

Application of Queueing Theory in Intelligent Ticketing and Inspection System of Subway Stations

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Abstract: Today, with the acceleration of the intelligent process of rail transit, how to optimize the equipment configuration has become a pain point in the industry. This study focuses on the optimization of terminal equipment layout in the automatic ticketing and checking system of subway stations. Taking Baiguoyuan Station of Xuzhou Metro Line 2 as a typical case, the research team broke through the traditional thinking pattern of equipment configuration and creatively constructed a dual-dimensional analysis framework. The most breakthrough in the research process is the innovative strategy of parallel M/M/S and M1/M2/c dual models. These two queueing theory models are like precise calculation scales, which take into account both the equipment input cost of the station operator and the time value cost of the passengers. Through the dynamic adjustment of the model parameters, the golden balance point that minimizes the total cost of society is finally found. The practical implications of this study go far beyond the data. For operation managers, the "time-segment elastic configuration scheme" and "forward-looking expansion guide" proposed by the research make equipment configuration leap from empirical decision to scientific decision.

Keywords: Queueing theory, Rail transit, Poisson distribution, Automatic ticketing and checking system

1. Introduction

Queueing Theory, or random service system theory, is to obtain statistical rules of these quantitative indicators (waiting time, queue length, busy period, etc.) through statistical research on the arrival and service time of service objects [1]. It then improves the structure of the service system or reorganize the service objects according to these laws. Therefore, the service system can not only meet the needs of service objects, but also meet the needs of service objects. It can also make the cost of the institution the most economical or some indicators the best [2,3].

Common ticketing methods in subway stations include ticket booth manual ticketing, automatic ticket machine (TVM) self-service ticketing, and mobile payment due to the development of the internet [4]. At present, most metro stations in cities have gradually replaced manual ticketing with self-service ticketing and mobile payment method can not only reduce the work intensity and pressure of ticket kiosks, but also help passengers save time and improve efficiency [5]. Therefore, in the subway station, the queueing theory involved in the configuration of the ticket machine is particularly important for the efficiency of the whole ticketing system.

At present, there are many literatures on queuing theory. Jiang et al have found that the configuration of subway automatic ticket machines poses a challenge [4]. If the number of machines is too low, it can easily become a bottleneck that restricts the overall capacity of the station. Conversely, if the number is too high, it will lead to increased additional investment. To address this issue, they have designed a simulation optimization experiment. By employing this method, the number of automatic ticket machines can be adjusted in real time to accommodate sudden surges in passenger flow and other unexpected situations [6]. Based on the current research situation at home and abroad, people analyzed the influencing factors of the layout and configuration number of automatic ticketing equipment, and then applied theories and methods to establish the evaluation model of the layout and configuration number of automatic ticketing equipment [7]. These include the description of its queuing system characteristics and the hypothesis of the model. Finally, the rationality of the layout and quantity of automatic ticketing equipment in Beijing South Railway Station is evaluated by simulation.

The paper is divided into five parts. The first part is introduction of the queuing theory in subway ticket vending machines. The second part is an analysis of the practical case and method of M/M/S model in Baiguoyuan station of Xuzhou metro line 2. The third part is an analysis and application of M/M/C/ ∞ model in simulating the entry and exit of passengers at real metro stations. The fourth part is an application study on configuration quantity of M/M/C model in the automatic ticket vending machines of the subway. The fifth part is conclusion which contain results and future prospects.

2. M/M/S model on Automatic Ticket Checking Machines

Taking Baiguoyuan Station of Xuzhou Metro Line 2 as the research object, the optimization problem of the number of automatic ticketing machines based on queuing theory is discussed [8]. The aim of this study is to improve subway operation efficiency, reduce construction and maintenance costs, and reduce passengers' waiting time by optimizing the number of automatic ticketing machines. Firstly, by means of video acquisition and manual observation, the data of passenger arrival and ticket check at Baiguoyuan Station during the morning rush hour (7:00-8:00) were collected. Through the analysis of the data, it is verified that the passenger arrival process conforms to the Poisson distribution and the service time conforms to the negative exponential distribution. On this basis, the M/M/S queuing model and cost model are established to solve the optimal number of automatic ticket checking machines.

2.1. Research Method

This paper shall start with the construction of queuing model called M/M/S model, which assumes that passenger arrivals follow a Poisson distribution and service times follow a negative exponential distribution. The Poisson distribution is given by

$$p(k) = \frac{(\lambda t)^k e^{-\lambda t}}{k!} \quad (1)$$

According to Eq. (1), Probability that the system has no passengers p_0 is calculated as

$$p_0 = \left[\sum_{k=0}^{s-1} \frac{1}{k!} \left(\frac{\lambda}{\mu} \right)^k + \frac{1}{s!} \cdot \frac{1}{1 - p_s} \cdot \left(\frac{\lambda}{\mu} \right)^s \right]^{-1} \quad (2)$$

Likewise, the Average queue length of passengers and Average length of passengers are $L_q = \frac{p_0 \rho^s \rho_s}{s!(1-\rho_s)^2}$ and $L_s = L_q + \frac{\lambda}{\mu}$, respectively. The Average waiting time and stay time $W_q = \frac{L_q}{\lambda}$, $W_s = \frac{L_s}{\lambda}$.

Data collection is conducted through video collection and manual observation, inbound passenger flow data during morning peak hours (7:00-8:00) on working days were collected. The statistical results show that the inbound passenger flow is 572 people /h, the average arrival rate $\lambda=0.16$ people /s, the average service rate $\mu = 0.25$ people/s. The X2 test verified that the passenger arrival rule was consistent with Poisson distribution and the test is offered by

$$x^2 = \sum_{i=1}^g \left(\frac{f_i}{n} - p_i \right)^2 \cdot \frac{n}{p_i} = \sum_{i=1}^g \frac{f_i}{F_i} - n. \quad (3)$$

2.2. Specific Application

The paper takes Baiguoyuan Station of Xuzhou Metro Line 2 as a practical case and analyzes the configuration of automatic ticket checking machine in the station. Through field observation and data analysis, the passenger arrival rate ($\lambda = 0.16$ persons/second) and service rate ($\mu = 0.25$ persons/second) are obtained. According to the calculation result of M/M/S model, the current number of automatic tickets checking machines in Baiguoyuan Station is 5, but the optimization calculation shows that the optimal number of automatic tickets checking machines is 2. As shown in Table 1, when $S = 2$, the combined cost incurred by subway operators and passengers is the least; when $S = 5,6$, the combined costs are basically the same.

Table 1: Calculation results of different service model parameters

S	ρ_s	P0	Lq	Ls	Wq	Ws	time
1	0.6400	0.3600	1.1400	1.7800	7.125	11.1250	0.7319
2	0.3200	0.5151	0.0730	0.7130	0.4563	4.4560	0.5598
3	0.2100	0.5262	0.0070	0.6470	0.0438	4.0440	0.7055
4	0.6100	0.5272	0.0010	0.6410	0.0063	4.0500	0.8703
5	0.1300	0.5273	0.0000	0.6400	0.0000	4.0000	1.0367
6	0.1100	0.5273	0.0000	0.6400	0.0000	4.0000	1.2034

To account for potential operational disruptions, such as adverse weather or equipment malfunctions, it is recommended to keep one machine offline permanently and operate 3-4 machines daily to meet passenger demand.

Through optimization calculation, it is found that the number of automatic ticket checking machines in Baiguoyuan Station is redundant. It is suggested to turn off one automatic ticket checking machine and turn on 3-4 machines daily to meet the needs of passengers. According to this optimization plan, Baiguoyuan Station can save 62,050 to 124,100 yuan in operating costs per year. The research verifies the validity and practicability of the model, and provides a theoretical basis for the configuration of automatic ticket checking machine in Xuzhou subway line in the future.

3. M/M/C/ ∞ model on Automatic Fare Collection System

The subway AFC terminal is equipped with automatic ticket vending machine (TVM), semi-automatic ticket vending machine (BOM), automatic ticket checking machine (GATE), and so on [9]. In the design of subway AFC system, the number and location of them are very important. Not only to ensure the volume of transport, but also to ensure the quality of service. The main contents of rail transit service quality are convenience, swiftness, comfort and economy. In addition to meeting the comfort of passengers, it also reduces the waiting time for passengers to enter and exit the station.

3.1. Conventional Methods

At present, the commonly used calculation method is based on the passenger flow in the morning and evening peak hours and the capacity of equipment to calculate. The following takes a typical station as an example to calculate the number of inbound and outbound ticket checking machines of AFC terminal equipment.

As shown in Table 2, the number of people getting on and off at a station in both directions during morning and evening peak hours is multiplied by the peak hour coefficient K .

Table 2: Statistics of morning and evening peak passenger flow of a typical station

Peak hour	People getting on(down)	People getting off(down)	People getting on(up)	People getting off(up)	People getting on(both)	People getting off(both)
Morning peak hour	1502	3581	3132	1244	4634	4825
Evening peak hour	1175	3015	3280	1077	4455	4092

Specifically, $K = 1.2$ can be used to obtain the passenger flow, as shown in Table 3.

Table 3: Morning and evening peak passenger flow table of a typical station multiplied by the peak hour coefficient.

Peak hour	People getting on	People getting off
Morning peak hour	5561	5790
Evening peak hour	5346	4911

In the morning and evening peak hour passenger flow, take the larger value of the number of people getting on and getting off, and evenly distribute it to the entrances and exits of the station hall in the four directions. If the formula is given by

$$N = 5561 \times 50\% \times A/t/B, \quad (4)$$

then the result is $N = 3$. However, the former also considers the train interval, mainly the outbound passenger flow is time-sharing centralized arrival, in order to meet the passengers in a certain time to be able to smoothly exit the station, and not to the next train arriving passengers and the passengers of this train congestion, the subway safety impact. The number of outbound ticketing machines is calculated as follows

$$N = 5790 \times 50\% \times A/t/B, \quad (5)$$

then the result is $N = 4$, by multiplying both the inbound and outbound ticketing machines calculated before by 2. It can be concluded that the number of inbound ticketing machines configured at the station is 6 channels and the number of outbound ticketing machines is 8 channels.

Queuing Theory, also known as stochastic service system theory, is a mathematical discipline studying the congestion phenomenon. It solves the optimal design and optimal control of various service systems by studying the probabilistic characteristics of queuing. It can be divided into three parts: input process, queuing rules and service organization. Common input processes include fixed-length input, Poisson input, and Erlan input.

3.2. Queuing theory Method

The basic model of queuing theory, commonly used models are represented: $M/M/1/\infty$, $M/M/C/\infty$, $M/G/1/\infty$ and so on. Where M represents the negative exponential distribution of passenger arrival time, C represents the number of service desks, and G represents the general distribution of service time. The random event flow can be seen as an approximate Poisson flow, and the queuing model is $M/M/C/\infty$, because the random event flow approximately follows the following three characteristics which are the stability, the no after-effect, and the commonness. Its probability distribution characteristics are as follows

$$P(k) = \frac{(\lambda t)^k e^{-\lambda t}}{k!}, k = 0, 1, 2, \dots \quad (6)$$

Under the condition of statistical equilibrium, one can get an average length is

$$\bar{N} = \sum_{k=0}^{\infty} k p_k = \frac{\rho}{1-\rho}, \rho < 1. \quad (7)$$

The average waiting time is: $\bar{W}_q = \frac{\rho}{\mu(1-\rho)}, \rho < 1$ and the average length of stay is: $\bar{W} = \frac{1}{\mu-\lambda}, \rho < 1$. It is calculated that when the number of tickets checking machine channels is 5, its related service quality index is obviously better than that when the number of channels is 4, but when the number of tickets checking machine channels is 6, its layout is more wasteful. The passenger flow data calculated by the conventional method is 6 channels, which is 1 channel more than the queuing theory method.

4. M/M/C Model on Ticket Vending Machines

Driven by the wave of intelligence, 93% of the domestic subway stations have completed the upgrade of the self-service ticket system [10]. This change has brought the problem of optimizing the configuration of ticket vending machines (TVM) to the forefront of research: how to find a balance between operational efficiency and cost control? When people are stuck in a long queue during the morning and evening rush hour, one may be witnessing a concrete interpretation of a classic queuing theory model in reality.

The first is about the dynamic balance model of ticketing system. Through three months of field observation, it is found that the subway ticket system presents typical Poisson distribution characteristics. Taking Chengdu Tianfu Square Station as an example, each ticket unit can be regarded as an independent service node, and its operation mode perfectly fits the $M/M/c$ queuing model. It should be noted that the c value in the model represents the number of parallel service terminals, and when $n \geq c$, the system enters the full load state - just like the subway station on holidays, where there is a long queue in front of all TVMS, at which time the system throughput reaches the theoretical peak of $c\mu$.

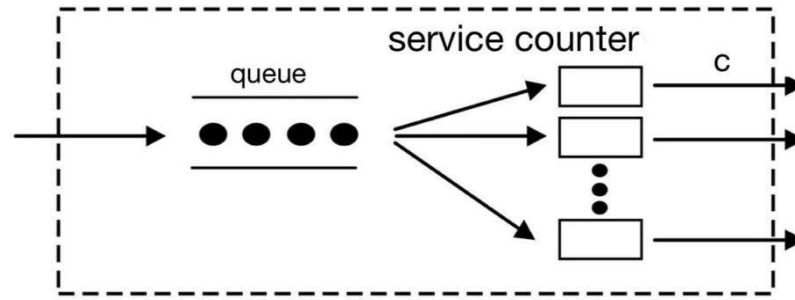


Figure 1: Single team -multi service desk model [10].

The second is about the dynamic analysis of system state transition. In the multi-service desk single-queue model shown in Figure 1, the state transition rate presents a stepped change feature. When the number of service terminals is increased from 2 to 14 (take Chengdu Line 1 Station A as an example), the system processing capacity is not linear growth. It is like adding tolls on a motorway: when there are fewer open gates than waiting traffic, each new gate increases efficiency significantly; But beyond the critical value, the marginal benefit will decline sharply.

Based on the modeling analysis of passenger flow data in 2018, it is found that the number of passengers per hour during peak hours from 18:00 to 19:00 is 6843, which is 3.2 times of that during normal peak hours. It is worth noting that when the TVM service intensity ρ value approaches 0.9 (critical value 0.89), the average waiting time of passengers shows a step increase. This alerts people to the need to reserve 20 for device configuration.

5. Conclusion

This study comparatively analyzes conventional and queuing theory-based methodologies for determining optimal AFC terminal configurations in metro systems. While conventional approaches relying on peak-hour traffic and average service time offer computational simplicity, they provide limited decision-support metrics. In contrast, queuing theory models requiring comprehensive statistical data (including arrival/service time distributions) enable precise calculation of critical indicators like queue length and service duration, proving particularly valuable for spatial arrangement optimization of TVMs and gates. Through empirical analysis at Baiguoyuan Station, the queuing optimization model achieved 62,050-124,100 RMB annual cost savings by strategically reducing operational gates from 4 to 3-4 during off-peak periods, demonstrating significant practical applicability. The validated model establishes a theoretical framework for future AFC planning in Xuzhou's upcoming Lines 4 and 6.

However, limitations warrant future exploration. The study's constrained time sampling and simplified assumptions (neglecting TVM ticket-dispensing efficiency and layout factors) highlight the need for extended data collection periods incorporating multi-dimensional variables. Furthermore, the current static model requires adaptation for long-term elasticity to accommodate passenger growth patterns. Future research directions should: 1) Develop dynamic queuing models integrating real-time passenger behavior analytics 2) Incorporate machine learning for predictive distribution pattern recognition 3) Establish multi-objective optimization frameworks balancing capital investment, maintenance costs, and passenger experience metrics. Such enhancements would enable responsive AFC configurations adaptable to smart city evolution, particularly as contactless payment technologies reshape traditional fare collection paradigms. This methodological evolution from deterministic to adaptive modeling promises to redefine metro operational efficiency standards.

Authors Contribution

All the authors contributed equally and the names were listed in alphabetical order.

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