

A Markov Chain Based Study of Teachers' Approaches to Improving Education Equality

Jiejun Zhu

*School of Mathematics and Information Sciences, Guangzhou University, Guangzhou, China
32215150058@e.gzhu.edu.cn*

Abstract: The Excellent education is an important foundation for national development. The effective delivery of high-quality instruction is a critical consideration that requires ongoing reflection and refinement by educators. Because of the good properties of Markov chains, many papers have used it as a tool to carry out research. This paper focuses on how teachers can use Markov chains as a tool to continuously improve the quality of education. To this end, the paper is carried out by summarizing the essential methods and applications of Markov chains in education. The application of Markov chains in teaching and learning has been found to focus on assessing the quality and methods of teaching and the research tools used in this area are usually quite homogeneous and dominated by Markov models. In addition, a small number of studies have investigated the issues of student attention allocation and student data privacy risk in combination with other research tools. This paper demonstrates to teachers the value and use of Markov models for improving the quality of education. Furthermore, it also encourages teachers to utilize practicable research methods to analyse and evaluate the teaching process scientifically and rationally, and to improve teaching methods and strategies accordingly.

Keywords: Markov model, Education, Transition Matrix, Quality teaching

1. Introduction

Quality education is an important foundation for national development, so it is really necessary to improve education constantly. Since teachers play a leading role in education, they should carry out scientific and reasonable teaching, and adjust their teaching strategies and methods timely based on students' learning performances to make sure a high quality of teaching. Other teachers' evaluation may be influenced by various factors, such as the relationship between each other and personal preference, so it cannot be a scientific and objective assessment method [1]. Evaluating the teaching efficiency by comparing students' grade neglects the differences in student base, so it also cannot be an appropriate method. Markov chain is widely used in statistics and probability theory for study, and it also can be a useful tool in studying improving teaching methods to enhance teaching quality through scientific and objective quantification analyses in the teaching process.

By carefully studying the existing related papers, the author summarized the application of Markov chain in education. First of all, some researchers used Markov chain to analysing students' behaviours and performances [2,3]. Based on the outcome of the analyses, teachers can adjust teaching methods in time to get a better teaching quality. Combined with eye movement experiment, Markov chain can be used to study the allocation of student's attention, the study finds that students will be

more focused when looking at the media and the teacher's board, and will be easily distracted when reading the textbook [4]. With Big Data applied in various areas, including education, schools or teachers have collected large amount of data about students, which raises privacy risks. Vatsalan Dinusha et al. quantified privacy risk in education by using Markov model [5]. This provides direction for developing mechanisms to improve student privacy safeguards.

The main purpose of this paper is to demonstrate the value and use of using Markov models in education, and the rest of the paper will be organized as follows: Firstly, the basic theory of Markov models will be introduced, then the application of each of the three types of Markov models in education will be presented, and finally, a brief summary will be given.

2. Basic Theory of Markov Model

2.1. Traditional Markov Model

A Markov chain is a stochastic process in which the future state depends only on the current state and is independent of the past states. The definition is as follows:

Let $\{X_n, n = 0, 1, 2, \dots\}$ be a stochastic process that takes on a finite or countable number of possible values. Unless otherwise mentioned, this set of possible values of the process will be denoted by the set of nonnegative integers $\{0, 1, 2, \dots\}$. If $X_n = i$, then the process is said to be in state i at time n . It is supposed that whenever the process is in state i , there is a fixed probability P_{ij} that it will next be in state j . That is,

$$P\{X_{n+1} = j \mid X_n = i, X_{n-1} = i_{n-1}, \dots, X_1 = i_1, X_0 = i_0\} = P_{ij}. \quad (1)$$

For all states $i_0, i_1, \dots, i_{n-1}, i, j$ and all $n \geq 0$. Such a stochastic process is known as a Markov chain.

The mathematical model of a Markov chain mainly consists of a state space and transfer probabilities. Transfer probability describes the probability of transferring from one state to another. By constructing the transition matrix, the transfer pattern between states can be clearly demonstrated. The stationary distribution reflects the stable proportion of the system that is in each state during long-term operation, which is important for analysing the long-term behaviour of the system. Consider an irreducible Markov chain. If the chain is positive recurrent then the long-run proportions are the unique solution of the equations

$$\begin{cases} \pi_j = \sum_{i \in I} \pi_i P_{ij} \\ \sum_{j \in I} \pi_j = 1, \pi_j \geq 0 \end{cases}. \quad (2)$$

The following are the steps in the application of Markov model in the assessment of the quality of different teaching method.

Firstly, determine the state space I of the model. Usually, the state space is distribution of students' grade. Categorize students' performance into five states: excellent, good, moderate, passing, and failing. Let $X_1 = (90, 100]$, $X_2 = (80, 90]$, $X_3 = (70, 80]$, $X_4 = [60, 70]$, $X_5 = (0, 60)$ denotes the above five states respectively, and $I = (X_1, X_2, X_3, X_4, X_5)$ denotes the state space.

Secondly, determine the transition probabilities P_{ij} of each teaching method and construct the transition matrix. The transition probability P_{ij} represents the probability that state i will be in state j in one step. The one-step transition matrix is

$$P = \begin{pmatrix} P_{11} & \cdots & P_{15} \\ \vdots & \ddots & \vdots \\ P_{51} & \cdots & P_{55} \end{pmatrix}. \quad (3)$$

Due to Chapman-Kolmogorov Equations, the n-step transition matrix can be obtained by multiplying the matrix P by itself n times.

Thirdly, Solve the balance equation. Assuming that this Markov Chain is aperiodic, if there exists a positive integer n such that for all $i, j \in I$, there is $P_{ij}^n > 0$, then the Markov chain is ergodic. Therefore, $\lim_{n \rightarrow \infty} P_{ij}^n = \pi_j$. If possibility distribution $\{\pi_j, j \in I\}$ satisfies Eq. (2), possibility distribution $\{\pi_j, j \in I\}$ is a stationary distribution of Markov chain. An irreducible ergodic Markov chain exists stationary distribution. Solution of the balance equation $\pi P^T = \pi$ is $\pi = (\pi_1, \pi_2, \pi_3, \pi_4, \pi_5)$, showing the stable probability of each state.

Fourthly, analyse the index C. Choose the median grade of each state. That is $N(95, 85, 75, 65, 55)$. The index C is the value of $N \times \pi^T$, then comparing each index C of each teaching method. Higher value of index means this teaching method has a better teaching result.

2.2. Improved Markov Model

Improved Markov model provides a new concept named degree of relative progress [6]. When determine the state place, better states are usually denoted by smaller number. If $i < j$, then P_{ij} means the probability of progress. If $i > j$, P_{ij} means the probability of regression. The following Figure 1 shows that the one-step transition matrix with an upper triangular part containing probabilities of regression and a lower triangular part containing probabilities of progress.

$$P = \begin{pmatrix} P_{11} & \cdots & P_{15} \\ \vdots & \ddots & \vdots \\ P_{51} & \cdots & P_{55} \end{pmatrix}$$

Figure 1: The feature of one-step transition matrix

This feature of the transition probabilities P_{ij} provides a new idea for the research [7]. Therefore, improved Markov model omits the complex process of solving the balance equation and analyses the result through new index, degree of relative progress.

3. Applications in Education

3.1. Analysing Students' Behaviours and Performances

The first application is analysing students' behaviours and performances which helps teachers optimise teaching strategies and improve teaching quality. Traditional Markov Chain and improved Markov Chain are the two main methods used to study this case. Because the traditional Markov model has been introduced in detail above, so this part will talk more about the improved Markov mode. The core concept of improved Markov model is degree of relative progress, which clearly demonstrates whether students in each state or whole class makes a progress after a periodic learning.

Following are some definitions of improved model: Assuming that there are 5 states in this model, let n_i denote the number of students in state i and n_{ij} denote the number of students transferring from

state i to state j . The relative learning progress matrix $A = \begin{pmatrix} a_{11} & \cdots & a_{15} \\ \vdots & \ddots & \vdots \\ a_{51} & \cdots & a_{55} \end{pmatrix}$ is defined, where each element is:

$$a_{ij} = (i - j) \frac{n_{ij}}{n_i}, i, j = 1, 2, 3, 4, 5. \quad (4)$$

Here, $a_{ij} \geq 0$ indicates overall progress or stability of student group, while negative value suggests regression. For each state i , the the degree of relative progress of students E_i is defined as: $E_i = \sum_{j=1}^5 (i - j) \frac{n_{ij}}{n_i}$. Comparing the degree of relative progress of students in different states shows the performances of students in different states. If *both* E_i and E_j are positive, it means students in state i and state j make a progress. And if $E_i > E_j$, it means students in state i make a bigger progress than students in state j . This is a more detailed study which cannot be done by traditional model. Adding each E_i can acquire a concept defined as degree of relative progress of students of whole class. That is

$$E = \sum_{i=1}^5 E_i = \sum_{i=1}^5 \sum_{j=1}^5 (i - j) \frac{n_{ij}}{n_i}. \quad (5)$$

Similar to E_i , index E shows the performance of whole class [7].

However, neither the traditional Markov model nor the improved Markov model accounts for the interferences from differences in exam difficulty and the characteristics of various disciplines. This is because both methods directly use students' grades as data.

Based on the above two methods, Jianhua Liu et al. proposed to standardize students' grade before determining the state space to deal with these interferences. Assuming that X , μ and σ^2 respectively represent the grade of a student, average grade of students in this course, variance of student grade in this course. According to central limit theorem, X approximately follows a normal distribution so that standardized grade Y is denoted by $Y = (X - \mu)/\sigma$ [8].

When the grade data is large, standardizing each grade could be a large task without the help of computer. Quantifying the difficulty of test maybe can help to relieve the interference of difficulty difference. Test difficulty coefficient allows difficulty difference to be visualized. Test difficulty coefficient is defined as $T = 1 - \sum_{i=1}^n \frac{\text{average grade of } i\text{th question}}{\text{full mark of the test}}$, n denotes the total number of questions [9,10]. The higher the value of T , the higher the difficulty. There is not quite different with previous models. The difference is that the index should be multiply by test difficulty coefficient T before analyzation. Taking improved Markov model as example, instead of analysing the E_i and E , analysing the new indexes $E_i \times T$ and $E \times T$.

3.2. Studying the Allocation of Student's Attention

The second application is studying the allocation of student's attention. In fact, this application could be classified as the first application since attention is also a type of behaviour. However, Markov Modelling in this application is only one of the primary tools, and a much more extensive set of tools is actually employed. Therefore, categorising this application separately emphasises to readers the importance of utilising the diverse tools available for the study. According to psychology research, there is a very close relationship between students' concentration and their absorption of knowledge. So, the study of students' attention allocation plays an important role in optimizing teaching methods and strategies. Since eye movement tracking is an important method for psychology research and 80 to 90 percent of external information is acquired by vision [11], Chaobo Yu et al. combined eye movement tracking with Markov chain to studying the allocation of college student's attention [4].

Researchers conducted an eye movement experiment using an eye movement meter to collect gaze data from students who were learning the content for the first time. The students participating in the experiment were required to take a test after class. This allowed the researchers to assess the students' mastery of the knowledge and subsequently divide them into two groups: high-achieving students and average-achieving students. Because the allocation and transition of college student's attention in class have the property of Markov and the time interval of data collection is fixed, so the gaze points data can be seen as a Markov chain. First step is classifying gaze states as attentive and inattentive. This experiment labelled pupil diameters smaller than the mean and gaze points sustained gaze for more than 5 s without shifting as inattentive, otherwise attentive. Researchers only used the attentive data in this experiment, and determined 7 gaze zones for multimedia, blackboard, teacher, textbooks, notebooks, mobile phones and other zones. These 7 zones consist of the state place of Markov model. The data of gaze points selected by eye movement tracking shows the transition probability of each transfer so that the one-step transition matrix could be acquired. Then the later steps are the same as the steps of traditional Markov model.

Analysing the stationary distribution for both high - achieving and average students reveals that they are both significantly more likely to look at multimedia and the blackboard than at textbooks and notebooks. This suggests that students' cognitive processing of classroom - taught knowledge mainly comes from multimedia and the teacher's board. Teachers should therefore leverage multimedia and the blackboard effectively. Besides, Wilcoxon test was conducted on the data of one-step transition probability of gaze in the concentration state of high and average students, which showed the high students tended to engage in multiple successive gazes and process the acquired information cognitively in depth for accurate comprehension. Therefore, when the teachers are giving a lecture, it is better not to ask students to look at many things in a short time.

3.3. Privacy Risk Quantification in Education Data

The third application is quantifying privacy risk in education data. With the development of big data, individual information is being collected and used in a variety of ways, and education is certainly no exception. However, along with the convenience of big data comes the issue of privacy risks for students. Educational data contains a large amount of sensitive information, such as students' family information, academic performance, behavioural habits, etc. These data may be at risk of privacy disclosure during collection, storage, and sharing. Dinusha Vatsalan et al. quantified the dynamics of privacy risk in educational data by applying Markov modelling and constructing a privacy risk quantification framework [5].

Quantifying privacy risks makes educators more aware of the importance of enhancing privacy protections for student data. In this paper, Dinusha Vatsalan et al. did not directly point to ways to enhance the protection of student data. Nevertheless, the dynamic nature of Markov models provides new ideas for privacy protection of educational data. Educational institutions can dynamically adjust their privacy protection strategies, such as cryptography, access control and data desensitization, according to the changes in data privacy risks. Besides, teachers should strengthen the development of students' awareness and ability to protect their privacy. For example, teachers can require students to clear their browser records after completing a computer course [12].

4. Conclusion

This essay begins with a brief introduction to the operational steps of Markov modelling so that the reader can better understand the article. Thereafter it describes three applications of Markov chains in education and suggests a possible improvement in the models of assessing the quality of teaching and learning. For primary and secondary school teachers, in addition to Markov modelling, they

should make good use of accessible tools such as SPSS, MATLAB, etc. to conduct research. Tools such as eye-tracking experiments may only be accessible to teachers in higher education. In any case, teachers should make good use of all available resources to continuously improve the quality of education. This work suggests that the Markov Chain is an essential tool for teachers to improve education equality.

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