

Enhancing Image Reconstruction under Low-Light and Long-Distance Conditions Using Maximum Likelihood Estimation

Jiaze Tang

Tongji University, Shanghai, China
2451474@tongji.edu.cn

Abstract: Single-photon sensors are crucial for image reconstruction, particularly under low light and close proximity, due to their high sensitivity to photon arrival and straightforward data processing. This paper introduces the use of Maximum Likelihood Estimation (MLE) to enhance image reconstruction by optimizing the processing of incomplete pixel data carried by photons. MLE is employed to construct a macroscopic probability model through Mixed Distribution, integrating and iterating over defective data within an image. Additionally, a Loss Function is introduced to train and evaluate this model, enhancing the robustness and accuracy of the reconstruction by extending the iteration time and integrating defective data repetitively. An interval checking mechanism is developed to ensure the accuracy of the reconstructed images, providing a standard to measure the reliability of the computed values and associated variables that correlate positively with the outcome. The application of these methods significantly enhances image parameters such as Peak Signal-to-Noise Ratio (PSNR) and Structural Similarity Index Measure (SSIM). Results indicate that MLE is highly effective in identifying the most probable segments of incomplete data, thereby enabling precise image reconstruction in challenging conditions.

Keywords: Maximum likelihood estimation, Single photon, Image reconstruction, Mixed distribution

1. Introduction

With the rapid advancement of computer vision systems, recent studies have highlighted that artificial networks possess the capability to extract millions of hidden details from each arriving photon and to analyze discrete portions of an image through deep learning, supported by consistent training using mathematical models tailored to reflect objective reality [1]. This approach not only effectively integrates large data models with computing platforms but also streamlines the construction of neural networks and reduces algorithm execution times, which are crucial for enhancing communication and information interaction across various fields.

However, existing methodologies exhibit significant shortcomings in managing the mixed distributions exhibited by single photons [2]. Additionally, while some studies consider issues of blurry retrieval and pixel-by-pixel screening, challenges such as dim lighting and long distances can reduce photon availability and increase inaccuracies due to the inherent indeterminacy of single photons [3,4]. Factors such as background distribution, processing noise, and atmospheric acoustic

disturbances further degrade the quality of images captured by single-photon detectors [5]. These limitations highlight a lack of universality in the application of current research, often resulting in fragmented approaches that fail to consider the interconnectedness of different distributions, thereby hindering comprehensive analysis.

To address these challenges, this paper proposes an integrated approach to image reconstruction that consolidates distribution groups rather than treating them as discrete entities. By creating a new gradient and multiple reconstruction models, the aim is to more accurately simulate real-world conditions. This research provides theoretical support for image reconstruction based on single-photon sensors and facilitates the transition from two-dimensional to three-dimensional imaging structures, increasingly employed in bioassays [6].

The primary objective of this study is to optimize the values of Peak Signal-to-Noise Ratio (PSNR) and Structural Similarity Index Measure (SSIM). Specifically, statistical models are utilized to restore initial image details and reduce noise, thereby narrowing the confidence intervals of parameters [7]. Additionally, through adjustments and enhancements to the loss function, a comparison between the likelihood values generated by the computational model and actual observed values is enabled, thus assessing the similarity between the original and reconstructed images.

2. Theoretical Background

2.1. Role of Poisson and Gaussian Distribution in Imaging

Poisson distribution acts as a pillar of the total data calculation. Under the dim and long-distance condition, the number of the arriving photon plunges dramatically, so Poisson distribution can describe this kind of randomness well. This can be simplified by the fact that Poisson distribution is a discrete distribution, which is applied to construct the time of occurrence of a random event in given space. The probability mass function can be written as $P(k; \lambda) = \frac{\lambda^k}{k!} e^{-\lambda}$, k represents the times of photon triggers the detector, and λ shows the average standard number of photons induced and degree of the photon departure or the characteristics in data distortion [8]. More importantly, it has close relationship with Gaussian distribution, which is a continuous distribution. The function of Gaussian is $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$, usually the average value $\mu = 0$ in order to unify format, and σ is known as the distinction between rebuilding images and initial images. Under the premise of $\lambda = 5$, The histogram shows that the pixel value equals to 3.7 when the function get maximum, and the total figure is skewed to the left (Figure 1). However, because the amount of the pixel value tends to infinity, the Poisson distribution approaches to Gaussian distribution gradually and the discreteness weakens accordingly [9].

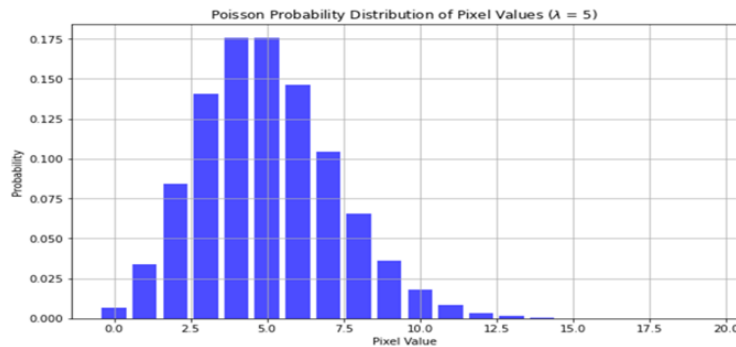


Figure 1: Poisson probability distribution of pixel values ($\lambda = 5$). (Picture credit : Original)

2.2. Fundamentals of Maximum Likelihood Estimation

Maximum Likelihood Estimation (MLE) is an important statistic methodology in evaluating unknown parameters by finding the specific value that maximize the likelihood function in terms of the observable data. To be specific, from a sample space, each element fits in a certain statistic function $f(x|\delta)$, δ in this function represents the parameter to be sought. This function is also called probability density function(PDF) [10]. After that, $\mathcal{L}(\delta)$ is defined as the likelihood function, a comprehensive consideration of discrete probability via continued product. The formation of the function can be written as $\mathcal{L}(\delta) = \prod_{i=1}^n f(x_i|\delta)$. The theory can be applied in the image reconstruction. Assuming that photon can be signed $Z_1, Z_2, Z_3 \dots Z_n$, then because the sensor can only find whether the single photon arrives or not, it only shows binary value: when photon arrives, $Z_n = 0$, on the contrary, $Z_n \neq 0$. By multiplying these two kinds of probabilities from 1 to N, the function will be built. Next, by taking the derivative of the function and make it equal to 0, the ultimate δ is got. This estimation relies on the initial value greatly, which may be confined into the local extremum. Hence, a lot of constraint conditions have to be introduced to calculate partial derivative for different conditions to obtain the optical solution [11].

2.3. Techniques in Image Reconstruction

Single-photon detector, which is based on the interaction between light and electricity, is one of the most important characters in image reconstruction. The theory about photoelectric effect underlines the detecting technique in this paper's research. When the photon is captured by the detector, it will stimulate a photoelectron, then via multiple acceleration by the electric field, the sensor will filter the impurity, amplify the tiny light information, judge the accuracy of the input photon, finally forms a obvious impulse (Figure 2). The purpose of the detector is providing useful information for denoising. Denoising is a technique that utilizes the sign with noise to separate the initial information by deleting the abnormal pixel value. In this paper, Gaussian noise and Poisson noise are two important noises which will be handled.



Figure 2: The obvious impulse. (Picture credit : Original)

3. Methodology

3.1. Loss Function Implementation

Under low-light and long-distance condition, loss function is often used to measure the difference between the true the calculated value. In this paper, the function is mainly introduced to modify the model built before. The function can be written as $MSE = \frac{1}{N} \sum_{i=1}^N (y_i - \hat{y}_i)^2$. By minimizing the function, it will fulfill the goal of transformation from images with low resolution to high resolution [12]. Simultaneously, the function this research builds captures the empty pixels which are caused by the lack of important information and amplify error message in order to obverse the error more conveniently and avoid the occurrence of non-physical information. One of the advantages of this function compared with others is that it can stimulate the image with different dimensions by

increasing the dimensions of arrays (Figure3). The three figures shows MSE loss function in three dimensions respectively. In the first figure, the assuming image value ranges from 720 to 1280 which is a long range because the large number of missing part. However, the previous image with true value is about 970, thus by finding the minimum of function which is the confluence of the axis $y \approx 970$ and the curve (the red point in figure). Similarly, by adding two parameter w_1 w_2 and expand the dimensions of array from two to three and setting the z –axis as loss value, functions are also obtained. The parameter of w_1 w_2 range from 1280 to 2560, 1270 to 2565 respectively. Finally, by finding depression area of the figure 3, the minimum value is got, which is the most reliable parameter groups.

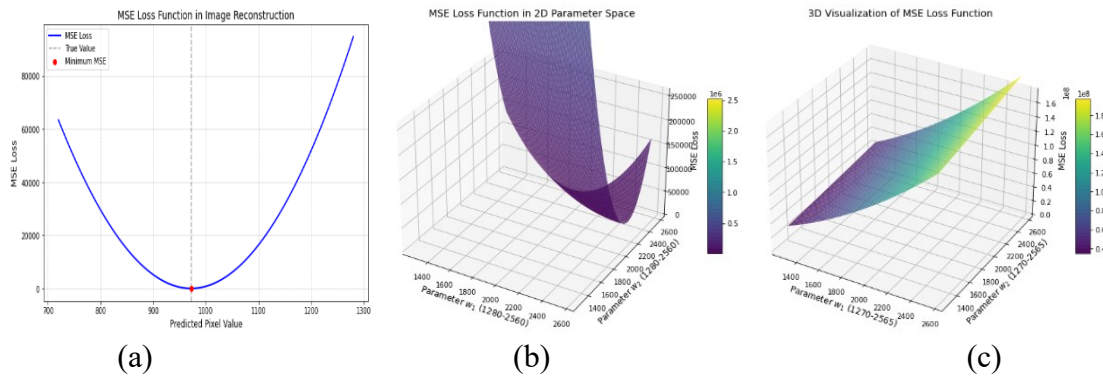


Figure 3: The image with different dimensions. (Picture credit : Original)

3.2. Analysis of Mixed Distribution Data

When only taking Poisson distribution into consideration, the fact is that the detector can only output the binary value simplifying the problem to Bernoulli distribution. Hence, ML estimation can be used to calculated the important parameter δ which has been mentioned in part 2.2. The steps are as follows [13].

Defining Z_n as a number taking values of either 0 or 1, make log:

$$\log \mathcal{L}(\delta | \mathbf{z}) = \log \left\{ \prod_{n=1}^N (1 - e^{-\delta})^{z_n} (e^{-\delta})^{1-z_n} \right\} \quad (1)$$

Define $S = \sum_{n=1}^N Z_n$, derivative:

$$\frac{d}{d\delta} \{S \log(1 - e^{-\delta}) - \delta(N - S)\} = \frac{S}{1 - e^{-\delta}} e^{-\delta} - (N - S) \quad (2)$$

Make it equals to 0, then

$$\delta = -\log \left(1 - \frac{S}{N} \right) \quad (3)$$

Substitute S:

$$\delta = -\log \left(1 - \frac{1}{N} \sum_{n=1}^N Z_n \right) \quad (4)$$

However, when putting Gaussian distribution into the model as well, handling the matter from the aspect of statistics is more feasible. Under the low-light condition, Gaussian noise occupies the main session of the whole noise, by keeping the light density and altering the distance between the light source and the photon detector, the difference between the single distribution and mixed distribution will be detected [14]. Meanwhile, compared the original image, Gaussian noise image and mixed image (Figure 4), which are added the noise with the same standard deviation 500, and the mean value 0, it is easy to find that Gaussian noise makes the main noise in image and will enhance the

exposure rate of the image. Poisson distribution makes contributions to the light and gray level, and make the image rougher. The integration of two distributions avoids the empty image set, and also is conducive to the convergence of the loss function built in 3.1 [15].

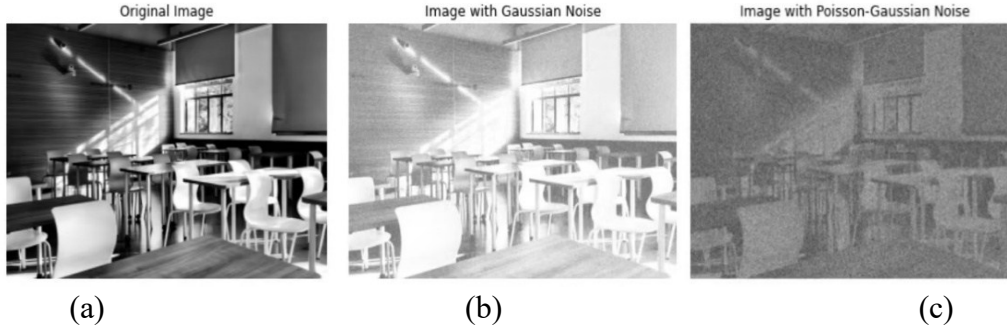


Figure 4: This caption has one line so it is centered. (Picture credit : Original)

3.3. Photon Increment and Integration

The methodology of Photon Increment and Integration gets rid of the limitation of the distance and the light condition completely. Every single photon can be recorded by the $N \times N$ linear matrix, by increasing the order of the certain matrix repeatedly, it store the vague data in different grid the matrix has separated, like a large storage box to classify different types of data. When the data is in need, the computer will output the related grid according to algorithm and the real acquirement and rearrange them to form a new matrix. The most important step in this research is extracting keyframes from the matrix, which will get through hundreds of iterations called keyframes increment and integration. In other word, only by integration can the incomplete message be considered totally and the noise point can be eliminated, which can be analogous to filtering. The figures display the different stage the keyframes are extracted ($T=5, 10, 20, 50, 90, 110$) (Figure 5). With the increment of T , the image will be more distinct, and the blank pixels induced by outer condition are compensated and be replaced by the predicted pixels.

3.4. Testing Confidence Intervals

Based on initial theory, in this paper, the testing confidence intervals are introduced to show the accuracy of the δ calculated before in 3.2. However, confidence intervals provide a specific range of the uncertainty of the predicted parameter and the size of the interval is negatively correlated with the accuracy of parameters [16]. The interval can be written as $\delta \pm \frac{z_{\alpha}}{2} K N_p^{-\frac{1\sqrt{\delta}}{2}}$, $\frac{z_{\alpha}}{2}$ is a constant according to the table of standard Gaussian distribution, K is the total amount of unit images(pixels), and N_p is the whole number of photon. Therefore, by increasing the value of K and N_p , the value of $\frac{z_{\alpha}}{2} K N_p^{-\frac{1\sqrt{\delta}}{2}}$ will be more small, which represents that the parameter δ is more believable.

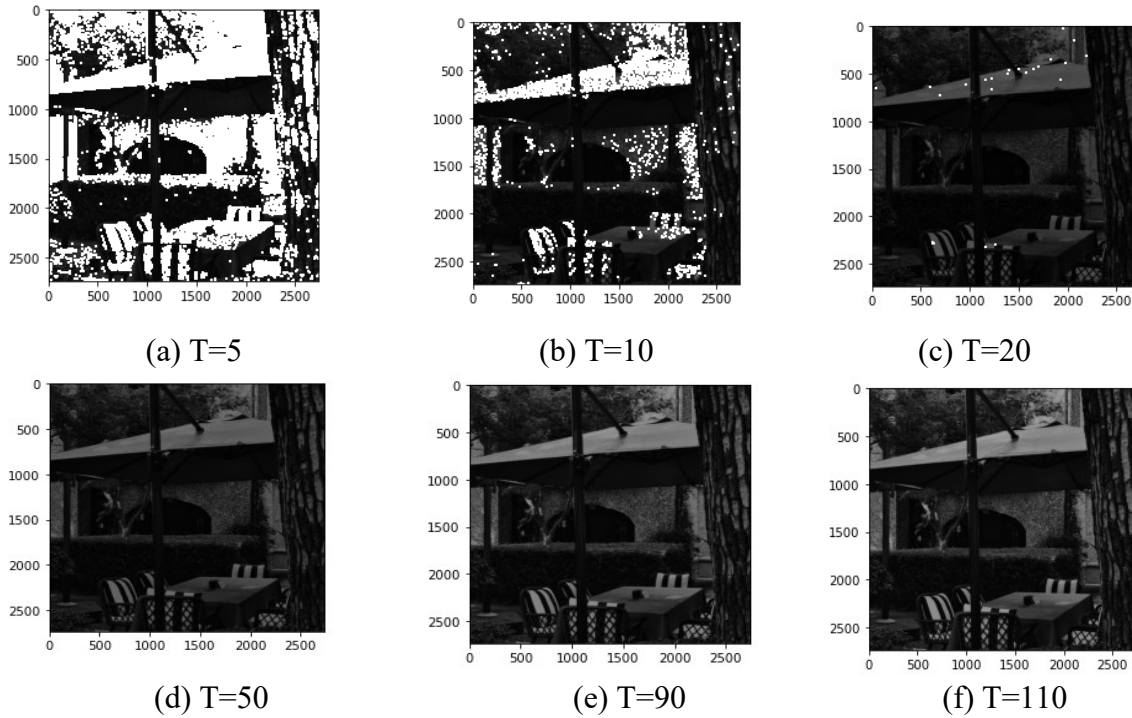


Figure 5: This caption has one line so it is centered. (Picture credit : Original)

4. Results

4.1. PSNR Analysis

Peak signal-to-noise ratio (PSNR) is an important value that measures the quality of the image reconstructed. In this paper, based on the research and methodology, it is necessary to test the effect of ML estimation. PSNR has the unit dB, which is the same unit as sound intensity level. The PSNR can be written as [17].

$$PSNR = 10 \cdot \log_{10}\left(\frac{MAX^2}{MSE}\right) \quad (5)$$

The larger the PSNR values are, the less important information the process miss, particularly under the extremely harsh conditions. In other words, it shows the ratio between initial session and the noises. For the six images showed in figure 5, it is wise to calculate the real value of PSNR as a compelling evidence to illustrate the feasibility of ML estimation combined with loss function, mixed distribution and the utilization of integration (Table 1). From the table, it is obvious that with the increase of the value of T, from 5 to 110, PSNR is rising slowly, from 4.41 to 6.68, which represents that the image via the ML estimation and combined treatment is getting close to the true image. Meanwhile, from the figure 6, it is also easy to analyze that the total tendency of the function related to T and PSNR is monotone increasing, though there are slow descents with small fluctuation. When the value is over 60, PSNR is flatten out around 7.00 instead of soaring in the value of T about 15 to 20, which proves the convergence of likelihood function and loss function built above.

Table 1: The value of different value of T and corresponding PSNR.

Value of T	PSNR (dB)
5	4.41
10	5.98
20	6.48
50	6.57
90	6.66
110	6.68

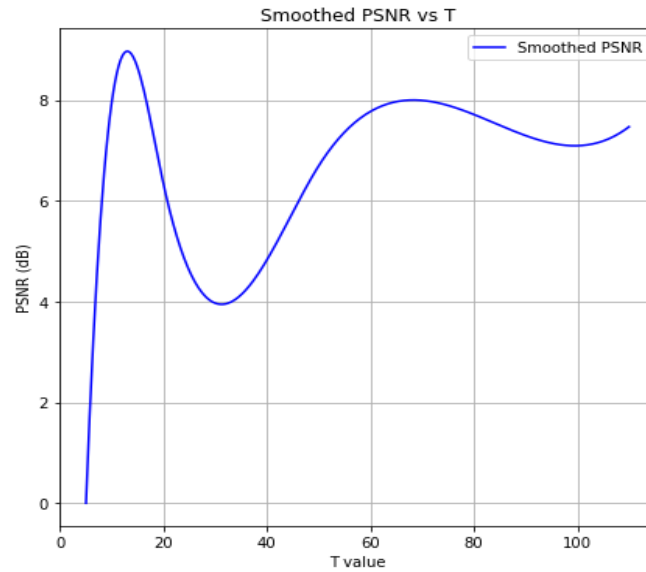


Figure 6: The real value of PSNR. (Picture credit : Original)

4.2. SSIM Values

Structural similarity index (SSIM) is also another vital standard that measure the similarity of two images by stimulating the visual system of humans on the feeling and perception of images. It makes more concentrations on the gray level and structures of the certain images.

The SSIM can be written as:

$$SSIM = \frac{(2\epsilon_x\epsilon_y+C_1)(2\eta_{xy}+C_2)}{(\epsilon_x^2+\epsilon_y^2+C_1)(\eta_x^2+\eta_y^2+C_2)} \quad (6)$$

ϵ_x, ϵ_y show the mean light density, η_x, η_y symbolize the similarity of structure, C_1, C_2 are constants in order to ensure the function with real meaning. The proper range of the SSIM in this research is from -1 to 1, the more the value is close to 1, the more similar the two images are, which is like the correlation coefficient [18].

Table 2: The value of different value of T and corresponding SSIM.

Value of T	SSIM
5	0.0891
10	0.1114
20	0.1218
50	0.1314
90	0.1359
110	0.1361

From the table 2, it is obvious to obtain the information that, with the increasing times of integration and iteration (T) from 5 to 110, SSIM is getting close to 1, from 0.0891 to 0.1361. Although the SSIM only fluctuates around 0.1, a small decimal compared to 1 because of the color distinction which are colors and grey respectively, it also illustrates the remarkable result of image reconstruction.

4.3. Visual Outcomes

Considering the visual effects output, it is obvious that the amount of the noise points plummets dramatically and the structure of the image has stronger three-dimensional effect. Meanwhile, with the growth of the information rebuilt, the image is getting more and more smooth instead of corner angle with high level of sharpening, which is the chief culprit of discomfort when people take look at first glance. In addition, the innate logic is clearer showing on the visual outcomes instead of a total mess [19].

5. Conclusion

In conclusion, this research effectively demonstrates the application of Maximum Likelihood Estimation combined with mixed distribution approaches to enhance image reconstruction under challenging conditions characterized by low light and long distances. By integrating advanced statistical models with computational methods, the clarity of reconstructed images has significantly improved, optimizing key parameters such as Peak Signal-to-Noise Ratio (PSNR) and Structural Similarity Index Measure (SSIM). The introduction of a sophisticated loss function and stringent confidence intervals plays a critical role in refining the parameter estimation process, ensuring more reliable and accurate image reconstruction. This approach not only reduces the impact of noise and other distortive factors but also enhances the precision of the reconstructed images, thus minimizing potential deviations that could affect practical applications.

The methodologies, particularly the innovative integration of Poisson and Gaussian distributions, address the non-universality and fragmentation observed in previous studies. By employing these mixed distribution models, the complexities of photon scarcity and indeterminacy, which are prevalent in long-distance imaging scenarios, have been successfully navigated.

References

- [1] Chen, Y. et al. (2023) All-analog photoelectronic chip for high-speed vision tasks. *Nature*.
- [2] Wang FL, Zuo B. (2016) Detection of surface cutting defect on magnet using Fourier image reconstruction. *Journal of Central South University*, 23(5), 1123-1131.
- [3] R. Saravana Ram, M. Vinoth Kumar, T. M. Al-shami, M. Masud, H. Aljuaid, M. Abouhawwash. (2023) Deep Fake Detection Using Computer Vision-Based Deep Neural Network with Pairwise Learning. *Intelligent Automation & Soft Computing*, 2(2), 2449-2462.
- [4] Mahgoub A. Salih, Taysir M. Elmahdi. (2024) Exploring the Implications of the Deformation Parameter and Minimal Length in the Generalized Uncertainty Principle. *Journal of Quantum Information Science*, 14(1), 1-14.
- [5] Ali Syed Saqlain, Fang Fang, Tanvir Ahmad, Liyun Wang, Zain-ul Abidin. (2021) Evolution and Effectiveness of Loss Functions in Generative Adversarial Networks. *China Communications*, 18(10), 45-76.
- [6] JIAO Fengyuan, YANG Zhixiu, SHI Shaojie, CAO Weiguo. (2024) Low-dose CT image denoising method based on generative adversarial network. *Journal of Measurement Science and Instrumentation*, 15(4), 490-498.
- [7] Hoeppli, M. E., Garenfeld, M. A., Mortensen, C. K., Nahman-Averbuch, H., King, C. D., & Coghill, R. C. (2023) Denoising task-related fMRI: Balancing noise reduction against signal loss. *Human Brain Mapping*, 44(17), 5523–5546.
- [8] Zhu, X., Liu, Z., Cambria, E., Yu, X., Fan, X., Chen, H., & Wang, R. (2025). A client–server based recognition system: Non-contact single/multiple emotional and behavioral state assessment methods. *Computer Methods and Programs in Biomedicine*, 260, 108564.

- [9] Wang, R., Zhu, J., Wang, S., Wang, T., Huang, J., & Zhu, X. (2024). Multi-modal emotion recognition using tensor decomposition fusion and self-supervised multi-tasking. *International Journal of Multimedia Information Retrieval*, 13(4), 39.
- [10] Wang, F., Ju, M., Zhu, X., Zhu, Q., Wang, H., Qian, C., & Wang, R. (2025). A Geometric algebra-enhanced network for skin lesion detection with diagnostic prior. *The Journal of Supercomputing*, 81(1), 1-24.
- [11] Zhao, Z., Zhu, X., Wei, X., Wang, X., & Zuo, J. (2021, June). Application of Workflow Technology in the Integrated Management Platform of Smart Park. In *2021 IEEE 4th Advanced Information Management, Communicates, Electronic and Automation Control Conference (IMCEC)* (Vol. 4, pp. 1433-1437). IEEE.
- [12] Zhang, Y., Zhao, H., Zhu, X., Zhao, Z., & Zuo, J. (2019, October). Strain Measurement Quantization Technology based on DAS System. In *2019 IEEE 3rd Advanced Information Management, Communicates, Electronic and Automation Control Conference (IMCEC)* (pp. 214-218). IEEE.
- [13] Grosdos Alexandros, Heaton Alexander, Kubjas Kaie, Kuznetsova Olga, Scholten Georgy, Sorea Miruna-Ştefana. (2023) Exact solutions in log-concave maximum likelihood estimation. *Advances in Applied Mathematics*.
- [14] Seifert J, Shao Y, van Dam R, Bouchet D, van Leeuwen T, Mosk AP. (2023) Maximum-likelihood estimation in ptychography in the presence of Poisson-Gaussian noise statistics: publisher's note. *Opt Lett*, 48(23), 6291. doi: 10.1364/OL.513661. PMID: 38039249.
- [15] Lin, J., Zhang, Z., Li, W., Pei, R., Xu, H., Zhang, H., & Zuo, W. (2024) UniRestorer: Universal Image Restoration via Adaptively Estimating Image Degradation at Proper Granularity. *ArXiv*, abs/2412.20157.
- [16] Li, S. (2021). Photon Counting Integrated Imaging Based on Element Image (Master's thesis, South China University of Technology). Master's.
- [17] HIMANSHU, REENA HOODA, VIKAS POPLY. (2024) Image Enhancement in Fourier Domain using MSE and PSNR[C]//Frontiers in industrial and applied mathematics : FIAM2022 : Mahendergarh, India, 22-23 Decembre 2022. 060001-1-060001-8.
- [18] RAJITH WEERASINGHE, ERIC YOUNG, RYAN SHAVER, et al. (2024) Analysis of the Impact of SSIM Parameterization for SAR to EO Translation Networks[C]//Algorithms for Synthetic Aperture Radar Imagery XXXI: At SPIE Defense + Commercial Sensing. 23-25 April 2024. National Harbor, Maryland, United States. 1303206.1-1303206.7.
- [19] Siyuan Yin, Shibao Wu, Zhanming Li, Haoran Lu, Zhiyao Wang, Zengquan Yan, Xianmin Jin. (2024) Photonic timestamped confocal microscopy. *Adv. Imaging*, 1, 021005.