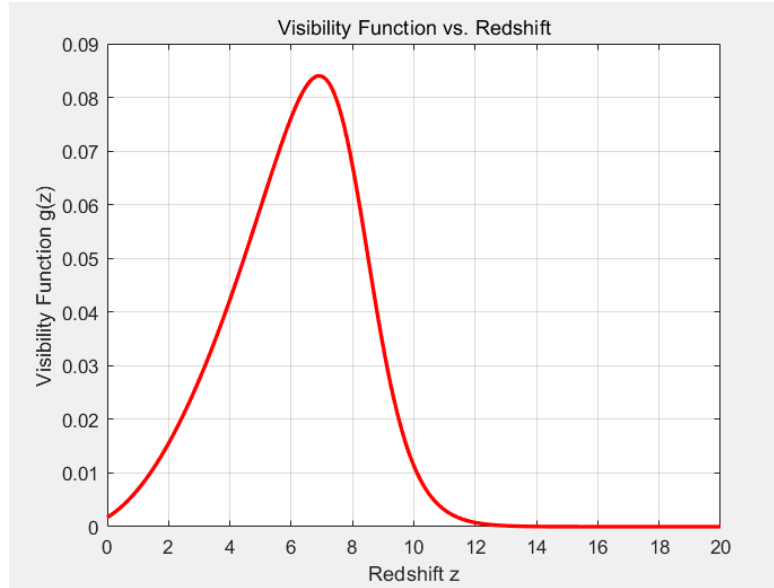


Figure 5. The unscaled visibility function obtained from equation (2.16) and (3.2), with effects from reionization, which shows a broad bump centred at $z \sim 7$

This visibility function encodes the probability of CMB photons being scattered at a given redshift. Today's value is roughly zero means that the probability of photons being scattered today is nearly zero. Instead of having this broad bump, the visibility function is a straight line near 0 without reionization.

The late time bump due to reionization leads to a non-zero late time contribution of some terms in original source function. From definition of source function in LoS in part 2, Doppler effects of CMB photons has a strong relation with the visibility function. The late-time enhancement in the visibility function due to reionization modifies the source function in the LoS formalism by reactivating scattering terms at low redshift. The Doppler term now has an early contribution and a late contribution. The early contribution comes from peculiar velocity of baryon-photon plasma at recombination, where photons interact with moving electrons at that time, as discussed in section 3.2. The late contribution comes from the broad bump of visibility function arises from the reionization, where photons interact with free electrons through Thomson scattering, which also generate anisotropies in polarization pattern in small angular scale. The effect of reionization on SW is weak compare to Doppler term, since SW relates to the gravitational potential well fluctuations from early universe but reionization occurs at small redshift. For ISW, on the other hand, reionization may play a crucial role. Reionization represents a transition from a transparent universe to opaque universe for CMB photons, the process of reionization can affect the evolution of gravitational potentials and the cosmological parameters that influence the ISW anisotropies.

3.5. Error propagation analysis in optical depth

Without effects from the reionization epoch, the optical depth can be defined as shown in (2.15), which has three main components: the scale factor a , the Thomson scattering rate σ_T and the number density of free electrons n_e . The error in optical depth then can be expressed as:

$$\Delta\tau = \sqrt{\left(\frac{\partial\tau}{\partial a} \Delta a\right)^2 + \left(\frac{\partial\tau}{\partial\sigma_T} \Delta\sigma_T\right)^2 + \left(\frac{\partial\tau}{\partial n_e} \Delta n_e\right)^2} \quad (23)$$

The uncertainty in Thomson scattering rate can be neglected since it is a constant that is calculated very precisely. Further approximating the integral in (15) by a sum:

$$\tau = \sum_i a_i \sigma_T n_{e,i} \Delta \eta_i \quad (24)$$

The error then can be expressed as:

$$\Delta\tau = \sqrt{\sum_i (\sigma_T n_{e,i} \Delta \eta_i \Delta a_i)^2 + (a_i \sigma_T \Delta \eta_i \Delta n_{e,i})^2} \quad (25)$$

If effects from the Reionization epoch is included, another factor $x_e(n, z')$ has to be included. During the Reionization epoch, x_e evolves rapidly, thus uncertainty in x_e has a strong relationship with uncertainty in optical depth. From previous part, the error in optical depth is then:

$$\Delta\tau = \sqrt{\int dz' \left(\left(\frac{(1+z')^2}{H(z')} \sigma_T n_{p,0} \Delta x_e(n, z') \right)^2 + \left(\frac{(1+z')^2}{H(z')} x_e(n, z') \sigma_T \Delta n_{p,0} \right)^2 \right)} \quad (26)$$

The uncertainty in the ionization fraction can be expressed as:

$$\Delta x_e = \sqrt{\left(\frac{\partial x_e}{\partial z_{re}} \Delta z_{re} \right)^2 + \left(\frac{\partial x_e}{\partial \Delta z} \Delta (\Delta z) \right)^2} \quad (27)$$

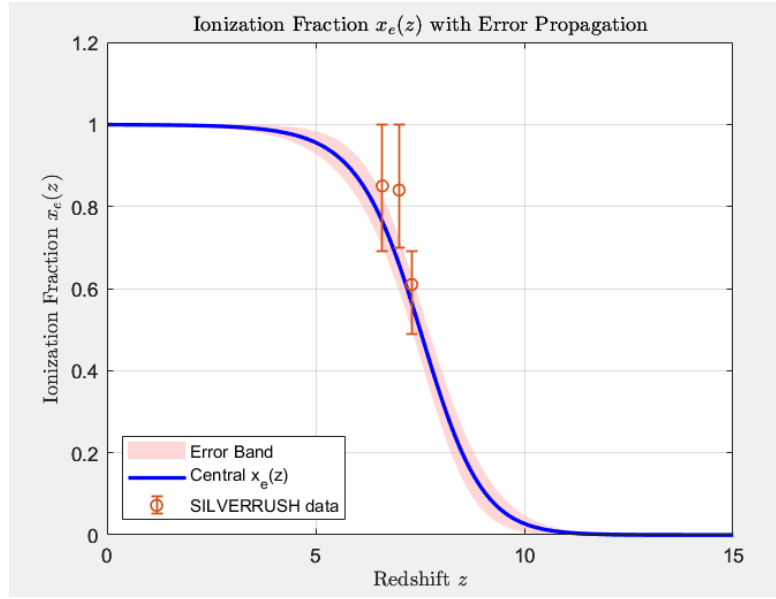


Figure 6. The central ionization fraction with error band using the tanh model with comparison, assume the uncertainty in z_{re} is 0.2 and uncertainty in Δz is 0.3. In comparison, the orange data points with uncertainty bar come from Table 2 of [23]

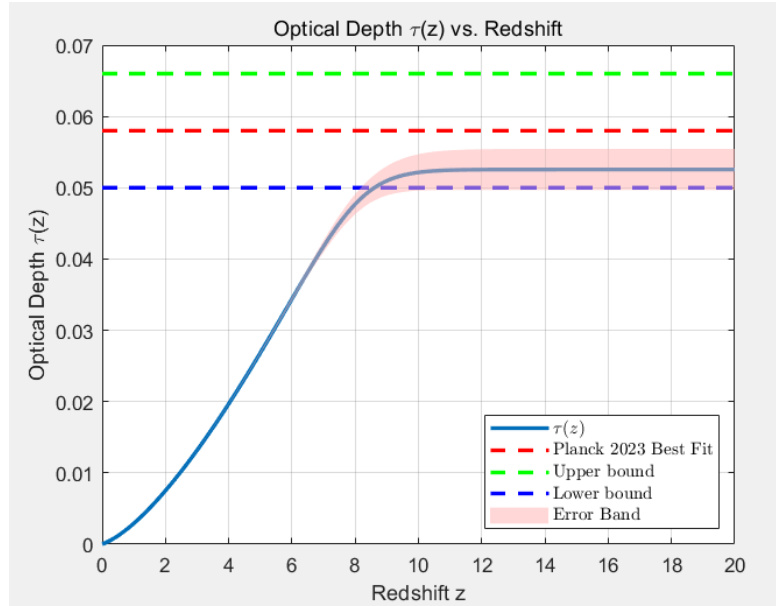


Figure 7. The uncertainty band of optical depth due to error in ionization fraction using tanh model

As shown in Figure 6, ionization fraction only has a large uncertainty during the Reionization epoch. That leads to an uncertainty band starts at reionization epoch, as shown in figure 7. The solid blue line in figure 7 illustrates the optical depth as a function of upper z limit in equation (21), the light red region shows the uncertainty band which only considers uncertainty from ionization fraction, as shown in figure 6. From figure 7, the optical depth would have a value of 0.0526 ± 0.0029 , which almost sits in the Planck 1σ band from 2023, which has a value of 0.058 ± 0.006 .

3.6. Why do people need to study the LoS and anisotropy of CMB

In studying cosmological perturbation theory, hundreds of coupled differential equations are required, which makes solving them computationally intensive. However, by making some assumption, people can reduce these differential equations into a single line of sight integral (19), which massively reduce computational work. At the same time, it clearly identifies three distinct contributions in (20), which is concordance with reality.

Studying anisotropy of CMB is like decoding blueprint for the early universe. On the one hand, CMB as relic radiation contains many information about the early universe such as deducing matter distribution in the early universe or testing inflation theory. On the other hand, CMB angular power spectrum encodes information about cosmological parameters such as Hubble parameter and density of different component of our universe. Different components in (20) encode different information about the universe. ISW could probe the evolution of gravitational potential, SW could probe the distribution of matter and Doppler could probe density variation in the universe prior to the recombination era. Refining the LoS to incorporate reionization could modify the CMB power spectrum, especially in Doppler term and ISW term. Without refining LoS to include the impact of reionization, some subtle but important features of the CMB power spectrum such as small angular scale polarization would be misrepresented, leading to potential biases in cosmological parameter estimation, particularly in constraints on the reionization redshift and the duration of reionization.

4. Conclusion

In this paper, the standard Boltzmann derivation of the line-of-sight solution is presented and shows that there are three independent contributions: SW, Doppler and ISW. Then from the result of the line-of-sight solution, this paper discuss mechanisms of these three components, where SW effect states the relationship between static gravitational potential and temperature fluctuation of free CMB photons, Doppler contribution shows the effect on photons scattered with moving electrons on last scattering surface and ISW illustrates the effect of time dependence of gravitational potential well on CMB anisotropy. Then a possible way to refine the LoS to incorporate effects from the Reionization is proposed, by varying the optical depth and the visibility function with error propagation included. This line-of-sight solution provides a simplification for hundreds of coupled differential equations when dealing with Boltzmann equation and Einstein equation straightforwardly.

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