

# ***Research on the Application of Linearization Model Methods in Low Altitude Airspace Planning***

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**Abstract:** This paper investigates the issue of conflict resolution between spatiotemporal conflict airspace requirements in low-altitude airspace planning. Regarding to the complex constraints of this multi-objective optimization problem, the quadratic constraints in the problem are relaxed by linear approximation method and the mixed-integer linear multi-objective programming model is established through absolute value linearization and multidimensional Boolean product linearization methods. Subsequently, specific scenarios are designed, and the user airspace application data is simulated. Using the solver to analyze the simulation data, and the results indicate that as the demand scale increases, the solving time increases exponentially, and the solving effectiveness gradually declines after exceeding a certain scale. Finally, through the analyzing of the linear relaxation method, combining with specific data, the deviation of the model from the actual problem is calculated. Due to the relatively small theoretical error, it is proven that this linearized model is applicable to solving the low-altitude airspace conflict resolution problem.

**Keywords:** Linear approximation, Mixed-integer linear multi-objective programming, Low-altitude airspace planning

## **1. Introduction**

With the rapid development of drone and small aircraft technology, the low-altitude economy has gradually become a hot topic. The low-altitude economy is an economic form derived from the low-altitude flight activities of civil manned or unmanned aircraft, encompassing aircraft research and development, manufacturing, commercial operations, and infrastructure construction.

The rapid development of the low-altitude economy and the significant relaxation of regulations in low-altitude airspace have provided aircraft with greater operational space, while also posing significant challenges to the planning and regulation of low-altitude airspace. Airspace management not only involves technical details such as the flight paths and altitudes of aircraft but also requires consideration of multiple dimensions including safety, efficiency, and environmental protection.

Low-altitude airspace planning encompasses multiple aspects: grid technology, remote sensing, communication and networking, aircraft route planning, and operational management [1]. This study primarily focuses on the construction of geofencing technology and airspace stratification technology within the airspace structure, specially the conflict resolution problem between spatiotemporal conflict airspace requirements. Although this technology segments the airspace and reduces its utilization efficiency, it offers unparalleled advantage of safety [2].

Generally, aircraft users submit airspace usage requests to the control center, which include the horizontal position area, the altitude range of the airspace, and the time range for usage. When two airspace applications are in conflict simultaneously in terms of time, altitude, and horizontal range, it is considered an airspace conflict. Specifically, this situation occurs when there is an overlap in all three dimensions of the airspace or when the distances between the ranges are less than the safety intervals.

Due to the large number of users, it is necessary to adjust conflicting user demands and optimize airspace resource allocation in order to maximize user needs and utilize airspace resources more effectively. The airspace applications in conflict are shown in Figure 1.

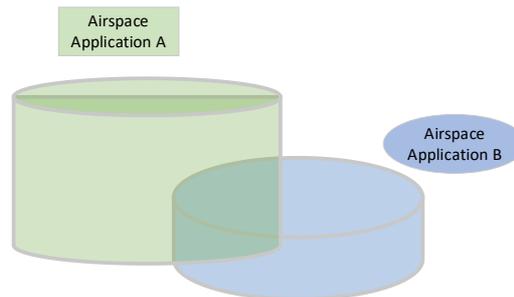


Figure 1: Airspace applications are in conflict

## 2. Literature review

Since the 1940s, researchers have conducted extensive studies on various issues in the field of airspace planning, leading to a plethora of algorithms and models. Numerous scholars have focused on airspace conflict detection and resolution planning based on aircraft trajectories and spatial grid partitioning systems.

Kuchar et al. [3] conducted a review of aircraft dynamic trajectory prediction, conflict detection, and resolution techniques since 2000, investigating 68 modeling methods for conflict detection and resolution. They classified and evaluated these methods based on the dimensions of state information, dynamic propagation methods, conflict detection thresholds, conflict resolution methods, maneuvering sizes and so on.

Pallottino et al. [4] studied the issue of conflicts between aircrafts flying in shared airspace. They established mixed-integer linear programming models for two scenarios: one allowing only changes in speed and the other allowing only changes in heading angle, and employed solvers to address the problem.

Ayuso et al. [5] proposed a hybrid 0-1 linear optimization model based on geometric transformations for airspace conflict detection and resolution. Knowing the initial coordinates, directional angles, and flight altitudes, conflict resolution is achieved by minimizing several objective functions (such as the variation in each dimension and the number of dimensions that changed) while enforcing a return to the original flight configuration in the absence of aircraft conflicts.

Tang et al. [6] developed a conflict detection and resolution algorithm for small fixed-wing aircraft based on a Spatial Grid Partitioning System (SGPS). This algorithm determines whether two trajectories pass through the same grid space within an overlapping time window. They also proposed techniques based on time scheduling and vertical adjustment to achieve conflict resolution.

Miao et al. [7] proposed a low-altitude flight conflict detection algorithm based on a multi-level grid spatiotemporal index. This algorithm transforms the traditional conflict computation based on trajectory traversal into a grid conflict state query within a distributed grid database. The algorithm first constructs a spatiotemporal coding model based on airspace, then uses grid coding to identify trajectories and no-fly zones, and subsequently establishes a multi-level grid spatiotemporal index

according to the table structure of the grid database. Finally, the algorithm designs an optimized query method to detect conflicts within the grid database.

These studies, in conducting airspace conflict detection and resolution, not only involve adjustments to the horizontal and vertical dimensions of the airspace but also include planning for the aircraft's heading and speed.

Due to the inherent non-linear nature of airspace conflict issues, it is difficult to establish a linear model, which increases the complexity of the solution. Some studies have linearized the non-linear problems to reduce the difficulty and directly applied solvers for resolution, while others have designed heuristic functions for solving. Tamas et al. [8] analyzed a lot of researches on the application of mixed-integer nonlinear optimization methods to air traffic management issues. They established a mixed-integer nonlinear model addressing airspace conflicts and resolution problems, and solved the model using a MINLP solver. They also examined the application of heuristic methods such as Branch and Bound, linearization methods, and neighborhood search in problem-solving.

### 3. Mathematical programming model

This study addresses the problem of airspace conflict detection and resolution by constructing a mixed-integer linear model. The conflict resolution is achieved through adjustments in the horizontal, vertical, and temporal dimensions. Different dimensions have different priorities, while minimizing adjustments to user applications and rejecting requests as little as possible. And solutions are obtained using a solver.

#### 3.1. Notations

The symbols used in the model of this article are explained as follows:

Table 1: Parameters definition

Parameters	Definition
$\delta_d$	The maximum value of the adjustment quantity in each dimension
$\lambda_d$	Safety distances for each dimension
$p_i$	The centroid of the airspace application i
$pr_{i,d}$	The radius of the airspace application i in each dimension
$bmin_d$	The lower bound of airspace application in each dimension
$bmax_d$	The upper bound of airspace application in each dimension

Table 2: Variables definition

Variables	Definition
$x_{i,d}$	The spatiotemporal translation amount for airspace application i
$dplus_{i,d}$	The positive deviation of the spatiotemporal adjustment amount for airspace application i
$dmini_{i,d}$	The negative deviation of the spatiotemporal adjustment amount for airspace application i
$y_i$	The airspace application i has been adjusted or not
$z_i$	The airspace application i has been rejected or not
$tbool_{i,j}$	The airspace applications i and j conflict in the time dimension or not
$hbool_{i,j}$	The airspace applications i and j conflict in the altitude dimension or not
$tempbool_{i,j,d}$	Boolean identifier
T	Total amount of space adjustment
S	Total amount of time adjustment
Adjustnum	Total number of adjusted airspace applications
rejectnum	Total number of rejected airspace applications

### 3.2. Equations

Based on the above symbol description, a mixed-integer nonlinear programming model is constructed for the low-altitude airspace planning problem as follows:

$$\min p1 * rejectnum + p2 * Adjustnum + p3 * T + p4 * S \quad (1)$$

S.t.

$$p_{i,d} + x_{i,d} > bmin_d + pr_{i,d} \quad (2)$$

$$p_{i,d} + x_{i,d} < bmax_d - pr_{i,d} \quad (3)$$

$$|x_{i,d}| < \delta_d \quad (4)$$

$$(1 - bool(|p_{i,1} + x_{i,1} - p_{j,1} - x_{j,1}| > \lambda_1 + pr_{i,1} + pr_{j,1})) * (1 - bool(|p_{i,2} + x_{i,2} - p_{j,2} - x_{j,2}| > \lambda_2 + pr_{i,2} + pr_{j,2})) * (1 - bool(temp > \lambda_3 + pr_{i,3} + pr_{j,3})) = 0 \quad (5)$$

$$|(p_{i,3} + x_{i,3}, p_{i,4} + x_{i,4}) - (p_{j,3} + x_{j,3}, p_{j,4} + x_{j,4})| = temp \quad (6)$$

$$x_{i,d} - dplus_{i,d} + dmini_{i,d} = 0 \quad (7)$$

$$dplus_{i,d} < \delta_d + z_i * M \quad (8)$$

$$dmini_{i,d} < \delta_d + z_i * M \quad (9)$$

$$dplus_{i,d} < y_i * M \quad (10)$$

$$dmini_{i,d} < y_i * M \quad (11)$$

$$T = \sum_{i \in N} dplus_{i,1} + dmini_{i,1} \quad (12)$$

$$S = \sum_{i \in N}^{d=2..4} dplus_{i,d} + dmini_{i,d} \quad (13)$$

Equation (2) and Equation (3) represent the marginal constraint. Equation (4) represents the constraint on the maximum adjustment amount. Equation (5) and Equation (6) indicates that there is no conflict between the airspace demands. Equation (7), Equation (8), Equation (9), Equation (10), Equation (11), Equation (12) and Equation (13) represent the relationship between variables.

## 4. Linearization

Clearly, the model has multiple nonlinear constraints. Therefore, it is considered to linearize the model and transform it into a mixed-integer quadratic programming model.

### 4.1. Multidimensional boolean product linearization

Introducing infinitesimals and Boolean variables into the variables to achieve the decomposition of equation (5).

$$|p_{i,2} + x_{i,2} - p_{j,2} - x_{j,2}| + M * hbool_{i,j} > \lambda_2 + pr_{i,2} + pr_{j,2} \quad (14)$$

$$|p_{i,1} + x_{i,1} - p_{j,1} - x_{j,1}| + M * tbool_{i,j} > \lambda_1 + pr_{i,1} + pr_{j,1} \quad (15)$$

$$temp > \lambda_1 + pr_{i,1} + pr_{j,1} + (tbool_{i,j} - 1) * M + (hbool_{i,j} - 1) * M \quad (16)$$

Proof.

$$|p_{i,2} + x_{i,2} - p_{j,2} - x_{j,2}| > \lambda_2 + pr_{i,2} + pr_{j,2} \Rightarrow hbool_{i,j} = \{0,1\} \quad (17)$$

$$|p_{i,2} + x_{i,2} - p_{j,2} - x_{j,2}| < \lambda_2 + pr_{i,2} + pr_{j,2} \Rightarrow hbool_{i,j} = 1 \quad (18)$$

$$minhbool_{i,j} \&\& |p_{i,2} + x_{i,2} - p_{j,2} - x_{j,2}| > \lambda_2 + pr_{i,2} + pr_{j,2} \Rightarrow hbool_{i,j} = 0 \quad (19)$$

$$|p_{i,1} + x_{i,1} - p_{j,1} - x_{j,1}| > \lambda_1 + pr_{i,1} + pr_{j,1} \Rightarrow tbool_{i,j} = \{0,1\} \quad (20)$$

$$|p_{i,1} + x_{i,1} - p_{j,1} - x_{j,1}| < \lambda_1 + pr_{i,1} + pr_{j,1} \Rightarrow tbool_{i,j} = 1 \quad (21)$$

$$mintbool_{i,j} \&\& |p_{i,1} + x_{i,1} - p_{j,1} - x_{j,1}| > \lambda_1 + pr_{i,1} + pr_{j,1} \Rightarrow tbool_{i,j} = 0 \quad (22)$$

if  $tbool_{i,j} = 0$  or  $hbool_{i,j} = 0$ , then  $|p_{i,1} + x_{i,1} - p_{j,1} - x_{j,1}| > \lambda_1 + pr_{i,1} + pr_{j,1} - M$ . if  $tbool_{i,j} = 1$  and  $hbool_{i,j} = 1$ , then  $|p_{i,1} + x_{i,1} - p_{j,1} - x_{j,1}| > \lambda_1 + pr_{i,1} + pr_{j,1}$ .

This method allows for the decomposition of equation (5) without altering the meaning of the constraints in it.

## 4.2. Quadratic function linearization

The linearization of the distance (squared term) between two points can be achieved by introducing an intermediate variable, which can transform the point information of that distance into interval information. Although this may introduce some bias in the precise solution of the model, it allows for the linearization of the constraints, effectively reducing the problem space. Sentence (6) can be linearized in this way.

$$temp^2 = (p_{i,3} + x_{i,3} - p_{j,3} + x_{j,3})^2 + (p_{i,4} + x_{i,4} - p_{j,4} + x_{j,4})^2 \quad (23)$$

$$temp^2 > 2 * min(|p_{i,3} + x_{i,3} - p_{j,3} + x_{j,3}|) * |p_{i,3} + x_{i,3} - p_{j,3} + x_{j,3}| - min(|p_{i,3} + x_{i,3} - p_{j,3} + x_{j,3}|)^2 + 2 * min(|p_{i,4} + x_{i,4} - p_{j,4} + x_{j,4}|) \quad (24)$$

$$* |p_{i,4} + x_{i,4} - p_{j,4} + x_{j,4}| - min(|p_{i,4} + x_{i,4} - p_{j,4} + x_{j,4}|)^2 \\ temp^2 > 2 * max(|p_{i,3} + x_{i,3} - p_{j,3} + x_{j,3}|) * |p_{i,3} + x_{i,3} - p_{j,3} + x_{j,3}| - max(|p_{i,3} + x_{i,3} - p_{j,3} + x_{j,3}|)^2 + 2 * max(|p_{i,4} + x_{i,4} - p_{j,4} + x_{j,4}|) \\ * |p_{i,4} + x_{i,4} - p_{j,4} + x_{j,4}| - max(|p_{i,4} + x_{i,4} - p_{j,4} + x_{j,4}|)^2 \quad (25)$$

Proof.

If  $c \in [a, b]$ , then  $c^2 > a^2 + 2a * (c - a) = 2ac - a^2$ ,  $b^2 > c^2 + 2c * (b - c) = 2bc - c^2$ .

Since the constraint on  $temp$  is greater than a certain value,  $temp$  can be replaced by the right-hand side of the equation (18) and equation (19).

Obviously,  $\max(|p_{i,3} + x_{i,3} - p_{j,3} - x_{j,3}|) = |p_{i,3} - p_{j,3}| - 2 * \delta_3$ ,  $\min(|p_{i,3} + x_{i,3} - p_{j,3} - x_{j,3}|) = |p_{i,3} - p_{j,3}| + 2 * \delta_3$ ,  $\max(|p_{i,4} + x_{i,4} - p_{j,4} - x_{j,4}|) = |p_{i,4} - p_{j,4}| - 2 * \delta_4$ ,  $\min(|p_{i,4} + x_{i,4} - p_{j,4} - x_{j,4}|) = |p_{i,4} - p_{j,4}| + 2 * \delta_4$ .

### 4.3. Absolute value linearization

For absolute value constraints, they can be implemented by adding a big M and Boolean variables to the model.

$$tempbool_{i,j,d} * M + p_{i,d} + x_{i,d} - p_{j,d} - x_{j,d} > 0 \quad (26)$$

$$mintempbool_{i,j,d} \quad (27)$$

Add  $tempbool * M$  to the right side of the inequalities, and in this way, the absolute value constraints will be linearized.

### 4.4. Auxiliary variable constraints

After linearization, the objective can be represented as  $p1 * rejectnum + p2 * Adjustnum + p3 * T + p4 * S + tempbool_{i,j,d} + dplus_{i,d} + dmini_{i,d}$ .

The Boolean variables introduced are defined in relation to two objects, and the relationship between these two is relative; it can be described using  $(i, j)$  as well as  $(j, i)$ . Based on the above characteristics, for each Boolean variable, add  $tempbool_{i,j,d} + tempbool_{j,i,d} = 1$ ,  $tbool_{i,j} + tbool_{j,i} = 1$  and  $hbool_{i,j} + hbool_{j,i} = 1$ . In this way, the sum of these Boolean variables can be effectively restricted to a fixed value without affecting the objective function.

Through the above processing methods, the original nonlinear model can be linearized, and the linearized model can be directly incorporated into the processor for solving.

## 5. Numerical experiments and results

The experiment was implemented using CPLEX 12.9 Studio, and all experiments were run on a computer with an AMD Ryzen Threadripper PRO 5975WX 32-Cores 3.60 GHz processor, 32.0 GB (31.8 GB available) of memory, and the operating system Windows 10.

The data is described in Table 3.

Table 3: Rules to format sections

$bmin_{3(4)}$	$bmin_2$	$bmin_1$	$bmax_{3(4)}$	$bmax_2$	$bmax_1$
0	30	0	1000	150	24
$\delta_{3(4)}$	$\delta_2$	$\delta_1$	$\lambda_{3(4)}$	$\lambda_2$	$\lambda_1$
10	1.5	0.5	10	3	0.5
$pr_{i,3(4)}$	$pr_{i,2}$	$pr_{i,1}$	$p_{i,3(4)}$	$p_{i,2}$	$p_{i,1}$
100	15	3	$U[0 - 1000]$	$U[30 - 150]$	$U[0 - 24]$

The experiment employs nine datasets with different demand quantities. The results are shown in Table 4.

Table 4: Rules to format sections

Target quantity	Opt sol or not	Time(s)	Gap
10	Yes	0.03	0
20	Yes	0.06	0
30	Yes	0.19	0
40	Yes	0.73	0
50	Yes	1.64	0
60	Yes	5.39	0

Table 4: (continued).

70	No	60.33	45.77%
80	No	603.11	41.42%
100	No	8514.50	90.81%

As shown in Figure 2, due to the rapid increase in computational complexity of the problem with the growing number of demands, the solver's performance tends to deteriorate when the scale is larger. The time required to solve the problem increases exponentially with the size of the problem. When the problem size exceeds 60, the solver is unable to obtain the optimal solution within an acceptable time period. Moreover, as the problem size increases, the gap value of the feasible solutions obtained by the solver becomes larger.

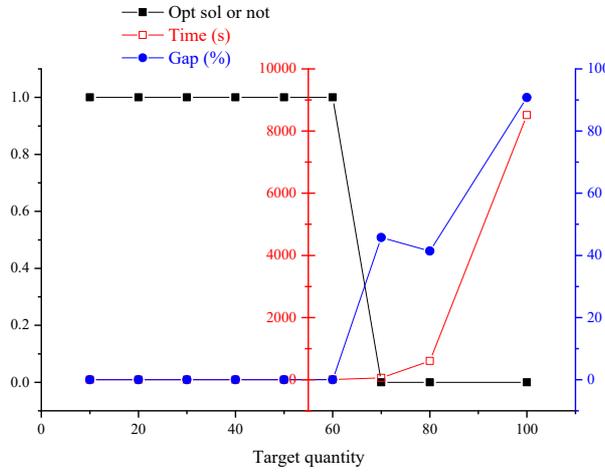


Figure 2: Line graph of experimental results

## 6. Conclusion

According to the linear estimation of the quadratic term: if  $c \in [a, b]$ , then  $c^2 > a^2 + 2a * (c - a) = 2ac - a^2$ ,  $b^2 > c^2 + 2c * (b - c) = 2bc - c^2$ . The maximum degree of relaxation of this linearization method for the problem is  $\frac{(a-b)^2}{(a+b)^2}$ . For this question, the degree of relaxation is determined by the maximum and minimum distances between the two airspace demands. The error between the optimal solution of this linearization method and the optimal solution of the actual problem will also be controlled within this maximum relaxation level.

Specifically, for this problem, when  $\max(|p_{i,3} + x_{i,3} - p_{j,3} + x_{j,3}|) < (\lambda_3 + pr_{i,3} + pr_{j,3})$ , the conflict cannot be resolved, and the relaxation of this constraint has no effect on the problem. When  $\max(|p_{i,3} + x_{i,3} - p_{j,3} + x_{j,3}|) > (\lambda_3 + pr_{i,3} + pr_{j,3})$ ,  $\min(|p_{i,3} + x_{i,3} - p_{j,3} + x_{j,3}|) > (\lambda_3 + pr_{i,3} + pr_{j,3} - 4\sqrt{2}\delta_3)$ , the degree of relaxation  $\frac{(a-b)^2}{(a+b)^2}$  will larger than  $\frac{(4\sqrt{2}\delta_3)^2}{(2(\lambda_3 + pr_{i,3} + pr_{j,3}) - 4\sqrt{2}\delta_3)^2}$ . As  $\delta_3 = 10$  and  $pr_{i,3} = 100$ , the small degree of relaxation has little impact on the problem.

In summary, the model proposed in this paper can be applied to the resolution of such problems. However, for large-scale problems, the solver's solving capability is not enough, necessitating more efficient algorithms.

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