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Research on Vehicle Routing Optimization Based on VRPTW

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Abstract: Urban delivery, as an important link in the logistics chain, plays an increasingly prominent role in supporting urban manufacturing, consumption economy, and other sectors. This study establishes a mathematical model aimed at minimizing total cost, exploring the Vehicle Routing Problem with Time Windows (VRPTW) considering factors such as time windows, cargo capacity, and the number of vehicles. The model and algorithm's effectiveness is validated through simulation examples. The results show that the model and algorithm successfully solve the VRPTW problem, providing a feasible path optimization method for industry practitioners.

Keywords: Delivery optimization, Vehicle routing optimization, Supply and demand matching, Cost management

1. Introduction

1.1. Research Background and Significance

1.1.1. Research Background

Economic development and the refinement of social division of labor have accelerated the growth of the logistics industry. However, logistics costs have not seen significant reductions. Transportation costs account for nearly half of the total logistics costs, making them the primary expense in logistics activities. Therefore, to promote the sustainable development of the logistics industry, it is crucial to reduce the costs of logistics activities, particularly by focusing on reducing transportation expenses. Transportation costs are closely linked to time constraints as well as customer shopping experience and satisfaction levels [1].

1.1.2. Research Significance

The choice of delivery routes directly impacts transportation costs and efficiency. Optimizing delivery routes and algorithms is of great significance and value, as it can help supermarkets and retailers find more efficient and cost-effective transportation methods. At the same time, it can effectively reduce logistics costs, optimize transportation timeliness, improve vehicle utilization rates, solve the issue of underutilized logistics resources, promote enterprise development, and enhance competitiveness and industry strength.

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1.2. Research Methods and Content

1.2.1. Research Methods

The objective function is to minimize cost. Data is obtained using Baidu Maps, processed and integrated using Python, and model construction and solving are done using Gurobi.

1.2.2. Research Content

The research takes the Zhongbai Storage (Nanhu Shopping Plaza Store) in Wuhan as the transportation origin and focuses on transporting goods to several universities and high schools in Wuhan. The study investigates the optimal vehicle routing path under the constraints of vehicle limitations, customer time windows, and vehicle cargo capacity, aiming to minimize transportation costs.

1.3. Innovations and Limitations

1.3.1. Innovations

Building upon the traditional Traveling Salesman Problem (TSP), this study introduces constraints such as the number of vehicles, vehicle capacity, and customer time windows. The model is validated through simulation examples to explore its solution effectiveness and practical value in real-world vehicle routing problems.

1.3.2. Limitations

Many real-world factors have not been incorporated into the model, such as time penalty costs, fuel costs, product loss costs, and the standardization of products, vehicles, and customers. Furthermore, the use of only the Gurobi solver, without employing other algorithms, results in longer solving times and reduced solving efficiency when processing large sample data.

2. Description of the VRPTW Problem

Warehouse distribution requires goods to be delivered to multiple customers. In the era of rapid internet development, the increasing number of orders poses greater challenges to transportation costs. Based on ensuring product quality and timely delivery, the logistics company aims to integrate relevant vehicle resources to increase vehicle load rates, reduce resource wastage, and lower logistics costs. In the VRPTW model, the salesman not only needs to deliver goods but also must consider customer time windows (both start and end times), vehicle number limitations, vehicle cargo capacity constraints, and service times at customer nodes [2].

3. Mathematical Model Construction

3.1. Model Assumptions

Factors to be considered in the distribution process include time costs, vehicle number limits, vehicle capacity constraints, customer service time windows, and service times. To simplify the problem and establish a mathematical model, the following assumptions are made [3]:

(1) The locations of the enterprise distribution center, the maximum number of distribution vehicles, and the vehicle capacity are known.

(2) The locations of customers, their demand, time windows, and service times are known.

(3) The goods in the enterprise's distribution center can be coordinated centrally, with vehicles departing from the distribution yard and ultimately returning to the original yard.
(4) The number of vehicles dispatched from the distribution center yard cannot exceed the maximum number of vehicles available.
(5) Each vehicle can only be scheduled once per task and must not exceed its maximum load capacity.
(6) Each customer must be visited and can only be visited by one vehicle.
(7) Only delivery situations are considered, meaning that the flow of goods is one-way.
(8) The vehicles carry the same type of goods.

3.2. Model Parameters Explanation

In this paper, the graph is represented as G(V, A), with other relevant parameters explained in the table below:

Table 1: Description of the parameters

Parameter	Description
V	$V = \{0,1,\ldots,n,n+1\}$ is the set of all nodes in the graph
A	A is the set of all arcs in the graph
K	Vehicle collection (maximum number of delivery vehicles)
Q_k	The maximum load of the k car
c_{ij}	Unit transportation cost of arc$(i\ ,j\ )$
x^k_{ij}	Does the k car service arc$(i\ ,j\ )$
t_{ij}	Transit time of arc$(i\ ,j\ )$
$serv_i$	Service duration of customer point i
s_i	The time when customer point i starts to be served
M	Defined maximum
e_i	Customer i time window starting point
l_i	Customer i time window end
q_i	Customer node demand

3.3. Model Construction

3.3.1. Cost Analysis

The objective function of this mathematical model is to minimize transportation costs. The total cost is calculated as the sum of x^k_{ij} for all $\forall\ k\ \in\ K$, multiplied by the transportation cost c_{ij} for the corresponding route. The specific expression is shown in Equation (1):

$$min \sum_{k\in K} \sum_{(i,j)\in A} c_{ij}x^k_{ij} \tag{1}$$

3.3.2. Vehicle Flow Analysis

Under vehicle constraints, the analysis ensures that: Vehicles first depart from the depot. Vehicle flow remains balanced. Vehicles return to the depot. The specific expressions are shown in Equations (2), (3), and (4) [4]:

$$\sum_{(0,j)\in A} x^k_{0j} = 1, \forall k \in K \tag{2}$$

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$$\sum_{(i,j)\in A} x_{ij}^k = \sum_{(j,i)\in A} x_{ji}^k, \forall k \in K, \forall i,j \in V \setminus \{s,t\} \tag{3}$$

$$\sum_{(i,n+1)\in A} x_{i,n+1}^k = 1, \forall k \in K \tag{4}$$

3.3.3. Customer Service Analysis

To ensure that each customer node is served, at least one vehicle x_{ij}^k must arrive at the node. Additionally, the service start times for adjacent nodes must maintain a proper sequence (avoiding loops). The service time s_i must also fall within the customer's time window. The specific expressions are shown in Equations (5), (6), and (7):

$$\sum_{k\in K} \sum_{(i,j)\in A} x_{ij}^k = 1, \forall i,j \in V \setminus \{s,t\} \tag{5}$$

$$s_i + t_{ij} + serv_i - M(1 - x_{ij}^k) \le s_j, \forall (i,j) \in A, \forall k \in K \tag{6}$$

$$e_i \le s_i \le l_i, \forall i \in V\{s,t\} \tag{7}$$

3.3.4. Vehicle Load Analysis

The total load for any vehicle must not exceed its capacity. This is ensured by summing the product of x_{ij}^k and q_i for all paths, which must be less than or equal to the vehicle capacity. The specific expression is shown in Equation (8):

$$\sum_{(i,j)\in A} x_i{}^{kgi} \le Q_i, \forall k \in K \tag{8}$$

3.3.5. Decision Variable Constraints

$$x_{ij}^k \in \{0,1\}, \forall (i,j) \in A, \forall k \in K, s_i \ge 0 \forall i,j \in V \setminus \{s,t\} \tag{9}$$

4. Simulation Case Study

4.1. Data Preparation

The latitude and longitude coordinates of the transportation origin and customer nodes were obtained through the Baidu Map Web Service API. To acquire accurate travel times between nodes, batch route calculations were performed using Baidu Map's data scraping service, ensuring that the transportation time (cost) between nodes aligns with real-world conditions.

Transportation origin: #0 Zhongbai Supermarket Nanhu Shopping Plaza, Wuhan.

Customer nodes (educational institutions): #1 Central China Normal University, #2 Zhongnan University of Economics and Law (Shouyi Campus), #3 Wuhan Textile University (Yangguang Campus), #4 Wuhan College of Arts and Science, #5 Zhongnan University of Economics and Law (Nanhu Campus), #6 Huazhong Agricultural University, #7 Wuhan University of Bioengineering, #8 Hubei University (Yangluo Campus), #9 Huazhong University of Science and Technology (East Campus), #10 Wuhan University of Technology (Mafangshan Campus), #11 Wuhan University of Technology (Jianhu Campus), #12 Wuhan University (College of Arts and Sciences), #13 China University of Geosciences, #14 South-Central Minzu University, #15 Wuhan University of Science and Technology, #16 Jianghan University (Wuchang Campus), #17 Wuhan Sports University, #18 Middle School Attached to HUST, #19 Wuluo Road Experimental Junior High School, Wuhan, #20 Wuhan No.2 High School, #21 Wuhan Guangshan Middle School, #22 Wuhan Optical Valley No.2 Senior Middle School, #23 Wuhan Middle School, #24 Hubei Wuchang Experimental High School, #25 Wuhan Hongshan High School, #26 Hubei Shuiguohu High School.

The vehicle capacity was set to 80. The fleet size (K) was adjusted during the experimental tests to find the optimal solution. The demand at each node was standardized as product quantities. [5] The time window for nodes started from zero, measured in minutes (min), and all deliveries were conducted within the customer's specified time window.

4.2. Model Algorithm Testing and Results

The aforementioned data were processed using Python. Gurobi was employed to define the decision variables $x^k{}_{ij}$ and t_i^k, as well as variable characteristics and constraints. The objective function was set to minimize the total cost, with constraints including vehicle starting points, route flow balance, vehicle endpoints, service demand, vehicle capacity, and time windows. The optimization results were exported to Excel, and the folium package was used for data visualization.

4.2.1. Model Results

After multiple iterations to adjust the number of vehicles (K) for the lowest cost, the study found that the optimal solution (minimum cost) was 185. The vehicle routing scheme achieving the minimum cost involved the following two scenarios:

(1) The delivery plan utilized six vehicles in total, with detailed delivery data shown in Table 2:

Table 2: Six vehicle distribution scheme

from_node_id	to_node_id	Vehicle	T_i	T_j
0	24	V_0	0	25
4	23	V_0	699	779
7	8	V_0	169	236
8	4	V_0	236	699
20	7	V_0	92	169
23	0	V_0	779	0
24	20	V_0	25	92
0	3	V_1	0	25
1	0	V_1	953	0
3	5	V_1	25	94
5	14	V_1	94	541
14	22	V_1	541	776
21	1	V_1	888	953
22	21	V_1	776	888
0	17	V_2	0	75
9	0	V_2	499	0
13	18	V_2	139	202
17	13	V_2	75	139
18	9	V_2	202	499
0	15	V_3	0	362
10	12	V_3	424	617
12	26	V_3	617	679
15	10	V_3	362	424
26	0	V_3	679	0
0	25	V_4	0	142

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Table 2: (continued).

2	0	V_4	802	0
19	2	V_4	244	802
25	19	V_4	142	244
0	11	V_5	0	420
6	0	V_5	580	0
11	16	V_5	420	486
16	6	V_5	486	580

(2) This delivery plan utilized a total of seven vehicles for transportation, with detailed data provided in Table 3:

Table 3: Seven vehicle distribution scheme

from_node_id	to_node_id	Vehicle	T_i	T_j
0	17	V_0	0	91
9	0	V_0	640	0
13	18	V_0	155	291
17	13	V_0	91	155
18	9	V_0	291	640
0	15	V_1	0	362
11	0	V_1	512	0
15	11	V_1	362	512
0	3	V_2	0	25
1	0	V_2	994	0
3	5	V_2	25	120
5	14	V_2	120	541
14	22	V_2	541	782
21	1	V_2	929	994
22	21	V_2	782	929
0	25	V_3	0	155
2	0	V_3	893	0
19	2	V_3	266	893
25	19	V_3	155	266
0	24	V_4	0	25
4	23	V_4	699	800
7	8	V_4	169	236
8	4	V_4	236	699
20	7	V_4	92	169
23	0	V_4	800	0
24	20	V_4	25	92
0	16	V_7	0	555
6	0	V_7	712	0
16	6	V_7	555	712
0	10	V_8	0	331

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Table 3: (continued).

10	12	V_8	331	675
12	26	V_8	675	741
26	0	V_8	741	0

4.2.2. Analysis of Results

Although both delivery plans achieve the lowest total cost (185), the second plan requires one additional vehicle, incurring opportunity costs. Therefore, the first plan is the optimal choice for this simulation. The results of the optimized model operation are shown in Figure 1.

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Explored 5662 nodes (295182 simplex iterations) in 8.28 seconds (8.27 work units)
Thread count was 8 (of 8 available processors)

Solution count 1: 185

Optimal solution found (tolerance 1.00e-04)
Best objective 1.850000000000e+02, best bound 1.850000000000e+02, gap 0.0000%
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Figure 1: Running Results of the Optimized Model

5. Conclusion and Outlook

5.1. Research Conclusions

This study explores the optimization of logistics delivery routes in real-world scenarios by combining vehicle delivery with the VRPTW. A target optimization model was constructed with the objective of minimizing time costs, constrained by factors such as time windows, customer demand, service duration, and vehicle capacity. Using the VRPTW algorithm, along with Python and the Gurobi optimization solver, this study successfully solved the logistics delivery route optimization problem for the simulation example.

5.2. Managerial Recommendations

Modern logistics has expanded from heavy cargo delivery to include diverse, instant delivery services for daily necessities such as flowers and fruits. This trend imposes higher demands on vehicle delivery and merchant platforms. This study offers optimization suggestions and methods that can be applied to various scenarios such as express delivery, fresh food delivery, and meal delivery. It provides theoretical value and practical significance for operational improvements and management optimization in the logistics sector.

5.3. Research Limitations and Future Outlook

Although this study accounts for constraints such as the number of vehicles, vehicle capacity, customer time windows, and service duration, it still has limitations in sample size and model scope. Compared to real-world transportation conditions, it lacks several constraints, such as fixed vehicle costs, product loss costs, and time penalty costs. Moreover, it does not account for a larger number of customer nodes and transportation depots.

This research is based on a single depot and 26 customers, focusing on the application and exploration of the VRPTW model algorithm. Future studies could expand the data to include more depots, additional customers, and more complex and diverse demands. With increasing constraints and a more intricate logistics environment, higher requirements for algorithms may arise, potentially

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necessitating the adoption of genetic algorithms, variable neighborhood descent searches, or simulated annealing algorithms.

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