

Research on crack development in rock with prefabricated holes based on various constitutive models

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Abstract. Under external loading, progressive damage caused by internal defects in rocks is the primary reason for rock failure. This paper establishes a numerical model for rock specimens containing prefabricated holes, compares the crack development patterns of rock specimens with different constitutive relationships, and analyzes the differences in crack development between the Von Mises and Drucker-Prager constitutive relations, comparing these with actual results. The research results show that: (1) Both constitutive models exhibit stress concentration and failure at the tips of pre-set cracks. (2) Cracks in both models first extend perpendicularly from the pre-set cracks and then vertically, with symmetrical cracks at both ends. (3) Both models develop vertical cracks through the sample from the tips of the pre-set cracks, but the vertical cracks differ between the models. (4) The vertical crack at the upper end of the Von Mises model is closer to the specimen's failure, while the vertical cracks at the lower end of both models are less aligned with the specimen's failure.

Keywords: Constitutive Relationships, Rock, Crack Development.

1. Introduction

With advancements in technology, the use of underground space has greatly increased, leading to a significant rise in the demand for tunnel excavation. Tunnel excavation causes a redistribution of surrounding rock stress, changing the rock stress state from a tri-axial distribution to bi-axial or uni-axial stress distributions, exceeding the rock's ultimate load capacity. Consequently, external loads induce minor internal defects in rocks, which progressively develop into cracks, leading to rock mass failure. Thus, analyzing the direction of crack propagation and the formation of cracks under external loads helps explore the modes of rock failure, thereby preventing or reducing engineering losses caused by rock damage.

In recent years, many studies have used numerical methods to analyze rock failure issues. Wang Shimin et al. [1] conducted numerical simulations of rocks with prefabricated cracks to analyze the impact of crack spacing on rock failure. Zhou Li et al. [2] performed numerical simulations on uniaxial compressed rocks to investigate the effects of crack inclination and aperture on rock mass. Ye Qiongyao et al. [3] analyzed rock crack propagation patterns using the discrete element program UDEC. Fan Xingyu et al. [4] examined the influence of rock joints on crack formation using the discrete element particle flow program PFC. Sun Xiaolei [5] investigated failure modes and crack formation in rocks with different cavities using PFC. Guo et al. [6] explored the effect of cutting rate on rock masses

through numerical simulations. Wang et al. [7] studied rock failure under load using a unit deletion algorithm.

This paper establishes a rock model with prefabricated holes and defines its constitutive relationships. Numerical simulations of cracks are conducted to compare the differences in rock cracks between Von Mises and Drucker-Prager constitutive models, and the results are compared with field test specimens to explore the distinctions in simulating rock crack patterns between different constitutive relationships.

2. Constitutive Relationships

2.1. Von Mises Criterion

The Von Mises criterion assumes that a material's plastic behavior is determined by equivalent stress and stress state criteria, which accurately describes the material's mechanical behavior under multi-axial stress states. Equivalent stress is derived from the stress tensor, and stress state criteria are derived from external stress, which can be expressed as follows:

$$J_2 = k_f^2 \quad (1)$$

Or:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 6k_f^2 = 2\sigma_s^2 \quad (2)$$

Here, k_f is a test constant with $k_f = \frac{\sigma_1 - \sigma_3}{2}$, J_2 is the second invariant of the deviatoric stress, $J_2 = \frac{1}{6}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$, and σ_s represents the material's yield point.

Here, σ_1, σ_2 , and σ_3 represent the first, second, and third principal stresses of the element, respectively ($\sigma_1 \geq \sigma_2 \geq \sigma_3$, where tensile stress is positive and compressive stress is negative).

The Von Mises criterion describes the stress state under certain failure conditions. It reflects the degree of shear stress experienced by rock elements under complex stress states. Equation (1) represents the geometric depiction of Von Mises yield in the principal stress three-dimensional yield space as a cylindrical surface with the space diagonal as the axis; yielding occurs when the stress reaches this cylindrical surface and continues to move along it.

2.2. Drucker-Prager Criterion

The Drucker-Prager criterion is an extension of the Mohr-Coulomb and Von Mises criteria. The Drucker-Prager criterion considers the effect of the intermediate principal stress and hydrostatic pressure, making it theoretically more reasonable. It is widely used in geotechnical engineering and can be expressed as:

$$\beta I_1(\sigma) + \sqrt{J_2(\sigma)} = k_f \quad (3)$$

Here, $\sigma = (\sigma_{xx} \ \sigma_{yy} \ \sigma_{zz} \ \sigma_{xy} \ \sigma_{xz} \ \sigma_{yz})^T$ represents the stress vector of the rock.

β and k_f are constants related to the physical properties of the rock material, where $\beta = \frac{\sqrt{3}\sin\varphi}{3\sqrt{3+\sin^2\varphi}}$ and $k_f = \frac{\sqrt{3}C\cos\varphi}{\sqrt{3+\sin^2\varphi}}$; where C and φ are the rock's cohesion and internal friction angle, I_1 is the first stress invariant, and J_2 is the second invariant of the deviatoric stress, given by

$$\begin{cases} I_1(\sigma) = \sigma_{xx} + \sigma_{yy} + \sigma_{zz} \\ J_2(\sigma) = \frac{1}{6}[(\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{yy} - \sigma_{zz})^2 + (\sigma_{zz} - \sigma_{xx})^2] + \sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{xz}^2 \end{cases} \quad (4)$$

Since this paper discusses the stress distribution of surrounding rock under axisymmetric conditions, the stress invariants I_1 and J_2 are

$$\begin{cases} I_1(\sigma) = \sigma_1 + \sigma_2 + \sigma_3 \\ J_2(\sigma) = \frac{1}{6}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] \end{cases} \quad (5)$$

Equation (3) represents the geometric representation of Drucker–Prager yield criterion as a positive cone in the stress space of the principal stress three-dimensional yield surface. When $\beta=0$, meaning hydrostatic pressure is not considered, Equation (3) reduces to Equation (1), which is the von Mises criterion.

3. Numerical Simulation of Rock Failure Under Uniaxial Compression

3.1. Model Setup

An isotropic model with dimensions $L \times W \times H = 60\text{mm} \times 20\text{mm} \times 120\text{mm}$ was established in ABAQUS. A hole with a diameter of 10mm was predrilled in the center, and two inclined cracks, each 12mm long, 2.5mm wide, and at a 45° angle, were predrilled on either side of the hole, as shown in Figure 1. The bottom of the model is fixed, and a displacement-controlled load is applied to the top surface for compression (see Figure 1, Table 1). The loading rate is 0.5mm/step, with a maximum displacement of 15mm at the top surface. The model is discretized into tetrahedral elements (see Figure 2).

Table 1. Rock Mechanics Parameters for the Model

Material Properties	Young's Modulus (MPa)	Poisson's Ratio ν	Density (t/mm^3)
Rock Matrix	2630	0.33	2.6e-09

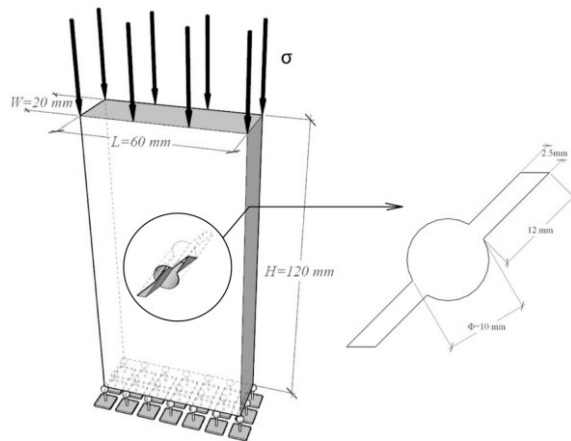


Figure 1. Computational Mode

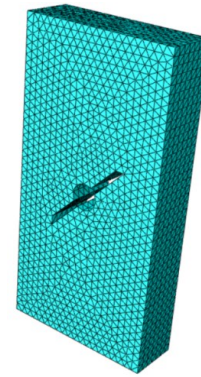


Figure 2. Mesh Model

3.2. Principle of Element Deletion Method

An element failure criterion is set; when an element meets this criterion, it is considered failed and deleted. The advantage of this method is that it allows for the selection of any element in the model and is suitable for any calculation scenario. The algorithm is straightforward and follows the principles of strict progressive structural optimization methods. However, its accuracy depends on mesh size, and deleted elements cannot be recovered, making it unsuitable for bidirectional and reverse progressive structural optimization methods.

Element damage failure focuses on describing the effect of damage on material stiffness. ABAQUS controls element deletion based on the value of state variables, meaning that an element fails if its plastic strain multiplied by the element's characteristic length exceeds a specified value. The VUMAT and UMAT methods are the most flexible, allowing for the definition of custom failure criteria and constitutive equations, and a state variable can be defined in the subroutine to represent material failure. This study employs the UMAT subroutine for analyzing the basic steps in ABAQUS and assessing whether the elements fail.

4. Results Analysis

4.1. Von Mises Criterion

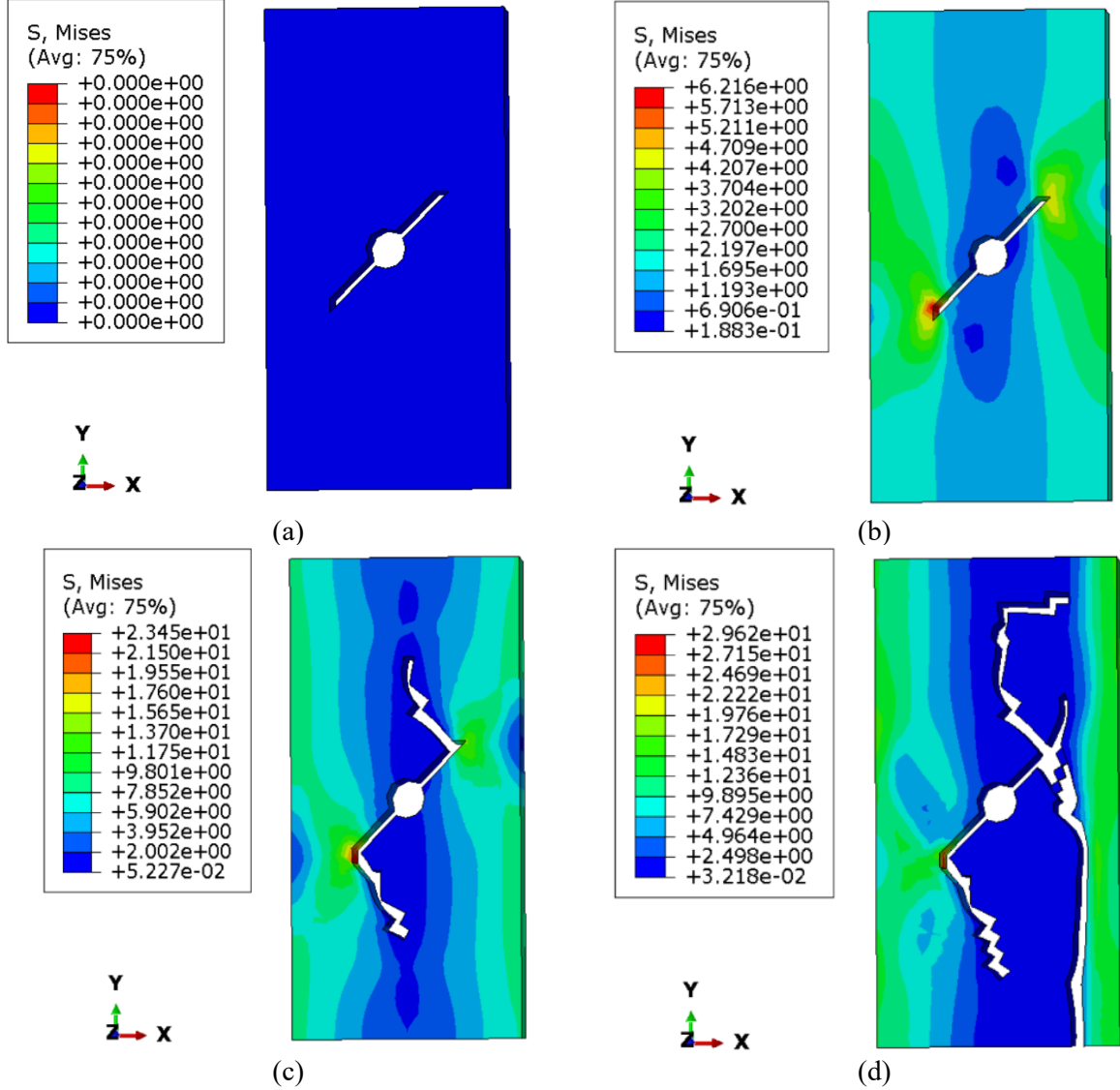


Figure 3. Rock Failure Process under Von Mises Constitutive Model (Units: MPa)

As shown in Figure 3, under vertical loading, the Von Mises model exhibits stress concentration at the crack tip. When the stress at the tip reaches the Von Mises yield condition, the element fails, leading to crack propagation. Initially, cracks perpendicular to the preset crack direction extend from both ends of the preset crack. Subsequently, the cracks propagate vertically, with the upper and lower cracks being roughly center-symmetric. Finally, the upper crack extends horizontally, while the lower right crack vertically penetrates downward.

4.2. Drucker-Prager Criterion

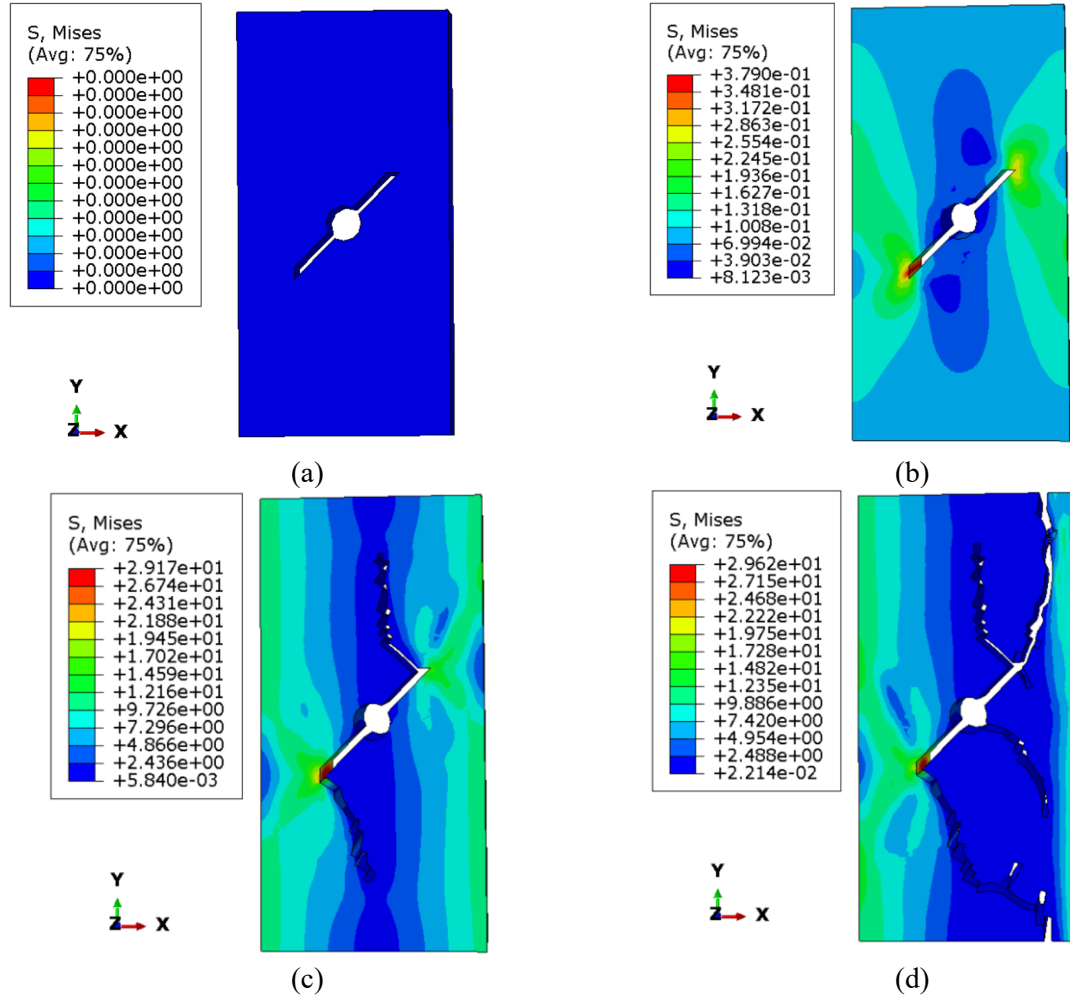


Figure 4. Rock Failure Process under Drucker-Prager Constitutive Model (Units: MPa)

As shown in Figure 4, initially, the crack propagation under the Drucker-Prager constitutive model is similar to that under the Von Mises model. Both first develop cracks perpendicular to the preset crack direction from both ends and extend vertically, with the upper and lower cracks being roughly center-symmetric. Subsequently, the Drucker-Prager model develops a through crack at the upper end of the preset crack and a diagonal crack extending to the lower right from the middle of the preset crack, while the diagonal crack from the lower part of the preset crack penetrates the model.

5. Summary

(1) Both constitutive models exhibit stress concentration and failure at the tip of the preset crack during the early loading phase. In the mid-loading phase, cracks first extend perpendicular to the preset crack direction and then vertically, with upper and lower cracks being roughly center-symmetric. In the late loading phase, diagonal cracks extending through the model originate from the tip of the preset crack, with upper and lower cracks no longer being center-symmetric.

(2) The main difference between the two constitutive models lies in the location of vertical cracks and their penetration through the model in the later loading phase. The Von Mises model develops vertical cracks extending downward from the upper end of the preset crack, while the Drucker-Prager model develops vertical cracks extending upward from the upper end of the preset crack and diagonal cracks extending from the lower end that penetrate the model.

(3) Compared with experimental failure diagrams, the Von Mises model better simulates the development of cracks 1a, 1b, and 3, while the Drucker-Prager model is better at simulating cracks 1a and 1b. The discrepancies in simulating cracks 2b and 4 may be due to the numerical model's insufficient accuracy or surface rock falling off near the vertex of the preset crack.

(4) The upper vertical crack in the Von Mises model is closer to the sample's actual failure, while the lower vertical cracks in both constitutive models are less accurate compared to the sample's failure.



Figure 5. (1) Mises Constitutive Model Failure Diagram

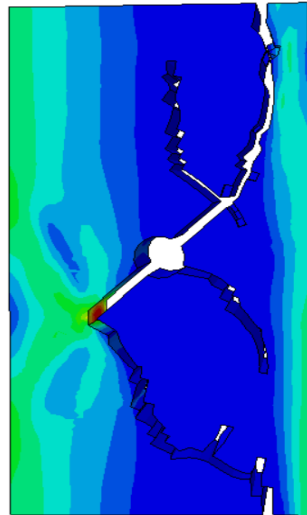


Figure 5. (2) Drucker-Prager Constitutive Model Failure Diagram

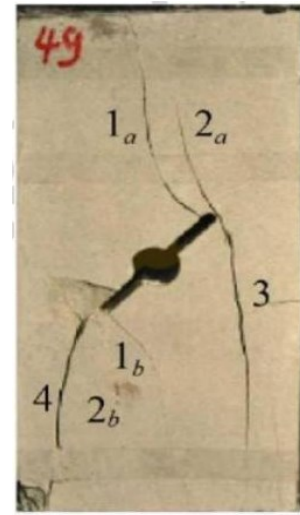


Figure 5. (3) Sample Failure Diagram

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