

Research on Optimizing the Profit Model of Live E-commerce Games Based on Mathematical Expectation

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Abstract: In recent years, the live broadcast e-commerce industry has risen rapidly. Especially driven by short video platforms such as Douyin, live broadcast has become one of the main sales methods for merchants. The traditional live broadcast sales method mainly relies on product introduction and price promotion. Users' purchasing decisions often depend on the cost-effectiveness of the product, which requires merchants to invest a lot of advertising costs to maintain sales. The emergence of live broadcast games, a live broadcast model with gamification design, has changed the traditional marketing model. This model allows consumers to "participate in the game" while shopping by constructing a probabilistic profit model, which increases the entertainment and uncertainty of shopping. Based on the principle of mathematical expectation, this study explores how to optimize the profit model of merchants by designing live broadcast games through theoretical research and case analysis, and verifies its commercial feasibility. The study shows that the reasonable application of mathematical expectation can help merchants design a live broadcast game model that can both stimulate users' desire to buy and ensure long-term profitability. However, this study has a certain degree of singularity. Future research can focus on the application of mathematical expectation in different industries.

Keywords: live broadcast e-commerce, mathematical expectation, gamification design, profit model

1. Introduction

In recent years, the live streaming e-commerce industry has risen rapidly. Especially driven by short video platforms such as Douyin, live streaming has become one of the main sales methods for merchants [1]. In 2024, the scale of China's live streaming e-commerce market has exceeded 4.9 trillion yuan, and it is expected to maintain rapid growth in the next few years [2]. The traditional live streaming method is mainly based on product introduction and price promotion. Users often base purchasing decisions on cost-performance, which requires merchants to invest heavily in advertising to sustain sales. Enhancing consumer purchase intent and increasing interactivity has become crucial for optimizing marketing strategies. Recently, a gamified live streaming model has gained popularity, allowing consumers to "play" while shopping. This model, such as the "39 yuan purchase for a lottery" approach, uses a probabilistic profit system. By applying mathematical expectation, merchants can control profits while leveraging consumers' "gambling psychology" to drive sales.

Mathematical expectation is widely used in economics and business decision-making [2]. Traditional business models rely on discounts and promotions to attract consumers, while live

streaming games use the principles of probability theory to cleverly adjust the probability of winning, product pricing, and prize settings to ensure the long-term profitability of merchants [3]. This paper will analyze how to use mathematical expectation to design profitable games or marketing strategies through feasibility analysis and case studies. The research goal of this paper is to explore how to optimize the profit model of live streaming games based on mathematical expectation and verify its commercial feasibility. Through mathematical modeling and case analysis, this paper will reveal how to use mathematical expectation to design a live streaming game that can both stimulate users' desire to buy and ensure stable profitability for merchants.

2. Overview of the Basic Theory of Mathematical Expectation

2.1. Definition and Basic Properties of Mathematical Expectation

Mathematical expectation refers to the weighted average of the possible values of a random variable, with the weight being the corresponding probability [4]. Let X be a discrete random variable that can take values $x_1, x_2, \dots, x_n$ with corresponding probabilities $P(X = x_1), P(X = x_2), \dots, P(X = x_n)$. Then, the expected value (denoted as $E[X]$) of the random variable X is defined as:

$$E[X] = \sum_{i=1}^n x_i P(X = x_i) \quad (1)$$

If X is a continuous random variable with probability density function $f(x)$, then its expected value is defined as:

$$E[X] = \int_{-\infty}^{\infty} x f(x) dx \quad (2)$$

Mathematical expectation can be understood as the average result of a large number of repeated experiments. For example, if the prizes of a game are 10 yuan, 50 yuan and 100 yuan, and the probabilities are 50%, 30% and 20% respectively, the mathematical expectation is calculated as follows:

$$E[X] = 10 \times 0.5 + 50 \times 0.3 + 100 \times 0.2 = 5 + 15 + 20 = 40 \quad (3)$$

This shows that if a user participates in the game multiple times, the average profit is 40 yuan. However, the actual profit of an individual user in a single game is still uncertain.

From this, it can be seen that the mathematical expectation has the following fundamental properties [5]

Linearity Property

For random variables X , Y and constants a , b :

$$E[aX + bY] = aE[X] + bE[Y] \quad (4)$$

This shows that the calculation of mathematical expectations is additive, that is, the expectations of multiple variables can be calculated separately.

Independence Property

If X and Y are independent random variables, then:

$$E[XY] = E[X]E[Y] \quad (5)$$

This property is particularly important when analyzing the expectation of multiple independent events, such as independent lottery draws.

Monotonicity Property

If a random variable $X \geq Y$ (for all possible values), then:

$$E[X] \geq E[Y] \quad (6)$$

This property is used to evaluate the expected returns of different business strategies and help merchants choose better strategies. These properties of mathematical expectation provide a theoretical basis for calculating the revenue of e-commerce live games. For example, merchants can use linear properties to calculate the total expected returns of multiple winning schemes and optimize the profit model by adjusting the probability distribution.

2.2. Application of Mathematical Expectation in Economics and Game Theory

Insurance companies use mathematical expectation to calculate the reasonable pricing of insurance products [6]. Assuming that the compensation amount of a certain insurance is 10,000 yuan and the probability of an accident is 0.1%, the insurance company can calculate the average compensation cost for each insured person through mathematical expectation:

$$E[X] = 10,000 \times 0.001 = 10 \quad (7)$$

Therefore, insurance companies add operational costs and profit margins on this basis to determine a reasonable premium.

In investment decisions, investors use mathematical expectation to evaluate the returns of different investment portfolios. For example:

Investment A: 70% probability of a 10% return, 30% probability of a 5% loss.

Investment B: 50% probability of a 15% return, 50% probability of a 10% loss.

The expected return is calculated as follows:

$$E[A] = 0.7 \times 10\% + 0.3 \times (-5\%) = 7\% - 1.5\% = 5.5\% \quad (8)$$

$$E[B] = 0.5 \times 15\% + 0.5 \times (-10\%) = 7.5\% - 5\% = 2.5\% \quad (9)$$

Since $E[A] > E[B]$, Investment A is superior in terms of long-term average returns.

Game theory studies the optimal strategy of decision makers in a competitive environment, and mathematical expectation plays a key role in it. Typical applications include casino games (such as roulette) controlling the house advantage through mathematical expectation to ensure long-term profitability. Roulette has 38 numbers (0-36 and 00). If a player bets on a single number, the odds are 35:1, while the actual probability of winning is $1/38$:

$$E[X] = 35 \times \frac{1}{38} + (-1) \times \frac{37}{38} = -\frac{2}{38} \approx -5.26\% \quad (10)$$

This implies that, over time, players will lose approximately 5.26 cents for every 1 yuan bet on average, ensuring consistent profits for casinos. In live-streaming sales games, merchants can implement bidding mechanics, such as “red envelope auctions,” allowing users to win high-value products at low prices. By leveraging mathematical expectation, merchants can calculate their average profits and fine-tune auction rules to optimize revenue. Unlike the clearly visible probabilities in traditional games like roulette, live-streaming sales games intentionally obscure the jackpot odds. This psychological design makes it harder for users to recognize the low probability of winning, ultimately boosting purchase conversion rates. Over time, the system reinforces itself. The game mechanics often engage users with small rewards, creating the illusion that a bigger win is just around the corner. This “next time will be better” mindset encourages repeated participation, leading to higher repurchase rates.

3. Analysis of mathematical expectation in business models under the background of traditional and live e-commerce

3.1. Mathematical expectation in traditional business models

The application of mathematical expectation in business models is crucial. It can help businesses improve profitability while controlling costs [2]. In the context of live e-commerce, mathematical expectation is used to analyze the benefits and risks of different marketing strategies and optimize user engagement. In traditional business models, sales mainly rely on fixed pricing, discount promotions, and full-reduction activities. The core of these models lies in price sensitivity and rational consumer decision-making. Merchants hope to attract consumers through promotions while ensuring overall profitability. The limitation of traditional business models is that traditional discount strategies cannot bring long-term user stickiness. For example, discount strategies are overly dependent on sales growth, but cannot stimulate users' buying impulse and maximize the value of a single transaction. Consumers pay more attention to the product itself rather than the purchase experience. This has prompted merchants to explore the live streaming game model to enhance user interaction and improve conversion rates.

3.2. Mathematical Expectation Analysis of the Live-streaming Game Model

Compared with traditional promotional methods, live-streaming games use probability mechanisms to stimulate consumers' sense of participation and consumption impulse, allowing merchants to increase sales while ensuring long-term profitability [7]. Mathematical expectation, as the core concept of probability theory, provides theoretical support for the profit model of such games.

In the design process of live-streaming games, merchants need to ensure that the mathematical expectation of the game is less than the participation fee paid by users to ensure long-term profitability. At the same time, in order to enhance the user experience, the game rules need to balance the probability of winning, the value of the prizes, and the psychological expectations of users, so that consumers are willing to participate multiple times and form continuous consumption habits. This section will explore the profit model of live-streaming games based on the theory of mathematical expectation, analyze its key parameter settings, and propose optimization strategies to ensure the sustainability of the game.

The core mechanism of live-streaming sales games involves setting game rules where users can participate in a lottery or game after purchasing products of a certain value. The rewards are distributed based on probabilities, with different prize tiers assigned varying chances of winning to ensure a positive expected profit for the merchant. The psychological element comes into play by obscuring the probability of winning the jackpot, tapping into users' optimism and motivating them to keep participating.

For Example, If assume the average order value is A

The cost price of the prize is $y_1, y_2, y_3, \dots, y_n$

$$p_i = P\{Y = y_i\} \quad (13)$$

$$E[Y] = \sum_{i=1}^n (A - y_i)p_i \quad (14)$$

When $E[Y] > 0$ 时, the merchant will make a profit in the long run. The larger the value, the more profit the merchant will earn. When the number of participations N, the merchant's profit is $N E[Y]$

The most common examples of this type of game are the wheel and the lottery. However, unlike the wheel, live-streaming shopping games often obscure the fact that the chances of winning the grand prize are very low. This makes it harder for customers to immediately recognize the small probability of winning, unlike in the wheel game where the area occupied by the grand prize is clearly visible.

As a result, customers' rational judgment is diminished, and their feelings of luck and greed are amplified, making them more likely to participate compared to traditional wheel games.

4. Case Study---The "Pair Matching" live-streaming game

The "Pair Matching" live-streaming game is based on dynamic deck management and a chain reaction mechanism, with the following rules:

The deck consists of 9 types of cards (marked as 1-9, with 1 representing a special card A chosen by the player). There are 7 cards of each type, totaling 63 cards.

In the initial stage, 9 cards are drawn randomly from the deck without replacement to form the starting hand.

The core process involves removing pairs: If at least one pair (two identical cards) is found in the hand, a pair is randomly selected for removal, and one new card is drawn from the remaining deck.

There is a special rule for card replacement: if the drawn card is A (type 1), an additional card is drawn (as long as the remaining deck is not empty).

The game continues until no pairs remain in the hand.

The statistical goal is to record the total number of pairs removed, with a maximum of 12 pairs (pairs greater than or equal to 12 are categorized as "12+"). Table 1 presents the Monte Carlo simulation results of 100,000 experiments, illustrating the probability of winning.

Table 1: Monte Carlo simulation of 100,000 experiments showing the probability of winning

Number of Pairs	Probability	Labelled Value (yuan)
0	0.15%	999
1	2.80%	299
2	7.50%	128
3	18.20%	39
5	33.60%	15
8	16.30%	49
9	11.00%	69
10	6.10%	120
11	3.20%	199
12+	1.80%	369

Computational mathematical expectations:

$$\begin{aligned}
 E[X] = & 0.15\% \times (39 - 999/3) + 2.8\% \times (39 - 299/3) + 7.5\% \times (39 - 128/3) \\
 & + 18.2\% \times (39 - 39/3) + 33.6\% \times (39 - 15/3) + 16.3\% \times (39 - 49/3) \\
 & + 11.0\% \times (39 - 69/3) + 6.1\% \times (39 - 120/3) + 3.2\% \times (39 - 199/3) \\
 & + 1.8\% \times (39 - 369/3) \approx 16.7
 \end{aligned}$$

With a reference monthly order volume of over 3,000, the merchant's expected revenue is approximately $3,000 \times 16.7 = 50,100$. This estimate pertains to a livestream room with only around 2,000 followers, underscoring the substantial profit potential that can be achieved through live-streaming sales games. By applying the appropriate formula, the expected value ($E(X)$) can be calculated. If $E(X) > 0$, the merchant stands to generate a profit, with this value representing the average return per transaction for the customer. As the number of transactions increases, total revenue is equal to the number of transactions multiplied by $E(X)$.

Live-streaming sales games have the potential to significantly enhance merchant profitability. However, empirical analysis indicates that for such outcomes to be realized, three critical conditions

must be met: a sufficiently low price point for the product, a grand prize with sufficient appeal, and the strategic obfuscation of the low probability of winning the grand prize. The low price serves as an incentive for customers to take risks, while the allure of the grand prize attracts customer engagement. Furthermore, the application of layered game mechanics (e.g., additional play features) helps obscure the inherent low probability of winning, thus enhancing the psychological engagement of participants.

5. Conclusion

This study explores the application of mathematical expectation in optimizing the profit model of live e-commerce games, and analyzes in detail how this concept can be used to increase consumer engagement and profitability. By applying mathematical expectation, merchants on online platforms can design games that balance prize probability, product pricing, and consumer psychology. This paper also shows through a case study of the "matching" live game that a reasonable game structure can bring considerable profits to merchants. This phenomenon illustrates the effectiveness of incorporating probability mechanisms into online live sales. That is, when mathematical expectation is properly applied, merchants can increase consumer engagement through risk management strategies.

However, this study has certain limitations. This paper only focuses on a single live game model and may not be representative of the entire live e-commerce strategy. Moreover, there are differences in consumer behavior across platforms and product categories, which means that the research results may not be universally applicable. For future research, it would be helpful to expand the scope of the case study to cover more types of live games in different industries and explore how different game mechanisms and prize distributions affect user engagement and profitability. Also, one promising avenue for future development is to combine artificial intelligence (AI) with mathematical expectations. By providing AI with expected mathematical outcomes, product costs, and target customer pricing, AI can generate new probability-based games optimized for specific business goals.

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