

Optimization of Enterprise Production Processes Based on Multi-Stage Decision-Making Models

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Abstract: This paper focuses on issues related to the inspection of spare parts, finished products, and the handling of defective items in enterprise production. Using methods such as sampling inspection and dynamic programming, a multi-stage mathematical model is developed to optimize production decisions. A statistical hypothesis testing scheme is designed for the sampling inspection of spare parts procurement, aiming to determine the optimal sample size. Dynamic programming models are introduced at each stage of the production process to explore the necessity of inspections and the impact of costs under six different scenarios, thereby formulating strategies. A multi-stage production decision model is constructed, treating production as a Markov decision process to solve for the optimal decisions. Considering fluctuations in the defect rate, confidence intervals are calculated using hypothesis testing, and Monte Carlo simulations are employed to generate the defect rate. The expected returns are then recalculated to increase enterprise profits.

Keywords: Multi-stage model, dynamic programming, Markov decision process

1. Problem Restatement

The quality of finished products in production is affected by the quality of spare parts, necessitating quality control at each stage and the proper handling of defective items. Optimizing inspection and treatment strategies to maximize profits is a key issue.

Problem 1: Suppliers provide nominal defect rates for spare parts. A sampling inspection scheme needs to be designed to allow the enterprise to determine whether to accept the parts with the fewest inspections. The decision should be made based on whether the defect rate exceeds the nominal value under 95% and 90% confidence levels, thereby deciding whether to accept or reject the parts.

Problem 2: Based on parameters such as the defect rate of spare parts, unit price, inspection costs, finished product assembly costs, defect rate, inspection costs, selling price, losses from rejecting defective items, and disassembly costs, an optimal decision-making plan needs to be formulated for each stage of production.

Problem 3: Given the defect rates of spare parts, semi-finished products, and finished products, the situation for m stages and n spare parts under general conditions, as well as for 2 stages and 8 spare parts under special conditions, is repeated from Problem 2. Specific solutions, reasoning, and criteria are provided for the special cases.

2. Problem Analysis

2.1. Analysis of Problem 1

The enterprise needs to procure spare parts and design a sampling inspection scheme to decide whether to accept the batch of spare parts from the supplier. This involves using hypothesis testing methods from mathematical statistics ^[2], and models such as the binomial distribution and normal distribution ^[1] can be considered. The goal is to determine the optimal sample size under specified confidence levels and test power requirements. The core challenge of this problem lies in how to minimize the number of inspections while satisfying the required confidence level, thereby reducing the enterprise's inspection costs.

2.2. Analysis of Problem 2

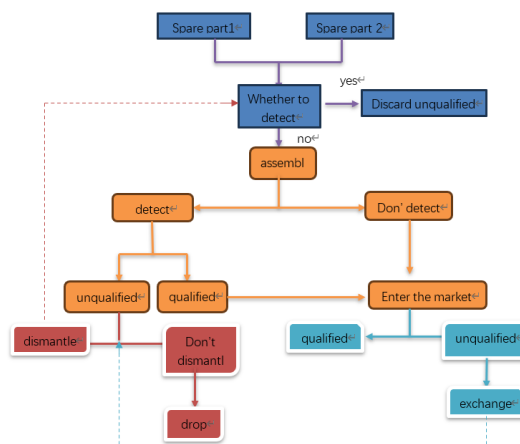


Figure 1: Flowchart of the decision-making process

The decisions faced at each stage of production include: whether to inspect spare parts (spare part 1 and/or spare part 2); if inspected, defective spare parts are discarded, and if not inspected, they proceed directly to the assembly stage. Whether to inspect the assembled finished products; if inspected, only qualified products enter the market, and if not inspected, all products enter the market. For defective finished products identified through inspection, a decision must be made whether to disassemble them; if disassembled, the process of inspecting the spare parts and finished products is repeated, and if not disassembled, the defective finished product is discarded. For defective products returned by customers after purchase, the enterprise will unconditionally replace them, incurring replacement losses, and repeat the process for defective finished products.

2.3. Analysis of Problem 3

In a production environment with multiple processes and spare parts, the enterprise must consider the interactions between processes, spare parts compatibility, quality control, and the costs of handling unexpected situations. A multi-stage production decision model can be developed, treating processes and spare parts as a multi-stage decision problem, each with its corresponding defect rate, inspection, and assembly costs. Since the state of each process affects the subsequent one, the production process decisions can be treated as a Markov decision process ^[3], with spare part inspection and assembly conditions defined as the state space, and corresponding decisions defined as the decision space. The optimal decisions can be found through the Bellman equation ^[5], aiming to maximize profit or minimize costs.

3. Model Assumptions and Symbol Representation

3.1. Basic Model Assumptions

It is assumed that the defect rates of spare part 1, spare part 2, and finished products in Problem 2 are constant and do not change with variations in the production process. The inspection cost, purchase unit price, assembly cost, replacement loss, and disassembly costs are all known and fixed values. Defective spare parts cannot be repaired and can only be discarded. Defective finished products can either be disassembled or discarded. The total replacement loss in the calculation process is the sum of the replacement loss (e.g., logistics costs) and the loss incurred from replacing defective products for customers (e.g., the production cost of the products).

3.2. Symbol Definitions

Symbol	Description
$1 - \alpha$	Confidence level
Z	Test statistic
x_{1i}, x_{2j}, d_{2j}	State variables for the first and second stages
R_p, R_h	Reutilization value of semi-finished product after being disassembled into parts, or of finished product after being disassembled into semi-finished products
$c_{bi}, c_{ti}, c_{gi}, c_{tj}$	Purchase and testing cost of part i Purchase and testing cost of semi-finished product j
p_i, p_{gj}	Defect rate of part i and semi-finished product j
p'_{gj}	Adjusted defect rate of semi-finished product j
c_{thi}	Testing cost for semi-finished product i in the h process
l	The sum of exchange losses and company losses resulting from replacing defective products

4. Model Construction and Solution

4.1. Model Construction and Solution for Problem 1

Under the assumption conditions, let the defect rate be p , the sample size be n , and the number of defective items detected be x . Then, we have:

$$\hat{p} = \frac{x}{n} \quad (1)$$

According to the binomial distribution, $x \sim \text{Binomial}(n, p)$, normal approximation can be used. Based on the Central Limit Theorem ^[4], when n is large enough, the binomial distribution can be approximated by a normal distribution. The standardized test statistic is:

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \quad (2)$$

A one-sided test model is constructed, and the null hypothesis (H_0): $p < p_0$ and alternative hypothesis (H_1): $p \geq p_0$ are established (for rejecting the acceptance condition).

Case 1: The null hypothesis is not satisfied, and the result falls into the rejection region, satisfying:

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \geq Z_{1-\alpha} \quad (3)$$

Expanding this inequality yields the following condition for n : $n \geq \frac{p_0(1-p_0)Z_{1-\alpha}^2}{\frac{\hat{p}-p_0}{(p-p_0)^2}}$. Let $\delta = \hat{p} - p$, we obtain: $n \geq \frac{p_0(1-p_0)Z_{1-\alpha}^2}{\delta^2}$.

Case 2: The null hypothesis is satisfied, and it does not fall into the rejection region, which satisfies:

$$Z = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \leq Z_{1-\alpha} \quad (4)$$

Expanding this gives the inequality for n : $n \leq \frac{p_0(1-p_0)Z_{1-\alpha}^2}{\frac{\hat{p}-p_0}{(p-p_0)^2}}$. Let $\delta = \hat{p} - p$, we obtain:

$$n \leq \frac{p_0(1-p_0)Z_{1-\alpha}^2}{\delta^2} \quad (5)$$

For case 1, at the 95% confidence level, using the critical value of the standard normal distribution, $Z_{0.95} = 1.645$, the sample size satisfies: $n \geq \frac{p_0(1-p_0)Z_{0.95}^2}{\delta^2}$. The minimum sample size obtained is 98. For case 2, at the 90% confidence level, using the critical value of the standard normal distribution, $Z_{0.90} = 1.28$, the sample size satisfies: $n \leq \frac{p_0(1-p_0)Z_{0.90}^2}{\delta^2}$. The maximum sample size obtained is 59.

4.2. Model Construction and Solution for Problem 2

Total profit = revenue - cost. Revenue primarily comes from the sales of finished products, while total costs include purchase cost, inspection cost, assembly cost, disassembly cost, and replacement loss. The calculation of revenue and costs takes into account the purchase quantity of parts, the cost of assembly, and whether the parts and finished products are inspected, as well as whether defective finished products are disassembled.

Total profit: $c_{zlr} = c_{zsy} - c_{zcb}$. The specific expressions for various revenues and costs are as follows:

(1) Total revenue c_{zsy} : $c_{zsy} = c_{bf} \cdot (1 - p_{cp} \cdot x_f) \cdot N$

The rate of qualified finished products assembled $p_{zp} = (1 - p_1 \cdot (1 - x_1)) \cdot (1 - p_2 \cdot (1 - x_2)) \cdot (1 - p_f \cdot (1 - x_f))$

The defect rate of all finished products assembled $p_{cp} = 1 - p_{zg}$

where p_1 , p_2 , and p_f represent the defect rates of parts 1, parts 2, and finished products, respectively; x_f is the inspection decision variable for finished products (1 indicates inspection, 0 indicates no inspection); and N is the number of assembled finished products.

(2) Total cost c_{zcb} : $c_{zcb} = c_{gm} + c_{jc} + c_{zp} + c_{cj} + c_{dh}$

c_{gm} , c_{jc} , c_{zp} , c_{cj} , c_{dh} represent the purchase cost, inspection cost, assembly cost, disassembly cost, and replacement cost, respectively.

(3) Purchase cost c_{gm} : $c_{gm} = c_{b_1} \cdot n_1 + c_{b_2} \cdot n_2$

Let n_1 and n_2 be the purchase quantities for parts 1 and 2, respectively, where $n_1 = \frac{N}{1-p_1 \cdot x_1}$, $n_2 = \frac{N}{1-p_2 \cdot x_2}$.

x_1 and x_2 are the inspection decision variables for parts 1 and 2 (1 indicates inspection, 0 indicates no inspection), and

c_{b_1} and c_{b_2} are the purchase unit prices for parts 1 and 2.

(4) Inspection cost c_{jc} : $c_{jc} = x_1 \cdot c_{t_1} \cdot n_1 + x_2 \cdot c_{t_2} \cdot n_2 + x_f \cdot c_f \cdot N$

x_1 , x_2 , and x_f are the inspection decision variables for parts 1, parts 2, and finished products, respectively (1 indicates inspection, 0 indicates no inspection); c_{t_1} , c_{t_2} , and c_{t_f} are the inspection costs for parts 1, parts 2, and finished products, respectively; c_{b_f} is the market selling price for finished products.

(5) Assembly cost c_{zp} : $c_{zp} = c_a \cdot N$, where c_a is the assembly cost for finished products.

(6) Disassembly cost c_{cj} : $c_{jc} = d_s \cdot c_s \cdot N \cdot p_{cp}$

d_s is the disassembly decision variable for finished products (1 indicates disassembly, 0 indicates no disassembly); c_s is the disassembly cost.

(7) Replacement cost c_{dh} : $c_{dh} = (1 - x_f) \cdot l \cdot N \cdot p_{cp}$, where l represents the replacement loss.

By traversing all possible strategy combinations and calculating the profit value for each combination, the strategy with the maximum profit is selected as the optimal solution. This problem involves four binary variables, so there are $2^4 = 16$ combinations. Python can be used to calculate the maximum profit and optimal decision for each row.

4.3. Model Construction and Solution for Problem 3

First, analyze the special case of 2 processes and 8 components, and then generalize it to the general case.

(1) Defining the Production Decision as Three Phases and State Definitions:

Phase 1: Component Procurement and Inspection Decision

State variable: $(x_{11}, x_{12}, x_{13} \dots x_{1n})$

$x_{1i} \in \{0,1\}$ represents whether component i is inspected (0 for no inspection, 1 for inspection), $i = 1, 2, \dots, 8$.

Phase 2: Semi-finished Product Inspection and Disassembly Decision

State variable: $(x_{21}, d_{21}, x_{22}, d_{22}, x_{23}, d_{23})$

Phase 3: Finished Product Inspection and Disassembly Decision

State variable: (x_3, d_3)

(2) Reuse Value After Disassembling Semi-finished and Finished Products:

Whether a semi-finished product is disassembled depends on its reuse value R_p , including the potential for reusing the components after disassembly, as well as the associated inspection costs, reassembly costs, and so on:

$$R_p = \sum_{i \in \text{Disassembled parts}} (c_{bi} - x_{1i} \cdot c_{ti}) \quad (6)$$

Whether a finished product is disassembled depends on its reuse value R_h , including the potential for reusing semi-finished products after disassembly, as well as the semi-finished products' procurement cost, inspection cost, reassembly cost, and so on:

$$R_h = \sum_{j \in \text{Disassembled semi-finished products}} (c_{gj} - x_{2j} \cdot c_{tj}) \quad (7)$$

Where, $c_{gj} = \sum_{i \in \text{Disassembled parts}} \frac{(c_{bi} - x_{1i} \cdot c_{ti})}{1 - p_i}$.

(3) Corrected Defect Rates for Semi-finished and Finished Products:

Defect rate of semi-finished products: $p'_{gj} = p_{gj} + \sum_{i \in \text{半成品}} (p_i(1 - s_{1i}))$

Defect rate of finished products: $p'_f = p_f + \sum_{j=1}^3 (p'_{gj}(1 - s_{2j}))$

(4) Applying the Dynamic Programming Model to Build the State Transition Equation:

We define the state variable $V(i, x_i)$, representing the optimal cost or benefit in state x_i at the i phase of production.

Phase 3:

$$V(3, x_3, d_3) = \begin{cases} (1 - p'_f)c_{bf} - p'_f \cdot (l + c_{bf}), x_3 = 0 \\ (1 - p'_f)c_{bf} - x_3 \cdot c_{tf} - p'_f \cdot x_3 \cdot d_3(c_s - R_h), x_3 = 1 \end{cases}$$

Phase 2:

$$V(2, x_{21}, d_{21}, x_{22}, d_{22}, x_{23}, d_{23}) = \sum_{j=1}^3 (1 - p'_{gj})V(3, x_3, d_3) - \sum_{j=1}^3 (x_{2j} \cdot c_{tj} + p'_{gj} \cdot x_{2j} \cdot d_{2j}(c_{sj} - R_p))$$

Phase 1:

$$V(1, x_{11}, x_{12}, \dots, x_{17}, x_{18}) = V(2, x_{21}, d_{21}, x_{22}, d_{22}, x_{23}, d_{23}) - \sum_{i=1}^8 (x_{1i} \cdot c_{ti} + c_{bi})$$

Thus, the generalization to the case of m processes and n components is as follows:

5. Conclusion

This paper constructs a multi-stage decision model to optimize component inspection, finished product inspection, and defective product handling in the production process. For component procurement, a hypothesis testing plan based on mathematical statistics was designed to determine the optimal sample size, minimizing the number of inspections and costs. In the various production stages, a dynamic programming model was introduced to analyze the necessity of inspections and the impact on costs under six different scenarios, resulting in the formulation of optimal decision plans. Furthermore, the production process was treated as a Markov decision process, and the optimal decision was solved using Bellman's equation to maximize profit or minimize costs. The model also accounts for fluctuations in defect rates, using hypothesis testing and Monte Carlo simulations to adjust defect rates and recalculate expected returns, thereby increasing enterprise profits. These models and strategies provide scientific decision-making support for enterprises, helping to improve production efficiency, quality control, reduce costs, and enhance market competitiveness. Practical application of the model has verified its effectiveness and practicality, offering significant theoretical and practical value for enterprise production management.

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