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Multi-Objective Model for Sustainable Tourism: Juneau and Shanghai Cases

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Abstract: This study examines sustainable tourism in Juneau, Alaska, and Shanghai, addressing overtourism and environmental degradation. In 2023, Juneau received 1.6 million cruise tourists, generating $375 million but straining infrastructure and accelerating glacier retreat. To tackle these issues, a mathematical model is developed using ESG criteria, SDGs, and the GRI framework. The model incorporates tourist volume, revenue, and regulatory tools (e.g., caps, taxes), with objectives to minimize environmental impact, maximize social benefits, and enhance community satisfaction. Constraints such as emission limits and seasonal tourist thresholds ensure practical relevance. Time-series data are analyzed using Fourier series and linear regression. Multivariable dynamic programming determines optimal monthly tourist numbers. Sensitivity analysis highlights the dominance of environmental and governance factors, underscoring the role of government in sustaining local quality of life. A fuzzy optimization approach addresses real-world uncertainties. A revenue allocation plan channels funds to environmental protection (30%), water (20%), waste management (20%), infrastructure (15%), marketing (10%), and community projects (5%), aligning with sustainability goals.

Keywords: Sustainable Tourism, Multi - objective Optimization Model, ESG Criteria, SDGs, Sensitivity Analysis, Fuzzy Optimization

1. Introduction

1.1. Problem background

Juneau, Alaska, with a population of 30,000, received 1.6 million cruise passengers in 2023, generating $375 million in tourism revenue [1]. While boosting the economy, this rapid growth has caused overcrowding, infrastructure strain, and environmental degradation—evident in the retreat of the Mendenhall Glacier. These impacts have raised serious concerns about the long-term sustainability of local tourism [2]. To address these issues, frameworks such as ESG criteria, the UN Sustainable Development Goals (SDGs), and the Global Reporting Initiative (GRI) offer structured approaches [3]. ESG emphasizes environmental protection, social responsibility, and effective governance. The SDGs promote climate action and responsible consumption, directly linked to tourism. The GRI framework provides standardized metrics for tracking sustainability performance. Together, these models offer theoretical support for balancing economic and environmental goals in tourism development, and guide efforts to create a more sustainable future for Juneau [4].

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1.2. Data collection

Table 1: Data collection

Data Detail	Data Website
number of tourists	https://industry.visitcalifornia.com
taxation situation in Juneau, Alaska	https://www.statista.com
Accommodation demand and accommodation prices	https://www.x-mol.com
price of carbon dioxide treatment	https://www.butiao.com
price of purchasing one ton of water	https://www.maigoo.com

2. Assumptions

This model is built upon the following assumptions regarding tourism in Alaska and Juneau. **Assumption 1**: Tourist volume will remain stable compared to 2021–2024. **Assumption 2**: Environmental impact is measured by carbon emissions, water use, and waste generation. **Assumption 3**: Economic impact is reflected in consumption and tax revenue. **Assumption 4**: Per capita consumption is directly proportional to tourist numbers. **Assumption 5**: Tax revenue positively affects community satisfaction. **Assumption 6**: However, a higher number of tourists may reduce community satisfaction. **Assumption 7**: A large share of tourism-related tax revenue is invested in infrastructure and public services. **Assumption 8**: A strong local consumption atmosphere can stimulate tourist spending. **Assumption 9**: Excessive tax rates may negatively impact tourist consumption behavior. **Assumption 10**: Environmental factors are limited to carbon emissions, water use, and waste. **Assumption 11**: Socioeconomic conditions, environmental factors, and community satisfaction are all quantifiable and together reflect tourism's overall impact.

3. Preliminary model : fourier series and linear regression

Fourier series expansion is used to model periodic fluctuations in time-series data, aiming to derive a mathematical function that describes monthly tourist variation. Based on a two-year dataset exhibiting clear periodicity, the series captures key temporal patterns and yields an analytical function linking time and tourist volume. This function serves as the input for Model I, providing essential data support for subsequent optimization and analysis.

On basis of the relationship between time and the number of tourists, a function is established, with time t as the independent variable and the number of tourists $N(t)$ as the dependent variable. In abstract form, this function can be expressed as:

$$N(t) = f(t) \tag{1}$$

where f represents an abstract mapping rule that reflects how the number of tourists changes with time.

The time series is modeled using a Fourier series, expressing tourist dynamics as a sum of sine and cosine functions across frequencies. Coefficients, fitted via linear regression, capture each component's amplitude and phase. The resulting analytical function effectively describes periodic trends and provides a foundation for subsequent optimization.

Fourier Feature Generation:

To extract the periodic features from the data, we first use a Fourier series to transform the time series into multiple sine and cosine terms. Each feature corresponds to a different frequency, with each frequency having both sine and cosine components. Since the dataset spans a period of 24 months (i.e., two years), we set the period to 24 and select a model with 10 Fourier terms, resulting

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in 20 features (10 sine terms and 10 cosine terms). The Fourier features are computed via the formulas shown in Equations (2) and (3).

$$Feature_i = sin(\frac{2\pi it}{24}) \tag{2}$$

$$Feature_{i+1} = cos(\frac{2\pi it}{24}) \tag{3}$$

where t is the time index, and where i is the frequency term number.

Linear Regression Model:

After generating the Fourier features, we fit the data via a linear regression model, which learns the coefficients of each Fourier term. The final fitted function is shown in Equation (4).

$$y = \text{interception} + \sum_{i=1}^{10} coef_{2i-1}\, sin(\frac{2\pi it}{24}) + coef_{2i}\, cos(\frac{2\pi it}{24}) \tag{4}$$

where the coefficients $coef_i$ are determined by the regression model.

Regression Results:

The fitted curve and the resulting equation are shown in Figure 1. which, almost excellently fit the current data.

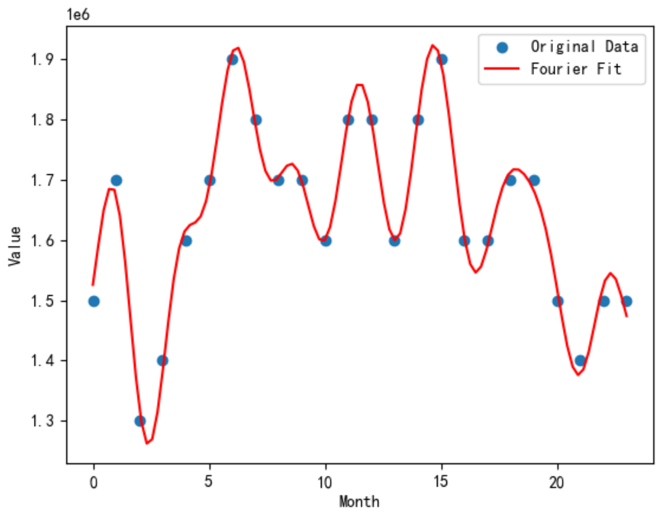

Figure 1: Preliminary model output

4. Model: sustainable tourism optimization model for Juneau

This study aims to promote sustainable tourism development in Juneau through a multi-objective dynamic optimization model that balances environmental, social, and economic outcomes. It

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integrates the Sustainable Development Goals (SDGs), Environmental, Social, and Governance (ESG) principles, and the Global Reporting Initiative (GRI) framework to provide comprehensive, data-driven support for tourism management. Specifically, the model aligns with key SDGs such as maximizing tourism revenue and job creation (SDG 8), reducing environmental impacts (SDG 12), mitigating climate effects (SDG 13), and protecting marine ecosystems (SDG 14). ESG principles guide environmental policy through carbon footprint monitoring, enhance social equity via community engagement and employment, and reinforce governance to ensure local well-being. The GRI framework quantifies sustainability performance by reporting tourism-related carbon emissions (GRI 305), water resource usage (GRI 303), and community impact (GRI 413). During implementation, the model addresses trade-offs across goals using techniques such as the weighted sum method, fuzzy weighting, and Pareto optimization to support strategic decisions for sustainable tourism in Juneau [4-6].

4.1. Objective functions

The optimization problem consists of three objectives (E, S, and G), which we aim to optimize simultaneously. These objectives are combined into a weighted sum, where the weights reflect the relative importance of each goal.

Objective 1: Minimizing Environmental Expenditure ($E(N)$)

On the basis of Assumptions 1, 2, 10, and 11, the primary objective of this research is to minimize the cumulative environmental cost stemming from carbon emissions, water utilization, and waste production. The mathematical relationship underpinning this objective can be rigorously defined by the equations (5), (6), (7) and (8).

Where, $N(t)$ represents the population number in month$a_o$represents the price for treating carbon dioxide (in US dollars); β_o represents the amount of carbon dioxide (in tons) produced by a single tourist in one month;λ_1 represents the price per ton of water resources, and λ_2 represents the amount of water resources (in tons) used by each person during one - month tourism. μ_1 represents the cost for treating each ton of garbage, and μ_2 represents the amount of garbage (in tons) generated by each person when traveling to the destination in one month.

$$min\, E\,(N(t)) = CO_2(N(t)) + WU(N(t)) + WP(N(t)) \tag{5}$$

$$CO_2(N(t)) = a_o \cdot \beta_o N(t) \tag{6}$$

$$WU(N(t)) = \lambda_1 \cdot \lambda_2 N(t) \tag{7}$$

$$WP(N(t)) = \mu_1 \cdot \mu_2 N(t) \tag{8}$$

Objective 2: Maximizing Social Improvement values$S(t)$:

On the basis of Assumptions 1, 3, 4, 7, 8, 9, and 11, the second objective is to maximize a function that gauges the impact on the socioeconomic sphere of tourism, taking into account the number of tourists and tax revenue over a specified time frame. We build an the interaction modelv [7] between population migration and regional consumption, as well as a relationship model between taxation and tourism revenue [8], The following presents an improved model, as shown in equations (9), (10), (11), (12) and (13).

In this model, $S(t)$ represents the total revenue generated by tourism $C(N(t))$ represents the average consumption per month, and $T(N(t))$ represents the average tax revenue related to the population. η represents other unexpected consumption.$\sigma_1 Y$ represents the monthly consumption of tourists driven by the local consumption situation;$\sigma_2 N(t)$ represents the monthly consumption of travelers. Additionally $\kappa$is the rate of tax and$\upsilon$is the adverse effect of taxation on consumption.

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$$S(t) = C(N(t)) \cdot N(t)^2 + T(N(t)) \cdot N(t), \tag{9}$$

$$C(N(t)) = \begin{cases} \eta + \sigma_1 Y + \sigma_2 N(t), & \sigma_1 > 1, t \neq t_0, \\ \eta + \sigma_1 Y + \sigma_2 N(t) & \sigma_1 = 0, t = t_0. \end{cases} \tag{10}$$

$$T(N(t)) = \kappa \cdot (C(N(t)) + \upsilon) \tag{11}$$

The goal is to maximize the integral of$S(t)$ over the time period $[a, b]$:

$$max \int_a^b S(t)dt \tag{12}$$

where $a, b \in [1,12] a, b \in N^*$.

Objective 3: Maximizing Community Satisfaction (G(N))

On Assumptions 1, 5, 6, and 11, the third objective is to maximize the community satisfaction function$G(N)$, which consists of two parts. One part is the government's infrastructure investment from tourist - sourced taxes. The other part is residents' dissatisfaction, which is affected by population - related factors as shown in Equations (14) and (15).

Whereγ represents the proportion of tax revenue used for infrastructure construction. $\psi(N(t))$ represents the impact brought by the tourist population. It has no effect when the number of tourists is lower than the average number of tourists within the cycle. When the number of tourists is higher than the average number within the cycle, it is proportional to the number of tourists, and the proportionality coefficient is ρ.

$$max\, G\,(N) = \gamma \times T(N(t)) \times N(t) - \psi(N(t)) \tag{13}$$

$$\psi(N(t)) = \begin{cases} 0, & N(t) \leq \dfrac{1}{a+b} \sum_a^b N(t), \\ \rho N(t), & N(t) \geq \dfrac{1}{a+b} \sum_a^b N(t). \end{cases} \tag{14}$$

4.2. Constraints

The optimization problem is subject to several constraints, which are defined as Equations (15), (16) and (17).

The total carbon emissions cannot exceed the monthly emission limit.,L_0 presents the monthly emission limit. This is defined as:

$$max_{t \in [1,12]} \leq L_0 \tag{15}$$

The population must remain within a specified range, where N_0represents the upper limit of the tourist population. This is defined as:

$$0 \leq N(t) \leq N_0 \tag{16}$$

The function must have an extreme value within a given time period$[a, b]$, which requires that there exist points where its derivative is zero or does not exist. This ensures that the data reflect the actual situation of tourists during both the off-season and the peak season. This is defined as:

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$$\exists t = t_0, s.t. \frac{dS(t)}{dt} = 0 or the derivative does not exist at t = t_0 \tag{17}$$

4.3. Combined objective function

The overall objective function is a weighted combination of the three individual objectives, where w_1 , w_2 and w_3 are the weights assigned to each objective function. $E(x)$ represents the environmental protection cost;$S(x)$ represents the economic benefit; and $G(x)$ represents the benefits (satisfaction) of local residents. This is defined as Equation (19):

$$max\ O\ bjective(x) = -w_1 \cdot E(x) + w_2 \cdot S(x) + w_3 \cdot G(x) \tag{18}$$

4.4. Model solution

4.4.1. Parameter selection

In the parameter selection section, we derived several key parameters through a comprehensive review of relevant literature and web-based data sources. Reasonable assumptions were made for parameters that were difficult to obtain. The selection criteria for each model parameter are explained as follows Table 2-5:

First, parameters for the environmental maintenance cost model were selected. On average, a tourist emits approximately 1.4 tons of CO_2 per month, and the treatment cost per ton of CO_2 is about \$500. A tourist consumes around 0.2 tons of water per month, with the water price at \$1 per ton. Additionally, each tourist generates about 1.1 tons of waste monthly, with a treatment cost of approximately \$80 per ton. These parameters collectively reflect the environmental impact of tourism and the associated management costs, forming the foundation of the environmental cost model. The selected parameters are summarized below.

Table 2: Parameter selection(E)

Parameters	Values
a_o	500\$
β_o	1.4(ton)
λ_1	1\$
λ_2	0.2(ton)
μ_1	80\$
μ_2	1.1(ton)

Second, each tourist incurs an average of \$50 in incidental monthly expenses. As a tourist destination, the city experiences an off-season in winter, during which the local economy has limited stimulatory effects. Tourist consumption is directly proportional to tourist numbers, with a coefficient of 0.3. The regular tax rate in California is 0.18, and since this is a tourist city, it is assumed that an increased tax rate does not reduce consumption, serving as a basis for the tax model.

Table 3: Parameter selection(S)

Parameters	Values
η	50\$
σ_1	0or1.2
σ_2	0.3
Y	50\$

Table 3: (continued).

t_o	10,11,12,1,2,3(month)
κ	0.18
υ	0
a, b	a =1, b =12(month)

Then, 85% of the tourism tax revenue is allocated to infrastructure construction, reflecting the city's investment in public facilities. The model uses a 12-month period, providing a stable temporal framework for simulating long-term tourism economic dynamics. The corresponding parameters are listed below.

Table 4: Parameter selection(G)

Parameters	**Values**
γ	0.85
ρ	0.2

Finally, based on data from Juneau, California and related literature, we determined the constraint-related parameters. The specific values are outlined below.

Table 5: Parameter selection(constraints)

Parameters	**Values**
L_0	1.73e8/12(ton)
N_0	1800000 (persons)

4.4.2. Model results

Through the above multi-variable dynamic programming, the optimal monthly tourist volume in California was determined. To clearly present the results, Figure 2 visualizes the findings. Based on relevant literature and data, approximately 20% of visitors to California travel to Juneau. Therefore, Table 6 presents the optimal monthly tourist volume in California, the corresponding tourist number in Juneau, and the estimated daily tourist number in Juneau, rounded to an appropriate precision.

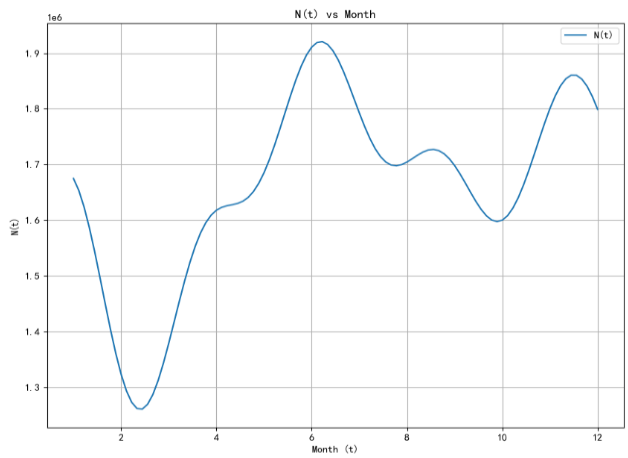

Figure 2: Model results (Alaska, Juneau)

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Table 6: Model results (Alaska, Juneau)

Month	Optimal Number of Tourists in Alaska	Optimal Number of Tourists in Juneau	Daily Number of Tourists in Juneau
1	1674947	334989	11166
2	1323354	264670	8822
3	1379083	275816	9193
4	1617904	323580	10786
5	1685474	337094	11236
6	1911007	382201	12740
7	1792408	358482	11949
8	1704509	340902	11363
9	1698027	339606	11320
10	1600152	320030	10667
11	1800825	360165	12005
12	1799105	359821	11994

5. Model validation and improvement

5.1. Sensitivity analysis

This paper conducts a sensitivity analysis to evaluate the robustness and generalizability of the model. By adjusting the weights of ESG factors, it identifies which factor is most sensitive in California and Juneau. The analysis includes an overall weight sensitivity test, individual weight sensitivity tests, and a test that controls for S (social factors) to assess the sensitivities of S and G. The results are illustrated in Figures 3, 4, and 5.

From these figures, it is evident that E (environmental) and G (governance) factors exhibit higher sensitivity, while S (social) factors show lower sensitivity. These findings suggest that in both California and Juneau, G and S—particularly the government's role in maintaining the quality of life in local communities—are critical components.

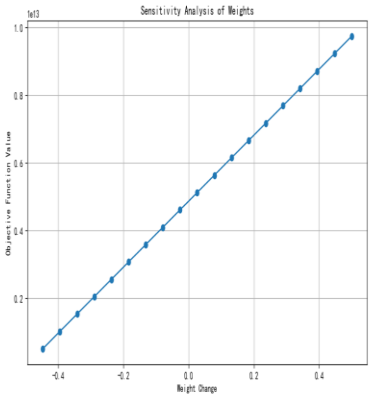

Figure 3: Overall weight sensitivity test

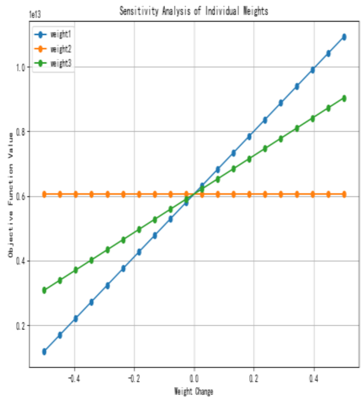

Figure 4: Individual weight sensitivity

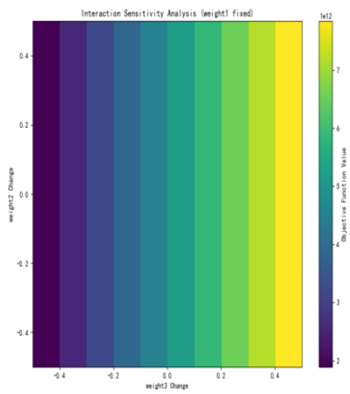

Figure 5: Controlling weight test

The above results can already be observed from the graphs of the sensitivity analysis in this paper. However, for the sake of rigor of the research, this paper adopts the normalization method to

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normalize E(t), G(t)and S(t), which makes the sensitivity analysis more accurate. As shown in from the following table 7, the results after normalization are consistent with the above results, which reflects the stability of the model.

Table 7: Dimensionless sensitivity

Variable ID	Before Dimensionless	After Dimensionless
Sensitivity of Weight1 (S)	9.736622094476209e+12	5.958677224288384
Sensitivity of Weight2 (E)	0.11	-1.635998886613271e-15
Sensitivity of Weight3 (G)	4.4580499023469305e+6	1.07

5.2. Fuzzy optimization: boosting models I in the real-world

Although sensitivity analysis has helped identify the key factors influencing the target region, the real world remains complex, dynamic, and full of uncertainty [7-8]. Traditional precise models and fixed-weight optimization methods often struggle to adapt to such complexities. For example, Models I and II discussed in this paper may fail to yield optimal results due to real-world variability.

To better address these complexities, we introduce fuzzy optimization. This approach offers distinct advantages in handling uncertainty. In practice, many problem boundaries and conditions are ill-defined, and it is often difficult to quantify the relative importance of objective functions. Fuzzy logic addresses this by using membership functions to map input variables (e.g., N(t)) into fuzzy sets such as "low", "medium", and "high", enabling a more flexible representation of ambiguity and making the optimization process more reflective of real-world conditions.

Moreover, fuzzy logic mimics human reasoning, which frequently involves imprecise descriptions. Fuzzy optimization allows subjective adjustment of objective weights under different scenarios, thereby enhancing alignment with real-world decision-making processes. It also improves adaptability by dynamically adjusting objective weights, overcoming the rigidity of fixed-weight methods.

In implementation, we fuzzify the weights. The fuzzy sets—"low", "medium", and "high"—are defined based on data and literature from Model I and set at 1,200,000, 1,500,000, and 2,000,000, respectively. These parameters are used to calculate the objective function via fuzzy membership degrees. The results are presented in Table 8.

Table 8: Fuzzy results

Month	Daily Number of Tourists in Juneau	Daily Number of Tourists in Juneau(fuzz)
1	11166	11166
2	8822	8822
3	9193	9194
4	10786	10786
5	11236	11237
6	12740	12740
7	11949	11950
8	11363	11364
9	11320	11320
10	10667	10668
11	12005	12006
12	11994	11994

The consistent results obtained after parameter selection and fuzzy optimization signify a remarkable level of stability and reliability in our mathematical model. This outcome strongly suggests that the model effectively captures the essential characteristics of the problem at hand, and that the fuzzy optimization method employed is highly suitable for the given scenario. The fixed optimal solution indicates that our approach is robust and immune to minor fluctuations in parameter settings. It provides a solid foundation for further in-depth research and practical applications. We can confidently rely on these findings to make accurate predictions and informed decisions in related fields.

6. Conclusion

This study presents a comprehensive multi-objective optimization model for promoting sustainable tourism in Juneau, Alaska, by integrating ESG criteria, the UN Sustainable Development Goals (SDGs), and the Global Reporting Initiative (GRI) framework. The model effectively captures the complex interplay between environmental, social, and economic factors, offering a data-driven approach to guide tourism management decisions. By incorporating time-series analysis, dynamic programming, and fuzzy optimization, the model accommodates real-world uncertainties while ensuring robust and practical solutions. The results demonstrate that environmental and governance factors exert the greatest influence on sustainable tourism outcomes, emphasizing the critical role of policy interventions and infrastructure investment. Moreover, the proposed revenue allocation scheme aligns fiscal priorities with sustainability objectives, ensuring long-term community benefits. Future research can expand this framework by integrating more granular social indicators, exploring real-time adaptive policies, and applying the model to other tourism-intensive regions.

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