

The AdS/CFT duality in Sachdev-Ye-Kitaev model: Clues from dynamics and thermal entropy

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Abstract. With the in-depth subjects chiasma among different research areas, many people are applying concepts and methods in theoretical high-energy physics to condensed matter systems. One of the most intensely studied tools is the Anti-de Sitter space/Conformal Field Theory (AdS/CFT) correspondence. It effectively permits a perturbative treatment of strongly coupled systems. Thus, it has been a focused topic in theoretical physics and high-energy physics. It represents a significant breakthrough in understanding quantum gravity. This paper focuses on the realisation of the duality in Jackiw-Teitelboim (JT) gravity and Sachdev-Ye-Kitaev (SYK) Model. To this end, the author derived the action, free energy, thermal entropy, and low-energy dynamics. The symmetry breaking pattern of the SYK model following previous works are also considered. These results were compared with the counterparts in Euclidean JT theory as a cut-out of the complete nearly AdS space in two dimensions. Based on the similarity in the mathematical form of these quantity and symmetry breaking process, the author concludes that the SYK model, as many believed, is a promising candidate for the holographic dual of the JT theory.

Keywords: AdS/CFT correspondence, Jackiw-Teitelboim gravity, Sachdev-Ye-Kitaev model, Quantum field theory.

1. Introduction

The holographic principle is recognized as a fundamental principle which provides insight to guide theoretical physics. It states that occurrences inside a closed surface can be represented by degrees of freedom on the surface itself. It originated from the fact that black hole entropy is proportional to its area instead of the volume. The Anti-de Sitter Space/Conformal Field Theory (AdS/CFT) correspondence is a concrete realisation of the holographic principle, particularly gauge/gravity duality. The original proposal related type IIB string theory on $\text{AdS}_5 \times S^5$ space to super-Yang-Mills theory with four supersymmetries and was later generalised. It demonstrates that a d -dimensional CFT can be described by gravity theory in the product of $(d + 1)$ -dimensional AdS space and a compact manifold.

The AdS/CFT correspondence, far from being a mere theoretical conjecture in high-energy physics, is a pivotal tool for comprehending and studying strongly coupled systems. This has led to numerous endeavors in identifying holographic pairs, where the CFT sides can be condensed matter systems, quark-gluon plasma, and fluid. For instance, in condensed matter, the near critical behavior of transport properties in coupled dimer antiferromagnets was studied holographically, demonstrating the profound

impact of the duality in this field [1]. Moreover, a plethora of problems in theoretical physics have been studied under the duality, further underscoring its significance.

While the original correspondence was postulated on $d = 4$ dimensions, the mathematics of such systems are highly complex. Firstly, higher dimensional AdS space typically has a more complicated geometry, not to mention the underlying compact manifold. Secondly, the dynamics on the CFT side are technically challenging to calculate due to the large number of degrees of freedom and strong coupling limit. For these reasons, it is hard to probe the basic principle and mechanism for the occurrence of the correspondence. Nevertheless, these problems can be resolved by studying the lower dimensional counterparts, typically $d = 3$ and $d = 2$.

There have been various studies of $\text{AdS}_2/\text{CFT}_1$ correspondence. The most prominent examples include the near horizon of the extremal black hole [2, 3], AdS_2 wormholes [4], and the black hole-like solution in type IIA supergravity [5]. Among these examples, the most studied one is the Sachdev-Ye-Kitaev (SYK) model proposed by Kitaev in 2015. The SYK model was proposed based on the Sachdev-Ye (SY) model, a Heisenberg model with infinitely long-ranged random interactions in fermion representation used to describe spin liquids. In his attempt to find the holographic duals of black holes, Kitaev observed that while a generic Heisenberg model with infinitely long-ranged random interactions does not produce black hole-like physics, a modified version of the SY model does. The SYK model, since its proposal, has remained a topic of intense study. Its generalisation to lattice has been applied to study non-Fermi liquid such as strange metal [6]. It is also the most studied model as candidates of the holographic dual to nearly AdS_2 theory. While there have been various attempts, there is no definitive evidence that the SYK model is a holographic dual of nearly AdS_2 gravity. This article, in continuation of research in this direction, invites further contributions to this exploration.

This paper is organised as follows. Section 2 presents a more technical introduction to AdS/CFT correspondences and study the nearly AdS_2 space. Section 2.2.1 provides a basic introduction to the JT theory, Section 2.2.2 derives the dynamics, particularly that of JT theory in the low energy region, and section 2.2.3 calculates its thermodynamics. Section 3 studies the SYK model by calculating its dynamics and thermodynamics and argue that it is a potential holographic dual of the JT theory based on the similarities appearing in the derivation.

2. AdS/CFT Correspondence and Nearly AdS_2 Space

2.1. AdS/CFT Correspondence

This section starts with an exhibition of the original proposal of the AdS/CFT conjecture following Ref. [7]. It also involves the recent progress, existing realisations, and applications.

In the background of N closely placed D3 branes lying in $(9 + 1)$ -dimensional Minkowski spacetime, there are two types of excitations in a type IIB string theory, i.e., closed strings as excitations of empty spaces and open strings describing D-brane excitations. In the string theory's low energy limit ($l_s \rightarrow 0$), only massless states can be excited, for which an effective action describes the interaction. In particular, the closed string in this limit is identified with type IIB supergravity. The open string, however, is identified with $n = 4$ $U(N)$ super-Yang-Mills theory. The total effective action is

$$S = S_{\text{bulk}} + S_{\text{brane}} + S_{\text{int}} \quad (1)$$

where the three terms describe the closed string (gravity), the open string containing the super-Yang-Mills theory, and the bulk-brane modes interactions. In this limit, the S_{int} and higher order derivative in S_{brane} vanishes, leaving only S_{bulk} and $U(N)$ gauge theory in S_{brane} . Thus, from the perspective of the open string field theory, the whole theory decouples to a free supergravity described by S_{bulk} and a CFT described by the leading order terms of S_{brane} .

In the low-energy limit of the supergravity, the two decoupled excitations (for observer at spatial infinity) are massless particles propagating in bulk with growing wavelength and particles confined to the near horizon, where the geometry is $\text{AdS}_5 \times S^5$. They correspond to free bulk supergravity and $\text{AdS}_5 \times S^5$ excitations. Since both perspectives contain the flat space supergravity, it is intuitive to

identify the other decoupled component in one description with the counterpart in the other. Therefore, $n = 4$ $U(N)$ super-Yang-Mills theory in $3 + 1$ dimension is the same as the type IIB superstring theory on $AdS_5 \times S^5$ space. It was generalised to the following situation. In the large N limit, the action of fields in $d + 1$ dimensional AdS space (in terms of boundary values) are of the form of generating functions of correlation functions of a d -dimensional CFT on the asymptotic boundary. A generic visualisation is depicted in Figure 1.

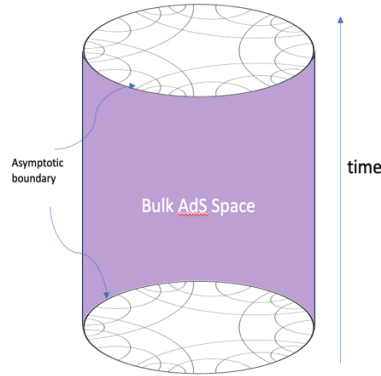


Figure 1. Visualisation of AdS_{d+1}/CFT_d correspondence where bulk AdS_{d+1} space is asymptotically flat and has negative scalar curvature.

Studies in AdS/CFT conjecture are distributed on various topics. On the theoretical side, there have been studies in integrability, quantum information, black hole physics, and quantum gravity [8]. On the practical side, the holographic computation of observables for known holographic systems has been intensely studied. Firstly, entanglement entropy can be calculated holographically [9]. Its holographic computations assist the research in quantum many-body systems and black holes and shed new light on the duality itself. Secondly, the shear viscosity of quark-gluon plasmas can be calculated via the black brane [10]. Lastly, superconducting phase transition was formulated as the holographic theory of a Maxwell field and a charged scalar field coupled to the AdS gravity [11].

2.2. Nearly AdS_2 Space and Jackiw-Teitelboim Theory

In this section, the author focuses on the description of the near horizon region of the extremal black hole based on the JT theory [12].

2.3. The Jackiw-Teitelboim theory

The quest of quantum gravity and quantum black hole physics started with the discovery of non-renormalisability of Einstein-Hilbert action (EH-action) in the $3 + 1$ dimension. One procedure to resolve the problem is inherited from Kaluza and Klein during 1910s: increasing the spacetime dimension. This leads to the string and M theories. Alternatively, one may consider a lower dimensional theory such that the gravity can be understood via a Euclidean path integral.

The JT theory arises from the second strategy, based on the modification to the simplest two-dimensional EH-action. the Lagrangian is the product of scalar curvature with square root of the doubly contracted metric. Since this simplest model is topological (i.e., EH-action is the Euler character of the manifold with vanishing Einstein tensor), it is a relatively static model and does not possess interesting dynamics. Thus, the easiest way of resolving this problem is to introduce a coupling with the scalar curvature, known as dilaton gravity. The most general dilaton gravity is defined by

$$I = -\frac{1}{16\pi G_N} \int_M d^2x \sqrt{g} [U_1(\tilde{\phi})R + U_2(\tilde{\phi})g_{\mu\nu}\partial_\mu\tilde{\phi} + U_3(\tilde{\phi})] \quad (2)$$

where $\tilde{\phi}$ is the dilaton field [13].

The JT theory has various applications and extensions. Some examples can be found in Ref. [13]. On one hand, it has been applied to study the phenomenon of entanglement, description of near horizon of extremal black holes, worm holes, EOW branes, two-dimensional cosmology, universe field theory, and quantum chaos. On other hand, it has been extended to supersymmetric theory and related to string theory. Most of these examples exploit the simplicity and solvability of the JT theory.

2.4. Action

The complete nearly AdS_2 spaces are described by JT theory, a solvable model which, when quantised, may describe quantum black hole. It can be obtained by redefinition of field, imposing a Weyl transformation, and take a linear dilaton potential to the generic model presented in Eq. (2). In terms of the warped product space action in Euclidean signature, it is arrived that

$$I = \frac{\phi_0}{16\pi G_N} \int d^2x \sqrt{-h} R_h + \frac{1}{16\pi G_N} \int d^2x \sqrt{-h} \phi (R_h + 2) \quad (3)$$

where $\sqrt{-h}$ is contracted metric, R_h is Ricci scalar, and ϕ is the scalar dilaton field. The spacetime metric is $ds^2 = -l_{AdS}^2(-dt^2 + dz^2)/z^2$ in Poincare patch and $ds^2 = d\rho^2 - \sinh^2(\rho) d\tau^2$ in Rindler patch with $l_{AdS} = 1$. The complete nearly AdS_2 space is described by combining the two charts for their corresponding patches, and it describes a near extremal black hole, such as Reissner–Nordström black hole.

To study the holographic field theory, where calculation is more easily done in Euclidean space, the author chose to study the Euclidean JT theory. Upon analytical continuation by Wick rotation, this space is described by a single metric

$$ds^2 = \frac{dt^2 + dz^2}{z^2} = d\rho^2 + \sinh^2(\rho) d\tau^2 \quad (4)$$

In the (ρ, τ) coordinates, $\tau \in [0, 2\pi)$ indicates that any holographic boundary must be a thermal field theory. Its existence is guaranteed by imposing a boundary term to Eq. (3): Dirichlet boundary condition described by the Gibbons-Hawking-York term. Thus,

$$I = -\frac{\phi_0}{16\pi G_N} \left[\int_M d^2x \sqrt{h} R_h + 2 \int_{\partial M} K \right] - \frac{1}{16\pi G_N} \left[\int_M d^2x \sqrt{h} \phi (R_h + 2) + 2 \int_{\partial M} \phi_b K \right] \quad (5)$$

Since any quantum field theory (QFT) must be convergent, where a critical renormalisation technique is UV cutoff, it is essential to understand the cut-out version of the Euclidean JT theory. The Einstein-Hilbert part (first part) of Eq. (5) is invariant for simply connected chunks under smooth deformations, corresponding to zero modes solution. Every cut-out can be obtained by fixing a Poincare coordinate and cutting it off along a boundary. With a boundary condition specified by ϵ , which plays the role of the boundary theory UV cutoff, the invariance of the first part is a reparametrisation invariance. It is lifted by the boundary term, manifested as the boundary term in Eq. (5). In particular, the cut-outs are invariant under translations and rotations, reducing the symmetry to $SL_2(\mathbb{R})$, which coincides with the conformal group of a fixed point CFT_1 . In particular, the zero modes are massless Euclidean Goldstone bosons associated with the symmetry breaking.

Since the dilaton field ϕ appears without derivative, and ϕ_0 is a constant, the variation of action is zero if and only if $R_h = -2$, signifying a constant scalar curvature AdS_2 space. Further reducing Eq. (3) to the Schwarzian action

$$I_{Sch} = -\frac{1}{8\pi G_N} \int_{\partial M} du \phi_r(u) Sch(t(u), u) \quad (6)$$

with solution $\tau(u) = 2\pi u/\beta$. Its periodicity shows that boundary theories are thermal field theories.

2.5. Thermal entropy

An important step toward a boundary theory is the calculation of thermal entropy based on $Z(\beta) \rightarrow \exp(-I_{gravity})$ in the limit of $G_N \rightarrow 0$. If a QFT or a quantum mechanical model has the thermal entropy of the same mathematical form, this theory may be a holographic dual. The solution to EL equation of eq.6 simplifies Schwarzian action to $I_{Sch} = -\pi\bar{\phi}_r/4\beta G_N$. However, it requires amendments in the entropy calculation, as the previously abandoned Einstein-Hilbert part contributes to it. Since JT theory describes extremal black holes whose low energy limit leaves the system with only the ground state described by CFT_1 , the holographic dictionary can be applied to relate the thermal entropy to number of states, which relates to the Einstein-Hilbert part with contribution S_0 [14], i.e.,

$$I_{gravity} = -S_0 - \frac{\pi\bar{\phi}_r}{4\beta G_N} \quad (7)$$

Hence, thermal entropy follows from derivative of the limiting partition function

$$S_{therm} = S_0 + \frac{\pi\bar{\phi}_r}{2\beta G_N} \quad (8)$$

3. Sachdev-Ye-Kitaev Model

Large N field theory is a large N limit QFT. It is motivated by the need of simplification in quantum chromodynamics due to its non-perturbative nature. Some examples are ϕ^4 theory and vector/matrix/tensor models. In general, a mathematically simple model does not contain sufficiently rich physics, while physically rich models are mathematically complex, typically unsolvable (e.g., tensor and matrix models) [15]. The SYK model may resolve such model.

3.1. Hamiltonian of SYK Model

The SYK model is a quantum mechanical model of an ensemble of finite-dimensional randomly interacting Majorana fermions. The original proposal models N Majorana fermions as ensemble of four-to-four interaction.

$$H = \sum_{i,j,k,l=1}^N J_{ijkl} \psi_i \psi_j \psi_k \psi_l \quad (9)$$

In this paper, the author rederives properties for its generalisation to q -to- q particle interaction, described by

$$H = i^{\frac{q}{2}} \sum_{1 \leq i_1 < \dots < i_q \leq N} J_{i_1 \dots i_q} \psi_{i_1} \dots \psi_{i_q} \quad (10)$$

where the coupling constant is randomly chosen from a Gaussian centred at zero. As its dominant Feynman diagram is the melon diagram, which is identical to tensor models, SYK is effectively a tensor theory with the coupling constant as the tensor field. A schematic illustration is presented in Figure 2.

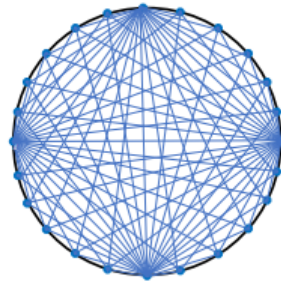


Figure 2. Illustration of the SYK model. The blue lines represent one-to-all interaction. In the graph, interactions at 4 points are specified, the rest are similar.

In the large N limit, the Schwinger-Dyson equation of correlation functions is exactly solvable,

$$G = (\partial_\tau - \Sigma)^{-1} \quad (11)$$

The thermodynamics of the model follows from the standard statistical mechanics, simplified by the application of the disorder average

$$\langle Z \rangle_J = \int DG D\Sigma \exp(-NI[G, \Sigma]) \quad (12)$$

In Eq. (12), N plays the role of $\hbar^{-1}$ in semi-classical expansion of quantum mechanics, and effective action is given by

$$I[G, \Sigma] = -\frac{1}{2} \ln |\det(\partial_\tau - \Sigma)| + \frac{1}{2} \iint d\tau d\tau' \left(\Sigma G - \frac{J^2}{q} G^q \right) \quad (13)$$

In the large N limit, the theory is completely classical.

This paper focused on the strong coupling limit $\beta J \gg 1$, where J is the root-mean-square of $J_{i_1 \dots i_q}^2$. It simplifies the integral form of Eq. (11) and grants it with a reparametrisation invariance valid in the infrared region $|\tau - \tau'| \ll J^{-1}$. Since it is identical to the two-point function of a CFT, the strong coupling limit is a “conformal limit” so that its solution has the same form as the CFT two-point function. The finite temperature two-point function can be obtained by parametrising the conformal solution with $\phi(\tau) = \tan(\pi\tau/\beta)$.

3.2. Thermal entropy

Since the action is known, the thermal entropy can be calculated similarly to section 2.2.1. Since it can be extracted from Helmholtz free energy, the first step is to calculate the partition function. The dominant contribution to the partition function originates from the saddle point configuration. Thus, the partition function simplifies from an integral to

$$\langle Z \rangle_J \sim \exp(-NI[G_*, \Sigma_*]) \quad (14)$$

where G_* and Σ_* are saddle point solutions. By standard thermodynamics, the free energy is approximately $NI[G_*, \Sigma_*]/\beta$. Thus, the calculation of entropy reduces to the calculation of G_* and Σ_* . The general strategy this section is mainly based on Ref. [16].

In the strong-coupling stochastic limit, one may take the conformal solution as an ansatz to Eq. (11)

$$G(\tau) = \frac{1}{2} \text{sgn}(\tau) \exp \left[\frac{g(\tau)}{q-1} \right] \quad (15)$$

$$\Sigma(\tau) = \frac{J^2}{2^{q-1}} \text{sgn}(\tau) \exp[g(\tau)].$$

The solution takes the form

$$\exp[g(\tau)] = \left\{ \frac{\cos\left(\frac{\pi\nu}{2}\right)}{\cos\left[\pi\nu\left(\frac{1}{2} - \frac{|\tau|}{\beta}\right)\right]} \right\}^2 \quad (16)$$

which is obtained by Fourier transform method, with ν as a coupling parameter $\beta J = \pi\nu/\cos(\pi\nu/2)$. This is formally similar to the two-point function of a finite-temperature CFT. Since any holographic dual of a finite-temperature CFT should be an AdS black hole, this solution indicates that the duality between the SYK model and JT theory is possible.

Substituting Eq. (16) into Eq. (15) and evaluating the action, one may obtain the free energy by Eq. (14), which yields

$$\frac{\beta F}{N} = -\frac{1}{2} \ln |2| - \frac{\pi\nu}{q^2} \left[\tan\left(\frac{\pi\nu}{2}\right) - \frac{\pi\nu}{4} \right] \quad (17)$$

In this limit, the parameter v expands to $1 - 2/\beta J + 4/\beta^2 J^2$. Taylor expanding Eq. (17) using this expression to the first order generates the large N entropy by direct differentiation, i.e.,

$$\frac{S_{therm}}{N} = \frac{1}{2} \ln|2| - \frac{\pi^2}{4q^2} + \frac{\pi^2}{q^2 \beta J}. \quad (18)$$

By similar methods, the ground-state entropy that is exact in q is

$$\frac{S_0}{N} = \frac{1}{2} \ln|2| - \pi \int_0^{\frac{1}{q}} dx \left(\frac{1}{2} - x \right) \tan(\pi x) \sim \frac{1}{2} \ln|2| - \frac{\pi^2}{4q^2} \quad (19)$$

which agrees with the first part of Eq. (18), verifying the validity of the limits taken and approximations made along the derivation.

The behaviour of entropy Eq. (18) is similar to that of an extremal black hole in Eq. (8). Firstly, both illustrate a residual entropy at zero temperature. In particular, the non-trivial ground state entropy originates from the chaotic nature of SYK. Secondly, Eq. (8) and Eq. (18) exhibit linear temperature dependence in the low-temperature regime. Based on the GKP-Witten relation, the holographic nature of thermal entropy and free energy in the SYK model and JT theory provides the second indication of their potential duality.

3.3. Low-energy dynamics and symmetry breaking: Schwarzian theory

In the conformal limit, the action Eq. (13) has reparametrisation invariance defined by

$$\begin{aligned} G(\tau, \tau') &\mapsto \phi'(\tau) \phi'(\tau')^{\frac{1}{q}} G(\phi(\tau), \phi(\tau')) \\ \Sigma(\tau, \tau') &\mapsto \phi'(\tau) \phi'(\tau')^{\frac{q-1}{q}} \Sigma(\phi(\tau), \phi(\tau')) \end{aligned} \quad (20)$$

Here, $\phi(\tau)$ is restricted to Möbius transformations, and the solution is form-invariant, indicating a spontaneous symmetry breaking to $SL_2(\mathbb{R})$. The symmetry breaking pattern in the SYK model is thus nearly identical to that of the JT theory, as discussed in section 2.2.1.

Consequently, the infrared-radiation dynamic of the SYK model is likely to be governed by a Schwarzian action, similar to the JT theory. This is the third indication that the SYK model is the holographic dual of 2D JT gravity. The author tried to refine this argument by constructing Schwarzian action for the SYK model. In terms of the dimensionless time $\theta = 2\pi\tau/\beta$, the second term of Eq. (13) is suppressed at least to the first order of $(\beta J)^{-1}$ so that it can be made parametrically small, diminishing its contribution.

Based on the derivation in Ref. [16], the Eq. (13) was modified by replacing the derivative, through which the reparametrisation of the saddle point propagator can be obtained. Expansion of the result in the conformal limit permits the explicit evaluation of the modified version of Eq. (13), reproducing the interesting result

$$I_{Sch} = \frac{\alpha N}{J} \int d\tau_1 Sch(\phi, \tau_1) \quad (21)$$

The free energy and thermal entropy are consistent with Eq. (17) and Eq. (18), indicating the validity of this approach. Therefore, the similarity between the symmetry-breaking pattern in JT theory and the SYK model could lead to similar low-energy dynamics indicated by the similarity between Eq. (6) and Eq. (21), completing the third sign of their correspondence.

4. Conclusion

This paper studies gauge-string duality in the context of the SYK model. It started by reviewing the development of the subject duality. Then, the author constructed the dynamics in nearly AdS_2 space and formulated the JT theory by considering its Euclidean cut-out version through which the free energy, thermal entropy, and infrared-radiation dynamics were calculated. In section 3, the author introduced

the SYK model and considered its stochastic generalisation. The paper calculated its propagator from which the action was derived. Then, its conformal limit was studied, where the same set of quantities were calculated. Based on the similarity in the symmetry breaking, the paper also formulated its infrared-radiation dynamics. The derivation led to three significant indications of the JT/SYK correspondence. Firstly, it is found that the conformal solution of the SYK model at finite temperature mirrors that of a CFT, a known holographic dual of black holes in higher dimensions. The fact that JT gravity describes extremal black holes suggests the possibility of the JT/SYK duality. Secondly, one observed a similarity in the low temperature behaviour of free energy and thermal entropy, a canonical signature of the AdS/CFT duality. Lastly, the author noted that the symmetry-breaking pattern and residual symmetry of the SYK model and JT gravity are identical, hinting at similarities in the IR dynamics. This led to construct a Schwarzian action similar to the JT theory. These findings are promising steps towards understanding the existence of the JT/SYK correspondence.

However, these are far from a complete verification of the correspondence. Firstly, the 4-point functions of both theories were not formulated in this work, preventing a complete comparison. Secondly, the duality is only approximately valid in the infrared-radiation regime. Thus, an imminent issue is the verification of ultraviolet behaviour, which has been ignored in most works. In summary, there are still various gaps to fill during the formulation of the duality. However, to the very least, JT/SYK is at least an approximate duality.

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