

Nonlinear active control of thin plate structures based on PVDF piezoelectric sheets

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Abstract. This paper studies how to actively control the nonlinear vibration of the plate using PVDF piezoelectric sheets as an actuator. By controlling the input voltage of the piezoelectric piece, the vibration displacement of the plate plane can be controlled. First, combining the structural geometric nonlinear theory and the isotropic assumption, the plate's large-amplitude vibration dynamics equation is obtained. Then, Galerkin decomposition is applied to the nonlinear dynamic equations to obtain the mode shape equations of the first four mode couplings. Next, negative velocity feedback control and fuzzy sliding mode control strategies are designed in terms of external disturbances in the plate. Finally, the feasibility of the research was supported through numerical simulation. The simulation results point out the fuzzy sliding mode controller and negative velocity feedback controller presented in this paper is effective; The fuzzy rule can adaptively regulate the sliding mode surface parameters to improve the time domain curve chattering phenomenon compared with classic sliding mode control.

Keywords: Thin plate, Nonlinear vibration, Modal coupling, Active control.

1. Introduction

In recent years, as flexible components in the aerospace field need to support many different working states, plate structures with the advantages of light mass and low cost have become a hot material in the aerospace field [1]. However, this structure has the characteristics of low damping, strong nonlinearity, small bending stiffness, and sensitivity to external loads, therefore, when subjected to external disturbance, the vibration decay time is long. Currently, techniques for nonlinear vibration control are primarily classified into two types: passive control and active control [18]. Passive control involves loading a damping device on the structure, such as installing a viscoelastic damper on a spacecraft [2] or applying a piezoelectric friction damper on a transmission tower model [3]. The controller passively generates control force as the structure vibrates. Traditional passive control is less successful in managing low-frequency vibrations compared to high-frequency conditions. [4]; It needs to add additional mass, that is, a damping device, which has great limitations in the aerospace field. In order to solve these problems, research on active control is needed. Compared to passive control, active control can track the error of control variable and adjust it in real time, and the control effect is mostly better [5].

Nowadays, scholars' research on active control of nonlinear vibration has become key points of current research [17]. Liu et al. used the nonlinear finite element method to obtain the nonlinear vibration

frequency of the structure and observed that the natural frequency of the structure will increase as the amplitude increases [6]. Meng et al. proposed a finite difference method to solve the vibration equation of the beam model under large deflection caused by external loads [7]. Compared with the geometrically accurate beam theory proposed by Hodge, the coupling relationship between different nonlinear terms can be revealed [8]. Sultan et al. analyzed the kinematic equations of a four-sided clamped rectangular membrane system and confirmed the controllability of the equations established [9]. Zhang et al. used PVDF actuators installed on the membrane and used H_∞ control to suppress the multi-mode vibration of the small-area membrane structure, effectively reducing the resonance peak [10].

This paper uses piezoelectric sheets made of PVDF to actively control the nonlinear vibration of a $1m \times 1m$ square thin plate. The control equation of the structure is approximated by the Galerkin decomposition method, and then dynamic equations of the nonlinear vibration of the thin plate is obtained; This paper designed a negative velocity feedback controller and a fuzzy sliding film controller, and proved the stability of the sliding mode controller through Lyapunov stability; Then this paper discussed the performance of negative velocity feedback control and no control, and the control effect under external disturbances is analyzed. The analysis results show that the use of fuzzy sliding mode control can effectively limit the vibration of the time domain curve.

2. Nonlinear Vibration Dynamics Equations of Plate Structures

This paper studies a $1m \times 1m$ square thin plate, as shown in **Fig. 1**. The plate plane is fixed by four-end clamping, so the pretensioners N_{0x} and N_{0y} are evenly spread perpendicular to the edges around the plate [13]. Since the piezoelectric sheet has the characteristics of small size, light weight, and fast response [11], it is used in this paper to suppress the vibration of the thin plate. Various parameters of thin plates and piezoelectric sheets can be obtained from [13].

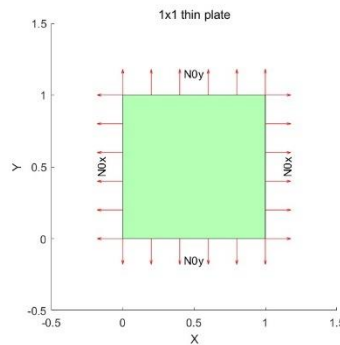


Figure 1. Diagram of film boundary pretension

Assuming that the thin plate is isotropic and neglecting the in-plane displacement of the structure, the dynamic equation of the large amplitude vibration of the thin plate can be obtained from [4]:

$$\begin{aligned} \rho h \frac{\partial^2 w}{\partial t^2} - h(\sigma_{x0} \frac{\partial^2 w}{\partial x^2} + \sigma_{y0} \frac{\partial^2 w}{\partial y^2} + 2\tau_{xy0} \frac{\partial^2 w}{\partial x \partial y}) - \frac{Eh}{2(1-\mu^2)} [(\frac{\partial w}{\partial x})^2 \frac{\partial^2 w}{\partial x^2} + (\frac{\partial w}{\partial y})^2 \frac{\partial^2 w}{\partial y^2}] \\ - \frac{Eh}{2(1-\mu^2)} [(\frac{\partial w}{\partial x})^2 \frac{\partial^2 w}{\partial y^2} + (\frac{\partial w}{\partial y})^2 \frac{\partial^2 w}{\partial x^2}] - \frac{Eh}{1+\mu} \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \frac{\partial^2 w}{\partial x \partial y} = l(x, y, t) \end{aligned} \quad (1)$$

Where h stands for thickness, m stands for length, n stands for width, σ_{x0} and σ_{y0} are the prestresses along x and y axes, τ_{xy0} is the pre-shear stress, $w(x, y, t)$ is deflection function along z axis, μ is Poisson's ratio, $l(x, y, t)$ is external load, and E is Young's modulus [4] [15].

Since the four ends of the plate plane are fixed, the vibration displacement and velocity on the plate boundary are both 0. According to [12], the vibration shape function is set as:

$$\varphi_{ij}(x, y) = \sin \frac{i\pi x}{m} \sin \frac{j\pi y}{n} \quad (2)$$

Then the deflection function w can be expressed as [13]:

$$w(x, y, t) = \sum_{i,j=1}^{\infty} \varphi_{ij}(x, y) q(t) \quad (3)$$

Where $q(t)$ represents the modal displacement, i, j are the modal orders [13]. This article considers the first four modes, *i.e.* ($i = 1, 2; j = 1, 2$), inserts Eq.(2) and Eq.(3) into Eq.(1) and uses Galerkin decomposition method, and obtains Eq.(4) according to [4] [13]:

$$\ddot{q}_{ij} + 2\zeta_0\omega_{ij}\dot{q}_{ij} + K_L q_{ij} + \sum_{i,j=1} \sum_{k,l=1} \sum_{s,t=1} a_{ijklst} q_{ij} q_{kl} q_{st} = L + \phi_{ij} \quad (4)$$

Where ζ_0 is the natural mode damping ratio, ω_{ij} is the nonlinear vibration frequency, K_L is the linear component of the structural frequency, a_{ijklst} is the nonlinear term coefficient. The nonlinear term is split into the coupling of different vibration mode displacements. L is the external load applied on the surface of the structure, and ϕ_{ij} is the control force exerted on the actuator [4]. This paper considers the first four modes and obtains the nonlinear coupled dynamic equations of the first four modes according to [4].

3. Control effect of fuzzy sliding mode controller and negative velocity feedback controller

3.1. Sliding Mode Controller

Due to the drastic changes in the controlled variable, traditional sliding mode control is difficult to eliminate chattering. However, this paper uses fuzzy sliding mode control to adaptively reduce chattering near the sliding mode surface by adjusting the gain of the sign function [14]. The control law in this paper is as shown in Eq.(5).

$$\begin{cases} s = ce + \dot{e} \\ e = q_d - q, \dot{e} = \dot{q}_d - \dot{q} \\ \dot{s} = -\varepsilon \operatorname{sgn}(s) - ks \end{cases} \quad (5)$$

Consider external disturbances ΔF_{ij} , where $|\Delta F_{ij}| \leq k_{fs} \in R^+$. Substitute Eq.(4) into Eq.(5) to obtain the control power ϕ_{ij} as:

$$\begin{aligned} \phi_{ij} = & \ddot{q}_{ija} + 2\zeta_0\omega_{ij}\dot{q}_{ija} + K_L q_{ij} + \sum_{i,j=1} \sum_{k,l=1} \sum_{s,t=1} a_{ijklst} q_{ij} q_{kl} q_{st} \\ & + c(\dot{q}_{ija} - \dot{q}_{ij}) + \varepsilon \operatorname{sgn}(s) + ks - L - \Delta F_{ij} \end{aligned} \quad (6)$$

Let Lyapunov function be set to $V = \frac{1}{2}s^2$,

$$\dot{V} = s\dot{s} = s(-\varepsilon \operatorname{sgn}(s) - ks - \Delta F_{ij}) \leq -\varepsilon|s| - ks^2 + k_{fs}|s|.$$

Make $\varepsilon > k_{fs}$, then $\dot{V} < 0$ is true, so the stability of fuzzy sliding mode controller is proven.

3.2. Fuzzy rule design

In order to reduce vibration displacement, the design control input $\Delta\varepsilon$ satisfies:

$$\begin{cases} \Delta\varepsilon > 0, & s\dot{s} > 0 \\ \Delta\varepsilon < 0, & s\dot{s} < 0 \end{cases} \quad (7)$$

Table 1. Fuzzy Rules

variables		fuzzy logic mapping rules [16]				
$s\dot{s}$	NBNM	NS	ZO	PS	PM	PB
$\Delta\varepsilon$	NBNM	NS	ZO	PS	PM	PB

This fuzzy rule takes $s\dot{s}$ as input and $\Delta\epsilon$ as output. The variable sizes include seven fuzzy sets. In the above table, “N” stands for negative, “P” stands for positive, “ZO” represents zero, “B” stands for big, “M” stands for medium, and “S” stands for small [16]. The input interval is $[-20,20]$, the output interval is $[-1.5,1.5]$. When $s\dot{s} > 0$, ϵ needs increases, that is $\Delta\epsilon > 0$, the system state is pulled towards the sliding surface, and vice versa.

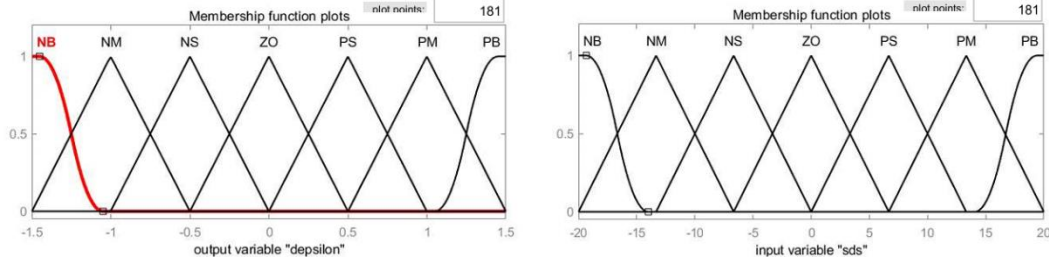


Figure 2. Membership Function

3.3. Negative velocity feedback control

According to [13], the use of negative velocity feedback can make the displacement converge quickly. The relationship between the control voltage and the control force is given below:

$$U_{ij}(t) = -G\dot{q}_{ij}(t) \quad (8)$$

$$\phi_{ij}^a(x, y, t) = M_{ij}^a(x, y)U_{ij}(t) \quad (9)$$

$$M_{ij}^a(x, y) = \frac{4E(h + h_p)}{\rho h m n} \left[d_{31} \left(\frac{i\pi}{m} \right)^2 + d_{32} \left(\frac{j\pi}{n} \right)^2 \right] \iint_{\Omega} \sin \left(\frac{i\pi x}{m} \right) \sin \left(\frac{j\pi y}{n} \right) dS \quad (10)$$

Where G is the gain coefficient, Ω is the surface of the thin plate, h_p is the thickness of the piezoelectric sheet, d_{31} , d_{32} are the piezoelectric coupling coefficients in x, y directions respectively. For safe operation of the actuator, the input voltage is controlled within $\pm 1000V$. The greater the G , the greater the control. To better achieve control effects, it is essential to optimize the position of $M^a(x, y)$.

3.4. Piezoelectric sheets position optimization

This paper uses four piezoelectric sheets to control the first four modes respectively. The size of each piezoelectric piece does not exceed $0.5m \times 0.5m$ and does not overlap with each other. It is noted that determining the size of $M_{ij}^a(x, y)$ is $F_{ij}(x, y) = \iint_{\Omega} \sin \left(\frac{i\pi x}{m} \right) \sin \left(\frac{j\pi y}{n} \right) dS$, so this article uses the particle swarm algorithm, the fitness function is set to $F_{ij}(x, y)$, and the position of the piezoelectric sheet is determined by finding the maximum value of the fitness function.

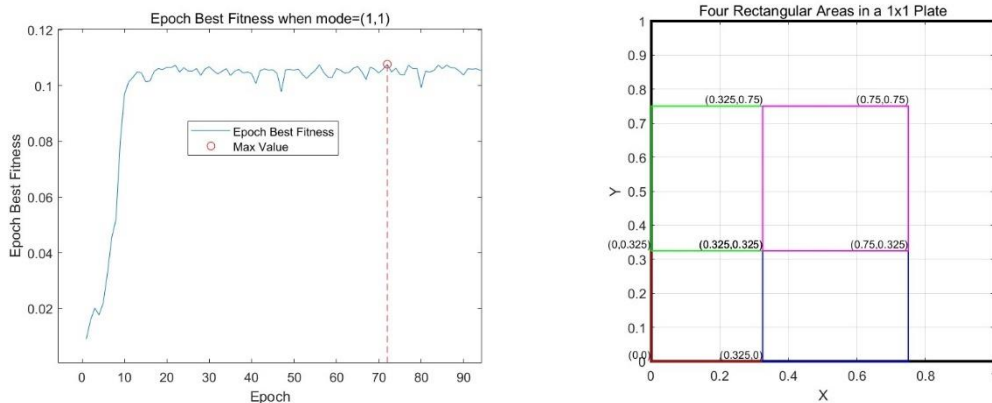


Figure 3. When mode=(1,1), best fitness per epoch(left); Position of sheets(right)

Position of four piezoelectric sheets is shown in **Fig.3** .

Table 2. Parametric Design of PSO

Parameter Name of PSO	Value
num_particles	50
num_narves	4
individual learning factors	2
social learning factor	2
inertia weight	0.9
num_iterations	100
maximum speed	0.1

3.5. Control effect comparison

Analyzing **Fig .4** and **Fig .5** , fuzzy sliding mode control has a faster response time than traditional sliding mode control, the adjustment time is slightly shorter than traditional controller, and the control voltage converges faster.

By observing the magnified **Fig .6**, there is obvious chattering in (2,2) mode of traditional smc. It can be concluded that fuzzy control can effectively suppress the chattering of the time domain curve, and the control effect can be improved by adaptively adjusting the sliding mode surface parameters.

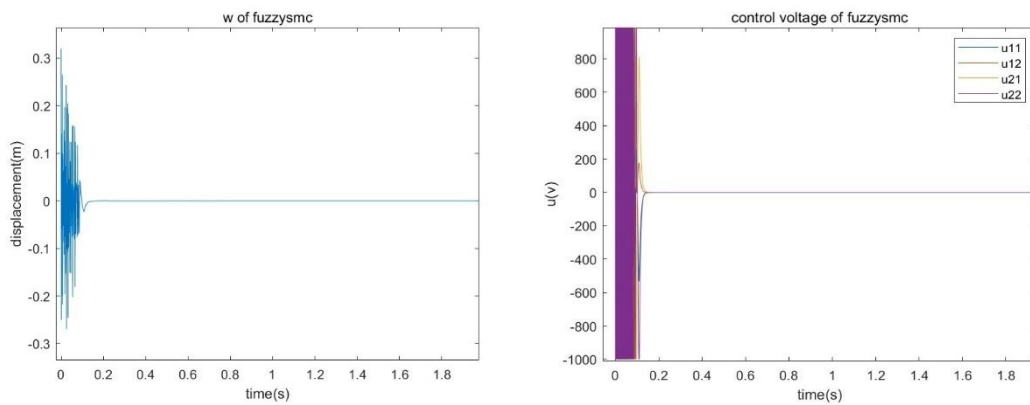


Figure 4. Displacement(left) and control voltage(right) curves of fuzzy sliding mode controller

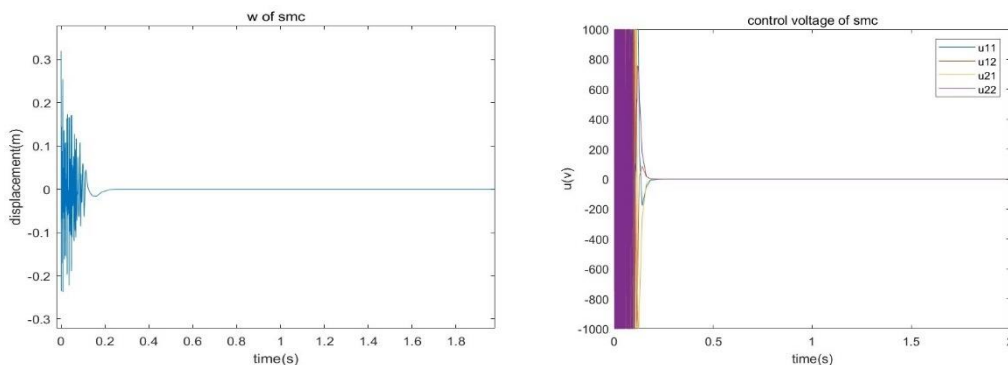


Figure 5. Displacement(left) and control voltage(right) curves of traditional sliding mode controller

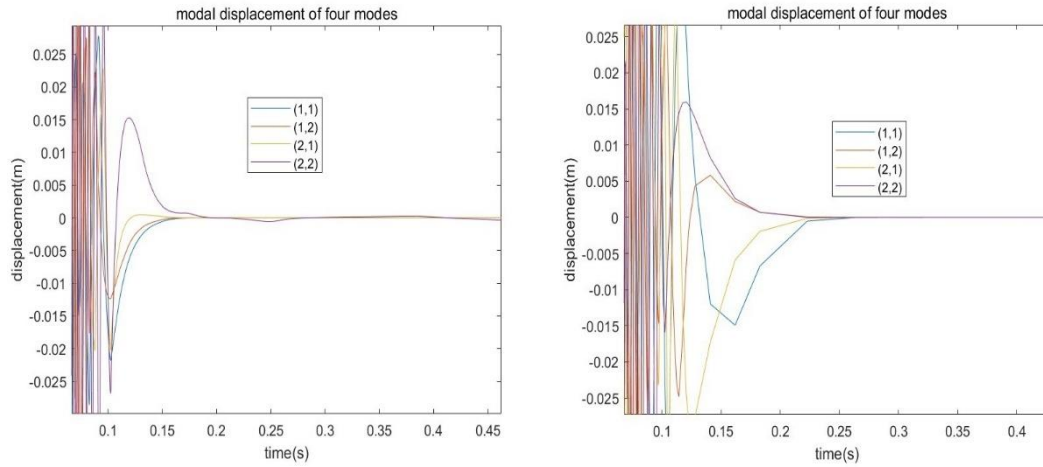


Figure 6. Displacement curves of each mode coupling (left: smc, right: fuzzysmc)

However, 'smcs' have a common problem. According to **Fig .4** and **Fig .5** , the control voltage reaches the voltage threshold multiple times, and the control voltage changes at high frequency before reaching the steady state. This high-frequency change and high-voltage characteristics have a greater impact on the performance of the actuator. Sacrificing the stability of the control voltage to suppress vibration displacement can easily cause damage to the actuator, requiring the piezoelectric sheet to have strong voltage input capability for support.

It can be conclude from **Fig .7** that the displacement curve of the uncontrolled system is difficult to converge to 0 under the influence of the inherent damping of the thin plate, thus illustrating the necessity of using an active control method; then, the adjustment time and response time of negative speed feedback controller(**nvfc**) are longer than those of sliding mode controller, But for control voltage, that of nvfc changes less frequently and does not reach the voltage threshold, which is beneficial to extending the service life of the actuator.

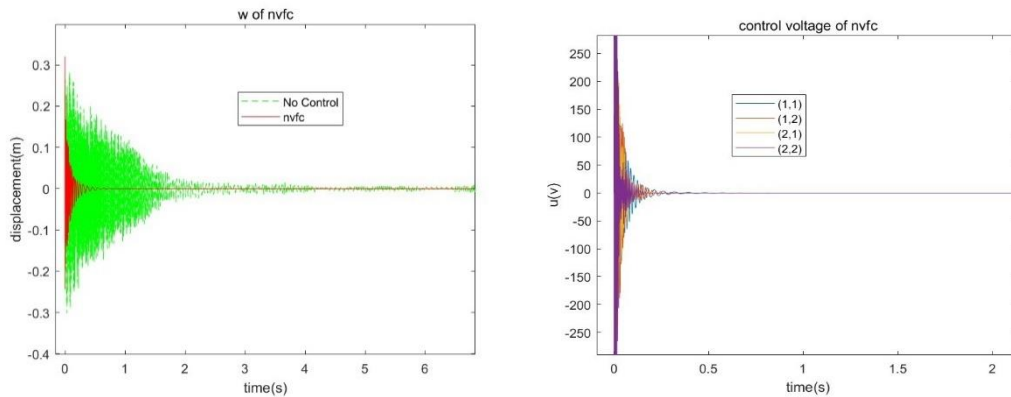


Figure 7. displacement of nvfc and No Control(left), control voltage of nvfc(right)

4. Conclusion

This paper conducts an active control simulation of nonlinear vibration of a thin plate, using three control methods: classic sliding mode control, fuzzy sliding mode control, and negative velocity feedback control, and compares it with an uncontrolled system, comparing pros and cons of each control method; One piezoelectric sheet controls one mode, and the optimal position of each piezoelectric sheet on the thin plate structure is determined using particle swarm algorithm. The main conclusions are summarized as follows: (1) Compared to traditional sliding mode control, fuzzy sliding mode control can effectively suppress the chattering of the time domain curve when external disturbances are applied, avoiding high-

frequency undesirable vibration against the sliding mode; (2) The adjustment time of negative velocity feedback is longer than that of sliding mode control, and the displacement curve converges slowly, but the control voltage changes more smoothly and does not exceed the voltage threshold, while the control voltage of the sliding mode controller has high-frequency changes. Both are suitable for control requirements in different scenarios; (3) Relying on the inherent modal damping of the thin plate structure, its vibration will be difficult to suppress, which illustrates the necessity and superiority of active control.

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