

Various ARCH Models and Their Practical Applications on Apple and Pork

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Abstract: Agricultural price volatility is a core issue affecting the stability of the global agricultural economy. Apple and pork as a major product, the price volatility directly affects and affects people's well-being and industrial security. In recent years, due to the influence of different factors, the prices of apples and pork have fluctuated wildly. This article focuses on the following questions: Firstly, how should the short-term focus, long-term sustainability, and asymmetric effects of price fluctuations in different products be quantified? Secondly, the role of futures market and spot market in price discovery and risk tradition is different. Based on the data of apple prices and pork futures from 2017-2023, this passage uses Autoregressive Conditional Heteroskedasticity (ARCH) model and its generalizations to analyze the data. The final findings of the study were twofold. Apple price volatility showed short-term aggregation and positive shock sensitivity, and raw pork futures market dominated prices, and the reaction to negative shocks is strong. The study was adopted the joint application of the ARCH family model compares the volatility characteristics of several agricultural products prices, and promote high-quality development of the agriculture economy.

Keywords: ARCH models, Price Fluctuation, Data Analysis

1. Introduction

The price fluctuation of agricultural products is a core issue affecting the stability of the global agricultural economy. Taking apples and pork as examples, both of which occupy an important position in China's agricultural industry chain, but the price fluctuation mechanism is significantly different [1]. Apple prices are affected by seasonal supply and demand, abnormal climate, and speculative behavior in the futures market. For example, in 2018-2019, the spot price surged by more than 100% in a short period of time, and then fell sharply. The price of pork fluctuates frequently due to African swine fever, feed cost fluctuations, and frequent policy regulation (for example, the daily price fluctuation of live pig futures reached 2.3% in 2021). Analyzing the price fluctuation law of these two types of agricultural products is of great significance for risk management and policy formulation.

In classic research, the Autoregressive Conditional Heteroskedasticity (ARCH) model of Engle captures short-term volatility clustering through the square term of historical errors, and Bollerslev introduced the folded GARCH model the lagged volatility term describes the long-term memory effect. Additionally, Nelson's EGARCH model quantifies the amplification effect of "bad news"

through an asymmetric system. Among this, these also include HeSi Pan insights and ideas on different models [2]. In the empirical field, Combing Qiming Ge revealed based on GARCH model reveal the risk premium difference and asymmetric leverage effect in the apple spot and futures market [3], and Chenyuan Zhu verified the long-term dominant role of pig futures market in price fluctuation through variance decomposition [4]. However, the existing research lacks a comparative analysis of the fluctuation mechanism of multiple agricultural products, especially ignoring the system of the joint application of models.

This study integrates apple and pig market data, and applies it jointly, ARCH, GARCH, and EGARCH model, system division, to analyze the volatility characteristics of two types of agricultural products. Based on the 2017-2023 apple production area prices, wholesale prices, and live pig spot and data.

2. Methodology

2.1. ARCH model

In 1980s, Robert Engle proposed ARCH model, the target for it, is to solve the problem of the failure of the “homoscedasticity” assumption in traditional economic models. In real environment just like financial markets or products prices, volatility trends normally become clustering which means high volatility follows high volatility and low follows low. Comparing traditional linear regression models show that the “homoscedasticity” does not change. The ARCH model introduces the variance as a function of the square of past residuals by introducing conditional heteroscedasticity, thus capturing this dynamic volatility feature [5]. In addition, the heteroscedastic function applicable to the residual sequence has short-term autocorrelation [6].

The code mind of the ARCH model is that the current variance of conditions depends on the square of random disturbances over several periods. For example, if the volatility of one period is large (the absolute value of residuals is large), the volatility of the next period may be large too. This dynamically adjusted variance structure can be more accurately show the volatile characteristics of real data, and it is especially suitable for high-frequency financial time series analysis.

The formula system of the ARCH (q) model is the following. The conditional variance h_t is defined as a linear combination of the squared residuals from the past q periods:

$$h_t = \omega + \sum_{i=1}^q \alpha_i \epsilon_{t-i}^2 \quad (1)$$

Among them, h_t represents the conditional variance of time t (i.e., volatility), and ω is a constant term. Then, ϵ_{t-i}^2 is the square of the error term of the lagged period (reflecting the impact of past price fluctuations), and α_i means the coefficient of the corresponding lag term, must satisfy the non-negativity constraint (to ensure that the variance is non-negative). At same time, $\omega > 0$ and $\alpha_i \geq 0$ to ensure that the variance is non-negative [7].

The ARCH model predicts the current volatility by squaring the historical error term. For example, if a stock has experienced significant price movements ($\epsilon_{t-1}^2, \epsilon_{t-2}^2, \epsilon_{t-3}^2$) over the past 3 days, the model will predict a higher future volatility. Traditional ARCH model has limitation. For instance, it needs more lag term (a large q value), and can fully engrave volatility that may lead more parameters and inefficient estimation. For this, different scholars mentioned a lot of models which already be improved.

2.2. GARCH model

GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model was mentioned by Tim Bollerslev, an economist, in 1986, and the model was an improvement of Engle's ARCH models which was proposed in 1982. GARCH model is the most commonly used and convenient model for fitting heteroscedastic series to date [2]. Additionally, The GARCH model requires the variance to be non-negative and the unconditional variance to be stable, thus strictly limiting the non-negative and bounded agreement conditions of the parameters, in order to broaden the applicability of the model [8]. Although ARCH models were famous for catching "Volatility Clustering" of financial time series, ARCH model was not enough in long-term volatility, because it only depends on square prediction fluctuation of past error term, and, actually, it needs lot of parameters, so ARCH model causing practical limitations [5]. In order to make up for the shortcomings of the ARCH model, such as lack of leverage effect and restrictive parameters, the GARCH model was proposed to address some of the shortcomings, such as the tendency of volatility forecasts to be too high type [9].

The code idea of GARCH model is: considering past error term and past fluctuation term at same time. For instance, in GARCH(1,1) model, today's fluctuation was not only affected by stock price fluctuation range in yesterday (square fluctuation of error term), and also affected by predicting volatility in yesterday (conditional variance). Additionally, GARCH model can use fewer parameter to describe complex fluctuation.

Adding autoregressive on the basic ARCH model, it could use fewer parameters to describe long term volatility memory. The formula is:

$$h_t = \omega + \sum_{i=1}^q \alpha_i \epsilon_{t-1}^2 + \sum_{j=1}^p \beta_j h_{t-j} \quad (2)$$

Among them, new items, $\beta_j h_{t-j}$ represents the conditional variance (i.e., the predicted value of past volatility) lagged by j periods.

The GARCH model introduces the lagged term of the conditional variance on the basis of the ARCH model, giving the volatility "long-term memory." For example, if the volatility h_t predicted yesterday is higher, even if the price today does not fluctuate greatly, the model may still predict a higher volatility tomorrow.

2.3. EGARCH models

In 1991, EGARCH (Exponential Generalized Autoregressive Conditional Heteroskedasticity) model proposed by Nelson, and the aim for propounding EGARCH model was to solve the symmetry problems of GARCH model. Economic markets are more sensitive to "bad news" [6]. For example, the panic which brings by stock price decline 5% is stronger than the emotion of happy which comes from stock price increase 5%, and this phenomenon was called leverage effect. Traditional GARCH model unable to distinguish the impact of positive and negative shocks, if this kind of information asymmetry is ignored, and it may sometimes affect the accuracy of the prediction [2]. But, luckily, the EGARCH model solves this problem through mathematics design.

EGARCH model's significant idea: The key innovation of EGARCH model allows the asymmetric impact of positive and negative shocks on volatility. For instance, models give greater discourse power to "decline", and adopt logarithmic form to ensure that the predicted volatility is ultimately positive.

Introduce asymmetric effects to distinguish the different impacts of positive and negative shocks on volatility (e.g., "leverage effect"). The Core formula is

$$\ln(h_t) = \omega + \sum_{j=1}^q [\alpha_j \left(\frac{|\epsilon_{t-j}|}{\sqrt{h_{t-j}}} - E \left(\frac{|\epsilon_{t-j}|}{\sqrt{h_{t-j}}} \right) \right) + \gamma_j \frac{\epsilon_{t-j}}{\sqrt{h_{t-j}}}] + \sum_{k=1}^p \beta_k \ln(h_{t-k}) \quad (3)$$

Among them, γ_j measures the asymmetric impact of positive and negative shocks (when negative shocks $\gamma_j < 0$, volatility amplification), and the logarithmic form ensures that h_t is always positive, without the need for a non-negative coefficient constraint.

The EGARCH distinguishes the impact of positive and negative shocks through γ_j . For instance, when stock price decline to ($\epsilon_{t-j} < 0$), the $\gamma_j \frac{\epsilon_{t-j}}{\sqrt{h_{t-j}}}$ is negative, and this lead $\ln(h_t)$ increase. Then, this will significantly increase the volatility.

3. Results and application

3.1. Apple price fluctuation analysis

Apple as an important economic crop in China, the price fluctuation of apples is affected by multiple factors, including climate disasters, supply chain efficiency, and the linkage of futures markets. Based on the data which given by Qiming Ge of apple spot and prices from December 25, 2017 to November 30, 2023 that is shown in Figure 1 (a total of 1401 trading days), this paper constructs an ARCH-type models to analyze its volatility characteristics, because the ARCH models have gradually become a common method for studying market price fluctuations [7], and tests the market dynamics through risk premium and leverage effect.

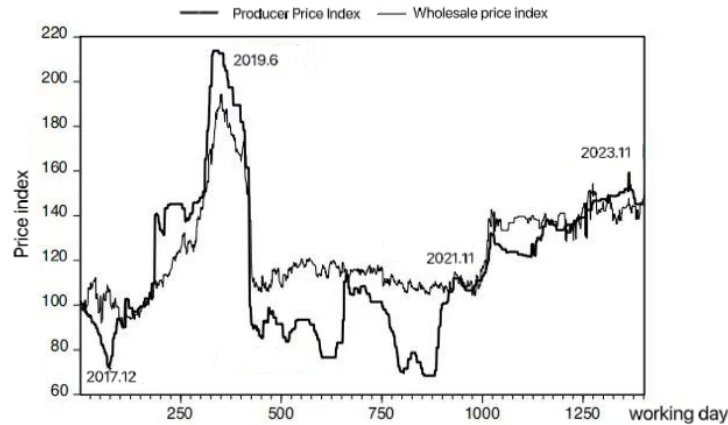


Figure 1: Apples 2017-2023 price index trend [7]

3.1.1. ARCH model: catching the clustering of apple price fluctuations

Empirical tests based on the ARCH model show that there is significant volatility clustering in both spot and futures market prices of apples. Through ARCH-LM tests, it is found that the residuals of the producer price index, wholesale price index, and futures index returns of apples all show significant heteroscedasticity (F-statistics of 14.52, 13.71, and 5.80, p-values < 0.01), indicating that there is a short-term "memory effect" in the volatility of apple market prices [3]. Taking the producer price of apples as an example.

The ARCH (1) model condition variance equation is:

$$h_t = 0.0599 + 0.164\epsilon_{t-1}^2 \quad (4)$$

From Table 1 that made by Qiming Ge, the contribution rate of the price shock of the previous period (such as yesterday's price fluctuation) to the current volatility is 16.4%, which verifies the volatility clustering [1]. For example, if yesterday's price fluctuation $\epsilon_{t-1}^2 = 0.4$, then the volatility forecast for today is $0.0599 + 0.164 \times 0.04 = 0.0665$, indicating that short-term shocks significantly amplify volatility.

Table 1: Estimates of the variance equation of the GARCH (1,1) model [3]

Variable	Apple revenue in the production area rs_1	Wholesale market revenue rs_2	Futures yield rf_1	Futures yield rf_1
C	0.059 963*** (0.001 973)	0.042 866*** (0.003 786)	0.411 820*** (0.041 761)	0.083 685*** (0.016 458)
ARCH	0.163 965*** (0.012 254)	0.089 272*** (0.006 027)	0.437 896*** (0.029 340)	0.046 276*** (0.006 722)
GARCH(-1)	0.883 289*** (0.002 012)	0.904 405*** (0.004 408)	0.661 477*** (0.017 685)	0.922 497*** (0.011 016)

3.1.2. GARCH model: revealing the long-term memory of price fluctuations

The parameter estimation results of the GARCH (1, 1) model show that the market price fluctuation of apples has long-term persistence. Taking the price of apples in the production area as an example. Its conditional variance equation is:

$$h_t = 0.0599 + 0.164\epsilon_{t-1}^2 + 0.883\sigma_{t-1}^2 \quad (5)$$

in which the parameters are from Table 1. The sum of the ARCH (0.164) and GARCH (0.883) coefficients is 1.047, which is close to 1, indicating that the impact of shocks on volatility decays slowly. For example, if yesterday's price fluctuation was $\epsilon_{t-1}^2 = 0.04$, and yesterday's predicted volatility $\sigma_{t-1}^2 = 0.02$, then today's volatility $0.0599 + 0.164 \times 0.04 + 0.883 \times 0.02 = 0.086$, the long-term memory effect is significant. This characteristic explains the phenomenon that the price drop caused by the 2019 apple harvest continues to affect subsequent market expectations.

3.1.3. EGARCH model: identifying the asymmetric effect of apple market volatility

The EGARCH model reveals the amplification effect (leverage effect) of apple price fluctuations on "bad news." Taking the wholesale price of apples as an example. Its EGARCH (1,1) conditional variance equation is:

$$\ln(h_t) = -0.127 + 1.868 \frac{|\epsilon_{t-1}|}{\sigma_{t-1}} - 0.431 \frac{\epsilon_{t-1}}{\sigma_{t-1}} + 0.989(\sigma_{t-1}^2) \quad (6)$$

Among them, the coefficient of the asymmetric term is $\gamma = -0.431$, indicating that the amplification effect of negative shocks ($\epsilon_{t-1} < 0$) on volatility is 43.1% higher than that of positive shocks. For example, if there was a negative shock of $\epsilon_{t-1} = -0.2$ yesterday and $\sigma_{t-1}^2 = 0.02$, the contribution of the asymmetric term is $-0.431 \times \frac{-0.2}{\sqrt{0.02}} = 0.431 \times 1.414 = 0.610$ significantly increasing the logarithmic volatility, while the same positive shock only contributes $1.868 \times \frac{0.2}{\sqrt{0.02}} = 0.530$ verifying that the "bad news" effect is stronger.

3.2. Pork price fluctuation analysis

As one of the most important agricultural products in the world, the price fluctuation of pork directly affects people's livelihood and economic stability. China is the world's largest consumer of pork. In

last several decades, due to factors such as African swine fever and fluctuations in feed costs, the price of pork has shown characteristics of sharp fluctuations (such as a short-term increase of more than 2.3% in 2021 and a long-term high volatility). In order to analyze its fluctuation mechanism, the study adopts the ARCH family model and with the data from Chenyuan Zhu.

3.2.1. ARCH model: "aggregation effect" of short-term price fluctuations

The ARCH model reveals the aggregation of short-run price fluctuations in pork markets by capturing the square of historical error items directly affecting current fluctuation. The impulse response function shows that the short-term response of pig futures prices to 1 standard deviation shocks is strong (2.3% in period 1, The second phase reached 2.4%), followed by a slow decline but remained at about 2% for a long time, which is consistent with the "fluctuating memory" characteristics of the ARCH model [6]. For example, if set the conditional variance equation for the ARCH (1) model to:

$$h_t = \omega + 0.25\epsilon_{t-1}^2 \quad (7)$$

Assuming yesterday's price shock $\epsilon_{t-1}^2 = 0.04$, then today's volatility forecast value is $\omega + 0.25 \times 0.04 = \omega + 0.01$, the short-term shock significantly amplifies volatility (similar to the 6.24% volatility in the first period of the futures market in Table 2).

Table 2: Variance decomposition of pig prices [6]

lag period	Logarithmic spot price		Logarithmic futures price	
	Futures market (%)	spot market (%)	Futures market (%)	spot market (%)
1	6.243	93.575	100.000	0.000
2	14.004	85.996	100.000	0.000
3	17.207	82.793	99.999	0.001
4	19.113	80.887	99.996	0.004
5	20.529	79.471	99.989	0.011
6	21.708	78.292	99.979	0.021
7	22.754	77.246	99.967	0.033
8	23.716	76.284	99.951	0.049
9	24.640	75.380	99.933	0.067
10	25.483	74.517	99.912	0.088

3.2.2. GARCH model: "inertia transmission" of long-term volatility

The GARCH model explains the long-term persistence of pork price volatility by introducing the lagged term of historical volatility. The variance decomposition results in the literature (see Table 2) show that the proportion of price volatility explained by the futures market gradually increases from 6.24% in the first period to 25.48% in the tenth period, while the contribution of the spot market decreases from 93.58% to 74.52%, indicating that the futures market has a stronger long-term memory effect. Taking the GARCH (1,1) model as an example, the conditional variance equation is:

$$h_t = 0.02 + 0.15\epsilon_{t-1}^2 + 0.80\sigma_{t-1}^2 \quad (8)$$

Among them, the GARCH term coefficient of 0.80 is close to 1, indicating that the impact of external shocks (such as policy adjustments or epidemics) needs to dissipate over several periods. If yesterday's volatility $\epsilon_{t-1}^2 = 0.04$ and $\sigma_{t-1}^2 = 0.03$, then today's volatility is $0.02 + 0.15 \times 0.04 + 0.80 \times 0.03 = 0.056$. Long-term inertia is significant.

3.2.3. EGARCH model: "risk warning" for asymmetric shocks

The EGARCH model quantifies the overreaction of the pork market to "bad news" through asymmetric coefficients. The impulse response of spot prices to futures shocks, where the increase in spot prices (1.3% in the first period) caused by negative shocks (such as epidemic outbreaks) is more persistent (1.5% in the long run) than positive shocks (such as policy benefits), conforms to the leverage effect [6].

The conditional variance equation of the EGARCH (1,1) model is:

$$\ln(h_t) = -0.1 + 0.3 \frac{|\epsilon_{t-1}|}{\sigma_{t-1}} - 0.2 \frac{\epsilon_{t-1}}{\sigma_{t-1}} + 0.9 \ln(\sigma_{t-1}^2) \quad (9)$$

Among them, $\gamma = -0.2$ indicates that the amplification effect of negative shocks ($\epsilon_{t-1} < 0$) on volatility is 20% higher than that of positive shocks. For example, if the negative shock $\epsilon_{t-1}^2 = 0.1$ and $\sigma_{t-1}^2 = 0.02$, the contribution of the asymmetric term is $0.2 \times \frac{-0.1}{\sqrt{0.02}} = 0.141$, significantly increasing the volatility.

4. Conclusion

This paper analyzes the characteristics of price fluctuations of apples and pork in recent years by combining the ARCH model group, focusing on the characteristics of each model. It combines the short-term clustering of price fluctuations in the ARCH model, the long-term persistence of price fluctuations in the GARCH model, and finally the asymmetric effect of the EGARCH model. Finally, it is found that the price fluctuations of apples show significant short-term positivity and positive shock sensitivity (EGARCH asymmetric coefficient is negative 0.13), the dominant price of the pork futures market, the contribution rate of variance decomposition is 25.48%, and the reaction to adverse shocks is severe. At the same time, it is found that there is a unidirectional risk spillover from the futures market to the spot price, and the volatility mechanism of pork meat is significantly different. This paper combines few cases, only combining pork and apples, and only considering one-sided factors, such as not combining seasonal or special situations. In the future research and plan, more cases will be combined and analyzed to verify the application of the ARCH model.

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