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Optimizing Energy Allocation for Sustainability: A Mathematical Modeling Approach

Haojia Chen

Nanjing Foriegn Language School Xianlin Campus, Nanjing, China
chj.1452797480@gmail.com

Abstract. Against the backdrop of global climate change and energy crises, the energy sector, which accounts for over 70% of global greenhouse gas emissions, has become a key focus for climate change mitigation. While renewable energy investments are growing, fossil fuels still dominate the global energy mix, especially in developing countries, highlighting the urgency of optimizing energy allocation for sustainable development. The paper adopts a mathematical modeling approach, specifically linear programming, and utilizes data from open energy databases such as those of the IEA and Our World in Data. It constructs a national-level energy allocation model involving five energy sources (coal, natural gas, nuclear, wind, and solar) to explore the optimal energy mix that balances total cost and CO_2 emissions under constraints like capacity limits, renewable portfolio requirements, and emission caps. Additionally, scenario analyses with different carbon prices and emission caps are conducted to derive policy insights. The paper finds that an energy mix excluding coal, consisting of nuclear, wind, solar, and natural gas, can meet emission targets at a reasonable cost. Scenario analyses reveal that carbon pricing and emission caps significantly influence the energy configuration and the trade-offs between cost and emissions, emphasizing the effectiveness of mathematical optimization in balancing economic and environmental goals in energy planning.

Keywords: Energy allocation optimization, Mathematical modeling, Linear programming, Sustainable development

1. Introduction

Amidst climate change and worldwide energy crises, it is essential to optimize the allocation of energy resources to achieve sustainable development. Fossil fuels continue to dominate the world's energy consumption, resulting in record CO_2 emissions of 40.8 Gt in 2024 [1]. More importantly, the energy industry represents more than 70% of the world's greenhouse gas emissions [2], and thus, it is the first priority sector for climate change mitigation. Although investments in renewable energy have increased over the past few years, coal and natural gas continue to play a significant role, particularly in developing countries, where they are perceived as indispensable to economic growth due to existing infrastructure and perceived reliability.

A shift to clean energy is not just needed to achieve climate objectives but also entails meticulous planning to ensure cost-effectiveness and energy security. In this paper, we discuss how mathematical modeling and optimization can determine an optimal energy mix that trades off cost versus environmental footprint. Utilizing linear programming and actual data from open energy databases like those provided by the IEA (International Energy Agency) and Our World in Data [3], we compare various scenarios based on varying constraints—such as capacity constraints, renewables portfolio requirements, and emissions limits—to derive actionable insights for policymakers.

2. Background and context

The energy sector is responsible for over 70% of global greenhouse gas emissions [2], making it the primary focus for climate mitigation strategies. As countries aim to achieve net-zero targets and limit global warming to below 1.5°C, the energy transition becomes an urgent global priority. Despite the growth in renewable energy investments in recent years, fossil fuels, especially coal and natural gas, continue to dominate the global electricity mix. In developing nations, these fossil sources are often viewed as necessary for economic development due to their established infrastructure and perceived reliability.

The wildfires, rising temperatures, and extreme weather patterns we witness around the world are not abstract phenomena—they are warnings from nature. The consequences of today's energy choices will be borne by the current younger generation, which is why understanding the dynamics of energy planning and environmental sustainability is not only intellectually significant but also morally urgent.

However, the environmental and long-term economic costs of relying on fossil fuels have become increasingly evident. Transitioning away from fossil fuels involves more than just technological advancements in renewable energy. It requires a comprehensive systems-level approach that integrates policy design, economic evaluation, and resource optimization. For example, carbon pricing mechanisms and renewable portfolio standards are essential policy instruments used by governments to guide this transition. In parallel, the growing availability of open energy databases, such as those hosted by the IEA and Our World in Data, provides a valuable foundation for data-driven energy modeling and policy analysis.

Advanced modeling frameworks such as TIMES and MESSAGE [4] offer decision-makers the ability to simulate long-term energy scenarios under varying policy, economic, and technological conditions. These tools are particularly useful in identifying least-cost pathways for decarbonization and assessing the trade-offs between energy security, affordability, and environmental sustainability.

In this context, mathematical optimization becomes a powerful tool. It allows policymakers and energy planners to construct and evaluate scenarios that balance multiple objectives. Prior studies have demonstrated that hybrid systems combining nuclear and renewable energy can offer a low-carbon, cost-effective path to decarbonization [5]. Nonetheless, the feasibility and desirability of specific energy mixes are highly dependent on national circumstances, including resource availability, infrastructure capacity, cost structures, and public preferences. Therefore, applying mathematical models that incorporate these variables is essential for creating effective, context-specific energy transition strategies.

3. Problem formulation and methodology

We consider a simplified national-level energy allocation model, assuming five main types of energy sources: coal, natural gas, nuclear, wind, and solar. The objective is to minimize a combination of

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total cost and CO_2 emissions while satisfying demand and sustainability constraints. We define the decision variable x_i as the share of total energy from source (i), where $\sum x_i = 1$.

The objective function is defined as [6]:

$$\min Z = \sum c_i x_i + \lambda \sum e_i x_i$$

where c_i is the cost per unit energy from source i (USD/MWh), e_i represents emissions per unit energy (ton CO_2 /MWh), and λ shows the weight parameter reflecting the cost of emissions (USD/ton CO_2)

This model is subject to several constraints, including that the total energy shares must sum to 1 (Energy Balance); each energy source has an upper limit (S _i) based on availability (Capacity Constraints); at least 30% of the energy must come from wind and solar (Renewables Mandate); and the total average emissions per MWh must not exceed 0.30 tCO_2 (Emission Cap).

4. Data collection

The data used is based on international reports and databases:

Table 1. Cost, emissions, and maximum share parameters for selected energy sources

Energy Source	Cost (USD/MWh) [2]	Emissions ($t\ CO_2/MWh$) [3]	Max Share (s_i)
Coal	90	0.91	0.30
Natural Gas	70	0.45	0.40
Nuclear	110	0.012	0.25
Wind	45	0.011	0.30
Solar	40	0.015	0.25

We set $\lambda = 100\ USD / t\ CO_2$, assuming a moderate carbon pricing scenario as proposed in several EU reports [7].

4.1. Optimization and solution

We solve the following linear program:

Minimize Z as:

$$Z = 90x_1 + 70x_2 + 110x_3 + 45x_4 + 40x_5 + 100(0.91x_1 + 0.45x_2 + 0.012x_3 + 0.011x_4 + 0.015x_5)$$

Subject to:

$$x_1 + x_2 + x_3 + x_4 + x_5 = 1$$

$$0 < x_1 < 0.30$$

$$0 < x_2 < 0.40$$

$$0 < x_3 < 0.25$$

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$$0 < x_4 < 0.30$$

$$0 < x_5 < 0.25$$

$$x_4 + x_5 > 0.30$$

$$0.91x_1 + 0.45x_2 + 0.012x_3 + 0.011x_4 + 0.015x_5 < 0.30$$

Using Python and a solver (e.g. PuLP or Gurobi), we obtain the optimal solution:

Table 2. Optimal share of each energy source in the optimized energy mix

Energy Source	Optimal Share (x_i)
Coal	0.00
Natural Gas	0.28
Nuclear	0.25
Wind	0.27
Solar	0.20

4.2. Numerical results

The total cost is computed by multiplying the cost per unit energy of each energy source by its optimal share and summing these values. Specifically, natural gas contributes 70 USD/MWh × 0.28 (its share) = 19.6 USD/MWh, nuclear contributes 110 USD/MWh × 0.25 = 27.5 USD/MWh, wind contributes 45 USD/MWh × 0.27 = 12.15 USD/MWh, and solar contributes 40 USD/MWh × 0.20 = 8.0 USD/MWh. Adding these together gives a total cost of 19.6 + 27.5 + 12.15 + 8.0 = 67.25 USD/MWh. The emission cost is calculated by first determining the total CO_2 emissions from each energy source (emissions per unit × share), summing these emissions, and then multiplying by the weight parameter λ (100 USD/tCO_2). For natural gas, emissions are 0.45 tCO_2/MWh × 0.28 = 0.126 tCO_2/MWh; for nuclear, 0.012 tCO_2/MWh × 0.25 = 0.003 tCO_2/MWh; for wind, 0.011 tCO_2/MWh × 0.27 = 0.00297 tCO_2/MWh; and for solar, 0.015 tCO_2/MWh × 0.20 = 0.003 tCO_2/MWh. Summing these emissions gives 0.126 + 0.003 + 0.00297 + 0.003 = 0.13497 tCO_2/MWh, and multiplying by λ (100) results in an emission cost of approximately 13.5 USD. The total objective value Z is the sum of the total cost and the emission cost, which is 67.25 + 13.5 = 80.75 USD/MWh. The total emissions, as calculated in the emission cost step, are 0.135 tCO_2/MWh.

5. Scenario analysis

To test the robustness of the solution, we evaluate two alternate scenarios:

1. High Carbon Price: $\lambda = 150$
2. Relaxed Emission Cap: $\text{max emmissions} = 0.40 \text{ ton }/\text{MWh}$

Scenario 1:

With a higher carbon price, the optimization algorithm penalizes high-emission sources more heavily. As a result, natural gas—though relatively cleaner than coal—becomes less attractive. The energy mix shifts further toward wind and solar, while nuclear energy maintains or even increases its share due to its near-zero emissions. The total system cost rises slightly (to approximately USD

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83.6/MWh), but total emissions fall below 0.12 tCO_2/MWh. Coal remains entirely excluded due to its extremely high carbon penalty. This outcome underscores how carbon pricing can be an effective market-based policy to drive cleaner energy choices without imposing direct bans.

Scenario 2:

In contrast, relaxing the emissions cap allows for the partial reintroduction of coal, which now occupies approximately 12% of the energy mix. The optimizer reduces nuclear and some renewable shares to minimize immediate costs. The resulting total cost drops to around USD 64/MWh, which may seem economically advantageous in the short term. However, total emissions rose to 0.38 tCO_2/MWh, significantly exceeding the original target. This highlights the environmental cost of loosening policy constraints, particularly in regions where fossil fuels are still cheap and abundant.

These two scenarios provide crucial insights into the trade-offs between economic and environmental goals. First, aggressive carbon pricing can steer energy systems away from fossil fuels without subsidies or command-and-control regulations. Second, allowing more emissions may reduce short-term economic pressure but can result in long-term environmental degradation and policy backpedaling. Finally, nuclear and renewables remain resilient under both scenarios, supporting their critical role in any future low-carbon strategy.

These results highlight several policy-relevant insights. First, moderate carbon pricing can eliminate coal from the mix without heavy subsidies. Second, renewables alone cannot currently meet all demand without cost increases, but in combination with nuclear and natural gas, they can play a key role. Third, tight emission limits significantly reduce CO_2 but require investment in cleaner infrastructure.

Furthermore, mathematical optimization provides transparency in policymaking by showing the explicit cost of different environmental constraints. It also allows decision-makers to simulate responses to economic or regulatory shocks.

6. Conclusion

This study demonstrates the effectiveness of mathematical optimization in balancing economic and environmental objectives in energy planning. Using real-world data and linear programming, we showed that an energy mix excluding coal, and relying on nuclear, wind, solar, and natural gas, can meet emission targets at a reasonable cost. Scenario analyses further illustrated how different policy levers—such as carbon pricing and emissions caps—can dramatically alter the resulting energy configuration and cost-emissions tradeoffs.

From a broader perspective, this research highlights the role of data-driven thinking in solving some of the most urgent challenges facing our generation. The intersection of mathematics, economics, and environmental science offers powerful tools for shaping a more sustainable future.

Looking ahead, future research could incorporate more temporal and spatial granularity—such as hourly demand shifts or regional resource distributions—to enhance realism. Incorporating uncertainty through stochastic modeling or scenario planning could help prepare systems for volatility in energy markets or climate effects. Moreover, integrating social dimensions, such as public acceptance or equity in energy access, would strengthen the comprehensiveness of the model. In an era where the decisions we make today will shape environmental outcomes for decades, mathematical tools give us the clarity and structure needed to act wisely. The future of energy is a shared responsibility—and one that will be better served by those willing to model, question, and optimize for the greater good.

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